Questions from Homework

② c) $\log_{10} 2 + \log_{10} 5$

$\log_{10}(2\times 5)$

$\log_{10} 10$

1

d) $\log_{10}\sqrt{\frac{1}{10}}$

$\log_{10}\left(\frac{1}{10}\right)^{1/2}$

$\log_{10}\left(10^{-1}\right)^{1/2}$

$\log_{10}\left(10^{-1/2}\right)$

$-\frac{1}{2}$

③ d) $4\log_2 x - \frac{1}{3}\log_2(x^2+1) + \log_2(x-1)$

$\log_2 x^4 - \log_2(x^2+1)^{1/3} + \log_2(x-1)$

$\log_2\left[\frac{x^4(x-1)}{(x^2+1)^{1/3}}\right]$

$\log_2\left(\frac{x^4(x-1)}{\sqrt[3]{x^2+1}}\right)$

Do I really understand??...

a) Express the following as a single logarithm... $2\log_2 3^2 + \log_2 6 - 3\log_2 3$

b) Evaluate the following... $\log_2 (32)^{\frac{1}{3}}$

c) Express the following as a single logarithm... $\frac{1}{2}[(\log_5 a + 2\log_5 b) - 3\log_5 c]$

d) Express as a single logarithm in simplest form...

$$\frac{3}{4}\left[12\left(\log_b x^2 - 2\log_b x\right) + 8\log_b \sqrt{x} - 4\log_b \frac{1}{x^7}\right]$$

$2\log_2 3^2 + \log_2 6 - 3\log_2 3$

$2\log_2 9 + \log_2 6 - 3\log_2 3$

$\log_2 9^2 + \log_2 6 - \log_2 3^3$

$\log_2 81 + \log_2 6 - \log_2 27$

$\log_2 \left(\frac{81 \cdot 6}{27}\right)$

$\boxed{\log_2 18}$

Let $x = \log_2 (32)^{1/3}$

$x = \log_2 (32)^{\frac{1}{3}}$

$2^x = (32)^{1/3}$

$2^x = (2^5)^{1/3}$

$2^x = 2^{5/3}$

$\boxed{x = \frac{5}{3}}$

$\frac{1}{2}[(\log_5 a + 2\log_5 b) - 3\log_5 c]$

$\frac{1}{2}[\log_5 a + \log_5 b^2 - \log_5 c^3]$

$\frac{1}{2}\left[\log_5 \left(\frac{ab^2}{c^3}\right)\right]$

$\frac{1}{2}\log_5 \left(\frac{ab^2}{c^3}\right)$

$\log_5 \left(\frac{ab^2}{c^3}\right)^{1/2}$

$\log_5 \sqrt{\frac{ab^2}{c^3}}$ or $\boxed{\log_5 b\sqrt{\frac{a}{c^3}}}$

$\frac{3}{4}\left[12\left(\log_b x^2 - 2\log_b x\right) + 8\log_b \sqrt{x} - 4\log_b \frac{1}{x^7}\right]$

$\frac{3}{4}\left[12\log_b x^2 - 24\log_b x + 8\log_b x^{1/2} - 4\log_b x^{-7}\right]$

$9\log_b (x^2) - 18\log_b x + 6\log_b (x^{1/2}) - 3\log_b (x^{-7})$

$\log_b (x^2)^9 - \log_b x^{18} + \log_b (x^{1/2})^6 - \log_b (x^{-7})^3$

$\log_b x^{18} - \log_b x^{18} + \log_b x^3 - \log_b x^{-21}$

$\log_b \left(\frac{x^3}{x^{-21}}\right)$

$\boxed{\log_b x^{24}}$

Logarithmic and Exponential Equations

Focus on...

- solving a logarithmic equation and verifying the solution
- explaining why a value obtained in solving a logarithmic equation may be extraneous
- solving an exponential equation in which the bases are not powers of one another
- solving a problem that involves exponential growth or decay
- solving a problem that involves the application of exponential equations to loans, mortgages, and investments
- solving a problem by modelling a situation with an exponential or logarithmic equation

General Properties of Logarithms:

If $c > 0$ and $c \neq 1$, then...

(i) $\log_c 1 = 0$

(ii) $\log_c c^x = x$

(iii) $c^{\log_c x} = x$

Did You Know?

The input value for a logarithm is called an argument. For example, in the expression $\log_6 1$, the argument is 1.

Example 1

Solve Logarithmic Equations

Solve.

a) $\log_6 (2x - 1) = \log_6 11$ **b)** $\log (8x + 4) = 1 + \log (x + 1)$

c) $\log_2 (x + 3)^2 = 4$

a) $\log_6(2x-1) = \log_6 11$

$2x-1 = 11$

$2x = 12$

$x = 6$

Check $x=6$

$\log_6(2(6)-1)$ | $\log_6 11$

$\log_6 11$

$x=6$ is a solution

b) $\log(8x+4) = 1 + \log(x+1)$

$\log(8x+4) - \log(x+1) = 1$

$\log\left(\frac{8x+4}{x+1}\right) = 1$ logarithmic

$\frac{10^1}{1} = \frac{8x+4}{x+1}$ Exponential

$10(x+1) = 1(8x+4)$

$10x+10 = 8x+4$

$2x = -6$

$x = -3$

Check $x=-3$

$\log(8(-3)+4)$ | $1+\log(-3+1)$

$\log(-24+4)$

$\log(-20)$

undefined

$x=-3$ is an extraneous root (not a solution)

c) $\log_2(x+3)^2 = 4$ Logarithmic

$2^4 = (x+3)^2$ Exponential

$16 = (x+3)(x+3)$

$16 = x^2+3x+3x+9$

$16 = x^2+6x+9$

$0 = x^2+6x-7$ ← simple trinomial

$0 = (x-1)(x+7)$

$x-1=0$ | $x+7=0$

$x=1$ | $x=-7$

Both $x=1$ + $x=-7$ are solutions

Check $x=-7$

$\log_2(-7+3)^2$ | 4

$\log_2(-4)^2$

$\log_2 16$

4

Check $x=1$

$\log_2(1+3)^2$ | 4

$\log_2 16$

4

Example 2

Solve Exponential Equations Using Logarithms

Solve. Round your answers to two decimal places.

a) $4^x = 605$

b) $8(3^{2x}) = 568$

c) $4^{2x-1} = 3^{x+2}$

a) $4^x = 605$ — Take the common logarithm of both sides.

$\log 4^x = \log 605$

$x \log 4 = \log 605$

$x = \frac{\log 605}{\log 4}$

$x \approx 4.62$ is a solution

Check $x = 4.62$

4^x	605
$4^{4.62}$	
605	

b) $\frac{8(3^{2x})}{8} = \frac{568}{8}$

$3^{2x} = 71$ — Take the common logarithm of both sides.

$\log 3^{2x} = \log 71$

$\frac{2x \log 3}{2\log 3} = \frac{\log 71}{2 \log 3}$

$x \approx 1.94$ is a solution

Check $x = 1.94$

$8(3^{2(1.94)})$	568
$8(71)$	
568	

c) $4^{2x-1} = 3^{x+2}$ — Take the common logarithm of both sides.

$\log 4^{2x-1} = \log 3^{x+2}$

$(2x-1)\log 4 = (x+2)\log 3$

$2x\log 4 - \log 4 = x\log 3 + 2\log 3$ — Group like terms

$2x\log 4 - x\log 3 = 2\log 3 + \log 4$ — Common factor of x

$x(2\log 4 - \log 3) = 2\log 3 + \log 4$

$x = \frac{2\log 3 + \log 4}{2\log 4 - \log 3}$

$x \approx 2.1407$ is a solution

Check $x = 2.1407$

$4^{2(2.1407)-1}$	$3^{2.1407+2}$
$4^{3.2814}$	$3^{4.1407}$
94.5	94.5

Example 4

Solve a Problem Involving Exponential Growth and Decay

When an animal dies, the amount of radioactive carbon-14 (C-14) in its bones decreases. Archaeologists use this fact to determine the age of a fossil based on the amount of C-14 remaining.
The half-life of C-14 is 5730 years.

Buffalo skull display, Head-Smashed-In buffalo Jump Visitor Centre, near Fort McLeod, Alberta

Head-Smashed-In Buffalo Jump in southwestern Alberta is recognized as the best example of a buffalo jump in North America. The oldest bones unearthed at the site had 49.5% of the C-14 left. How old were the bones when they were found?

Given:

Base = (0.5)

Exp. = $\frac{t}{5730}$

Initial Amount = A_i

Final Amount = $0.495A_i$

$$\frac{0.495 A_i}{A_i} = \frac{A_i (0.5)^{\frac{t}{5730}}}{A_i}$$

$$0.495 = (0.5)^{\frac{t}{5730}}$$

$$\log 0.495 = \log (0.5)^{\frac{t}{5730}}$$

$$\log 0.495 = \frac{t}{5730} \log 0.5$$

$$\frac{5730 \log 0.495}{\log 0.5} = \frac{t \log 0.5}{\log 0.5}$$

5813 years = t

Key Ideas

- When solving a logarithmic equation algebraically, start by applying the laws of logarithms to express one side or both sides of the equation as a single logarithm.
- Some useful properties are listed below, where $c, L, R > 0$ and $c \neq 1$.
  - If $\log_c L = \log_c R$, then $L = R$.
  - The equation $\log_c L = R$ can be written with logarithms on both sides of the equation as $\log_c L = \log_c c^R$.
  - The equation $\log_c L = R$ can be written in exponential form as $L = c^R$.
  - The logarithm of zero or a negative number is undefined. To identify whether a root is extraneous, substitute the root into the original equation and check whether all of the logarithms are defined.
- You can solve an exponential equation algebraically by taking logarithms of both sides of the equation. If $L = R$, then $\log_c L = \log_c R$, where $c, L, R > 0$ and $c \neq 1$. Then, apply the power law for logarithms to solve for an unknown.
- You can solve an exponential equation or a logarithmic equation using graphical methods.
- Many real-world situations can be modelled with an exponential or a logarithmic equation. A general model for many problems involving exponential growth or decay is

$$\text{final quantity} = \text{initial quantity} \times (\text{change factor})^{\text{number of changes}}$$

Key Ideas

- Let P be any real number, and M, N, and c be positive real numbers with $c \neq 1$. Then, the following laws of logarithms are valid.

Name	Law	Description
Product	$\log_c MN = \log_c M + \log_c N$	The logarithm of a product of numbers is the sum of the logarithms of the numbers.
Quotient	$\log_c \frac{M}{N} = \log_c M - \log_c N$	The logarithm of a quotient of numbers is the difference of the logarithms of the dividend and divisor.
Power	$\log_c M^P = P \log_c M$	The logarithm of a power of a number is the exponent times the logarithm of the number.

- Many quantities in science are measured using a logarithmic scale. Two commonly used logarithmic scales are the decibel scale and the pH scale.

Homework

page 412 #1, 2, 4, 5, 7, 8, 11, 15

8.4 Logarithmic and Exponential Equations, pages 412 to 415

1. a) 1000 b) 14 c) 3 d) 108
2. a) 1.61 b) 10.38 c) 4.13 d) 0.94
3. No, since $\log_3 (x - 8)$ and $\log_3 (x - 6)$ are not defined when $x = 5$.
4. a) $x = 0$ is extraneous.
   b) Both roots are extraneous.
   c) $x = -6$ is extraneous.
   d) $x = 1$ is extraneous.
5. a) $x = 8$ b) $x = 25$ c) $x = 96$ d) $x = 9$
6. a) Rubina subtracted the contents of the log when she should have divided them. The solution should be
   $$\log_6 \left(\frac{2x + 1}{x - 1}\right) = \log_6 5$$
   $$2x + 1 = 5(x - 1)$$
   $$1 + 5 = 5x - 2x$$
   $$6 = 3x$$
   $$x = 2$$
   b) Ahmed incorrectly concluded that there was no solution. The solution is $x = 0$.
   c) Jennifer incorrectly eliminated the log in the third line. The solution, from the third line on, should be
   $$x(x + 2) = 2^3$$
   $$x^2 + 2x - 8 = 0$$
   $$(x - 2)(x + 4) = 0$$
   So, $x = 2$ or $x = -4$.
   Since $x > 0$, the solution is $x = 2$.
7. a) 0.65 b) −0.43 c) 81.37 d) 4.85
8. a) no solution ($x = -3$ not possible)
   b) $x = 10$ c) $x = 4$ d) $x = 2$ e) $x = -8, 4$
9. a) about 2.64 pc b) about 8.61 light years
10. 64 kg
11. a) 10 000 b) 3.5%
    c) approximately 20.1 years
12. a) 248 Earth years b) 228 million kilometres
13. a) 2 years b) 44 days c) 20.5 years
14. 30 years
15. approximately 9550 years
16. 8 days
17. 34.0 m
18. $x = 4.5$, $y = 0.5$
19. a) The first line is not true.
    b) To go from line 4 to line 5, you are dividing by a negative quantity, so the inequality sign must change direction.
20. a) $x = 100$ b) $x = \frac{1}{100}, 100$ c) $x = 1, 100$
21. a) $x = 16$ b) $x = 9$
22. $x = -5, 2, 4$