

Questions from Homework

Remember!

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\textcircled{8} \quad f(x) = \frac{4x^2}{3x+2}$$

$$f(x+h) = \frac{4(x+h)^2}{3(x+h)+2}$$

$$f(x+h) = \frac{4(x^2+2xh+h^2)}{3x+3h+2}$$

$$f(x+h) = \frac{4x^2+8xh+4h^2}{3x+3h+2}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\frac{4x^2+8xh+4h^2}{3x+3h+2} - \frac{4x^2}{3x+2}}{h}$$

Multiply everything by $(3x+2)(3x+3h+2)$

$$= \lim_{h \rightarrow 0} \frac{(3x+2)(4x^2+8xh+4h^2) - 4x^2(3x+3h+2)}{h(3x+2)(3x+3h+2)}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{12x^3} + \cancel{24x^2h} + \cancel{12xh^2} + \cancel{8x^2} + \cancel{16xh} + \cancel{8h^2} - \cancel{12x^3} - \cancel{12x^2h} - \cancel{8x^2}}{h(3x+2)(3x+3h+2)}$$

$$= \lim_{h \rightarrow 0} \frac{12x^2h + 12xh^2 + 16xh + 8h^2}{h(3x+2)(3x+3h+2)}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{h}(12x^2 + \cancel{12xh} + 16x + 8h)}{\cancel{h}(3x+2)(3x+3h+2)} = \frac{12x^2 + 16x}{(3x+2)^2}$$

↑
Slope of the
tangent

Remember!

If $f(x) = x^2 + 7x$, find $f'(3)$

Hint: find the derivative first then substitute 3 into that

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x) = x^2 + 7x$$

$$f(x+h) = (x+h)^2 + 7(x+h)$$

$$f(x+h) = x^2 + 2xh + h^2 + 7x + 7h$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{7x} + 7h - (\cancel{x^2} + \cancel{7x})}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{2xh + h^2 + 7h}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\cancel{h}(2x + \cancel{h} + 7)}{\cancel{h}}$$

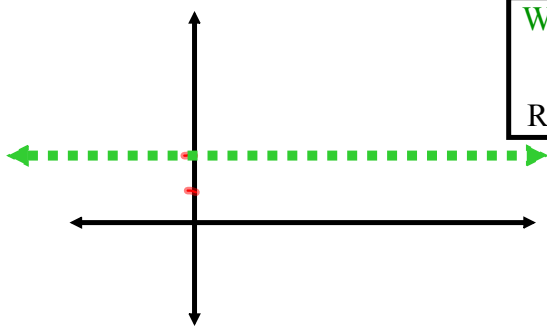
$$f'(x) = 2x + 7 \rightarrow \text{slope of the tangent}$$

$$\begin{aligned} f'(3) &= 2(3) + 7 \\ &= 6 + 7 \\ &= 13 \end{aligned}$$

Differentiation Rules

I. Constant Functions

- Sketch the function $y = 2$



What is the slope of the tangent to this graph?

Recall: slope of the tangent is the derivative

The derivative of a constant will always be equal to "0".

$$f(x) = 6$$

$$f'(x) = 0$$

$$f(x) = 3x$$

$$f'(x) = 0$$

$$f(x) = 6x + 5$$

$$f'(x) = 6 + 0$$

$$f'(x) = 6$$

Formal Proof:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{c - c}{h}$$

$$= \lim_{h \rightarrow 0} 0 = 0$$

II. Power Functions

We want to come up with a rule to differentiate functions of the form $f(x) = x^n$, $x \in \mathbb{R}$

Using the definition of a derivative to differentiate $f(x) = x^4$ ($f(x+h) = (x+h)^4$) would lead to ...

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{(x+h)^4 - x^4}{h} \\ &= \lim_{h \rightarrow 0} \frac{x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 - x^4}{h} \\ &= \lim_{h \rightarrow 0} \frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4}{h} \\ &= \lim_{h \rightarrow 0} (4x^3 + 6x^2h + 4xh^2 + h^3) = 4x^3 \end{aligned}$$

Other examples we have looked at so far

$$f(x) = x^2$$

$$f'(x) = 2x$$



$$f(x) = x^3$$

$$f'(x) = 3x^2$$

Do you see a pattern emerging?

The Power Rule (General Version) If n is any real number, then

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

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Let's practice using the power rule...

Differentiate each of the following functions:

1. $f(x) = x^{25}$

$$f'(x) = 25x^{24}$$

2. $f(x) = x^{-5}$

$$f'(x) = -5x^{-6}$$

$$f'(x) = \frac{-5}{x^6}$$

3. $f(x) = \frac{1}{x^{10}}$

$$f(x) = x^{-10}$$

$$f'(x) = -10x^{-11}$$

4. $f(x) = \sqrt{x}$

$$f(x) = x^{1/2}$$

$$f'(x) = \frac{1}{2}x^{-1/2}$$

$$f'(x) = \frac{1}{2x^{1/2}}$$

$$f'(x) = \frac{1}{2\sqrt{x}}$$

Constant Multiples

- The following formula says that the derivative of a constant multiplied by a function is the constant multiplied by the derivative of the function:

The Constant Multiple Rule If c is a constant and f is a differentiable function, then

$$\frac{d}{dx} [cf(x)] = c \frac{d}{dx} f(x)$$

EXAMPLE 4

(a) $\frac{d}{dx} (3x^4) = 3 \frac{d}{dx} (x^4) = 3(4x^3) = 12x^3$

(b) $\frac{d}{dx} (-x) = \frac{d}{dx} [(-1)x] = (-1) \frac{d}{dx} (x) = -1(1) = -1$

Examples:

1. $f(x) = 4x^3$

$$f'(x) = 12x^2$$

2. $f(x) = \frac{8}{x^2}$

$$f(x) = 8x^{-2}$$

$$f'(x) = -16x^{-3}$$

$$f'(x) = \frac{-16}{x^3}$$

3. $f(x) = 5x^{\frac{6}{5}}$

$$f'(x) = 6x^{\frac{1}{5}}$$

4. $f(x) = (3x^2)^2$

$$f(x) = 9x^4$$

$$f'(x) = 36x^3$$

⑤ $f(x) = 12x$

$$\begin{aligned} f'(x) &= 12x^0 \\ &= 12 \end{aligned}$$

Recall the derivative of a function is equal to the slope of a line that is tangent to the function.

Find the slope of the tangent line to the function at the given "x" coordinate!

$$f(x) = 3x^2 \quad \text{at } x = 4$$

$$f'(x) = 6x$$

$$f'(4) = 6(4)$$
$$= 24$$

$$\textcircled{3} \text{ a) } f(x) = 2x^3, \quad x = \boxed{\frac{1}{3}}$$

$$f'(x) = 6x^2$$

$$f'\left(\frac{1}{3}\right) = 6\left(\frac{1}{3}\right)^2$$

$$= 6\left(\frac{1}{9}\right)$$

$$= \frac{6}{9}$$

$$= \frac{2}{3}$$

Homework

#4 To find an equation of a line:

① Slope

② Point

③ $y - y_1 = m(x - x_1)$

a) $f(x) = x^5$ $(\underline{2}, 32)$

① $f'(x) = 5x^4$

② $f'(2) = 5(2)^4$
 $= 5(16)$
 $= 80$

③ $y - y_1 = m(x - x_1)$

$$y - 32 = 80(x - 2)$$

$$y - 32 = 80x - 160$$

$$-80x + y + 128 = 0$$

$$\boxed{80x - y - 128 = 0}$$