Questions from Homework

$y-y_1=m(x-x_1)$

③ Find the equation of the tangent line to the curve at the given point:

b) $y=7\sqrt{x}-3x$, $(\underset{x_1}{1},\underset{y_1}{4})$

① Differentiate

$y=7x^{1/2}-3x$

$y'=\frac{7}{2}x^{-1/2}-3$

$y'=\frac{7}{2\sqrt{x}}-3$

② sub x=1

$y'(1)=\frac{7}{2\sqrt{1}}-3$

$y'(1)=\frac{7}{2}-\frac{6}{2}=\frac{1}{2}$

m

③ Find Equation:

$y-y_1=m(x-x_1)$

$y-4=\frac{1}{2}(x-1)$

$2\cdot y-2\cdot 4=2\cdot\frac{1}{2}x-2\cdot\frac{1}{2}$

$2y-8=x-1$

$\boxed{0=x-2y+7}$

(x,y)

⑧ Find the point on the curve $y=x\sqrt{x}$ where the tangent line is parallel to $6x-y=4$
(same slope)

① Find slope of $6x-y=4$

$-y=-6x+4$

$y=6x-4$

(slope) m=6

② Differentiate

$y=x\sqrt{x}=x(x^{1/2})=x^{3/2}$

$y'=\frac{3}{2}x^{1/2}=\frac{3\sqrt{x}}{2}$

③ Solve for x

$\frac{3\sqrt{x}}{2}=6$

$3\sqrt{x}=12$

$\sqrt{x}=4$

$\boxed{x=16}$

④ Solve for y

$y=x\sqrt{x}$

$y=(16)\sqrt{16}=64$

∴ The point is (16, 64)

Questions from Homework

$y - y_1 = m(x - x_1)$

③ Find the equation of the tangent line to the curve at the given point:

c) $y = x + \frac{6}{x}$, $(2, 5)$

x_1 , y_1

① Differentiate:

$y = x + 6x^{-1}$

$y' = 1 - 6x^{-2} = 1 - \frac{6}{x^2}$

② Sub $x = 2$

$y'(2) = 1 - \frac{6}{(2)^2}$

$y'(2) = 1 - \frac{3}{2}$

$y'(2) = \frac{2}{2} - \frac{3}{2} = -\frac{1}{2}$

m

③ Find equation

$y - 5 = -\frac{1}{2}(x - 2)$

$2 \quad y - 5 = -\frac{1}{2}x + 1$

$2y - 10 = -x + 2$

$\boxed{x + 2y - 12 = 0}$

⑥ b) $y = 2x^2 - 6\sqrt{x}$ at $(4, 20)$

x_1 y_1

① Differentiate:

$y = 2x^2 - 6x^{1/2}$

$y' = 4x - 3x^{-1/2}$

$y' = 4x - \frac{3}{x^{1/2}}$

$y' = 4x - \frac{3}{\sqrt{x}}$

② Sub in x-value

$y' = 4(4) - \frac{3}{\sqrt{(4)}}$

$y' = 16 - \frac{3}{2}$

$y' = \frac{32}{2} - \frac{3}{2} = \boxed{\frac{29}{2}}$

Slope of tangent "m"

③ Find equation:

$y - y_1 = m(x - x_1)$

$y - 20 = \frac{29}{2}(x - 4)$

$y - 20 = \frac{29x}{2} - \frac{116}{2}$

$2 \cdot\ y - 20 = \frac{29x}{2} - 58$

$2y - 40 = 29x - 116$

$\boxed{0 = 29x - 2y - 76}$

⑦ c) $g(x) = 4x^3 - \frac{6}{x^2} + 14x$

$g(x) = 4x^3 - 6x^{-2} + 14x$

$g'(x) = 12x^2 + 12x^{-3} + 14x^0$

$g'(x) = 12x^2 + \frac{12}{x^3} + 14$

Warm Up

Differentiate the following:

$$f(x) = -4x^2 - 5x\left(x^3+7\right)^2 + 2\sqrt[5]{x^9} - \frac{5}{x^{10}} + \frac{7x^2}{\sqrt{x}}.$$

$$f(x) = -4x^2 - 5x(x^6 + 14x^3 + 49) + 2x^{9/5} - 5x^{-10} + 7x^2(x^{-1/2})$$

$$f(x) = -4x^2 - 5x^7 - 70x^4 - 245x + 2x^{9/5} - 5x^{-10} + 7x^{3/2}$$

$2 + (-\frac{1}{2})$

$$f'(x) = -8x - 35x^6 - 280x^3 - 245 + \frac{18}{5}x^{4/5} + 50x^{-11} + \frac{21}{2}x^{1/2}$$

Differentiation Rules

Product Rule:

The Product Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

$$[f(x)g(x)]' = f(x)g'(x) + f'(x)g(x)$$

Express the product rule verbally if you are considering a function of the form...

$f(x)$ = (First) X (Second)

In words, ***the Product Rule*** says that the ***derivative of a product of two functions is:*** ***the first function times the derivative of the second function, plus the derivative of the first function times the second function***

$$(fg)' = fg' + f'g$$

Get in the habit of verbalizing the rule as you differentiate...it will help when the functions get more complicated.

Examples:

$y' = f'g + fg'$

$y = \boxed{(2x^3+5)}\boxed{(3x^2-x)}$

$f(x) = 2x^3+5$

$f'(x) = 6x^2$

$g(x) = 3x^2 - x$

$g'(x) = 6x - 1$

$y' = 6x^2(3x^2-x) + (2x^3+5)(6x-1)$

$y' = 18x^4 - 6x^3 + 12x^4 - 2x^3 + 30x - 5$

$\boxed{y' = 30x^4 - 8x^3 + 30x - 5}$

$f(x) = \sqrt{x}(2-3x)$

$f(x) = \sqrt{x} = x^{1/2}$

$f'(x) = \frac{1}{2}x^{-1/2}$

$g(x) = 2-3x$

$g'(x) = -3$

$y' = \frac{1}{2}x^{-1/2}(2-3x) + (x^{1/2})(-3)$

$2 \quad y' = x^{-1/2} - \frac{3}{2}x^{1/2} - 3x^{1/2}$

$2y' = 2x^{-1/2} - 3x^{1/2} - 6x^{1/2}$

$2y' = 2x^{-1/2} - 9x^{1/2}$

$2y' = \frac{2}{x^{1/2}} - \frac{9x^{1/2}}{1}$

$2y' = \frac{2}{x^{1/2}} - \frac{9x}{x^{1/2}}$

$\left(\frac{1}{2}\right) 2y' = \frac{2-9x}{x^{1/2}} \left(\frac{1}{2}\right)$

$\boxed{y' = \frac{2-9x}{2\sqrt{x}}}$

Examples: $(fg)' = f'g + fg'$

$$f(x) = (7x^3 - x^2 + 5)(x^9 + 3x - 5)$$

$$f'(x) = (21x^2 - 2x)(x^9 + 3x - 5) + (7x^3 - x^2 + 5)(9x^8 + 3)$$

$$h(t) = (t^3 - 5t)(6\sqrt{t} - t^{-5})$$

Homework