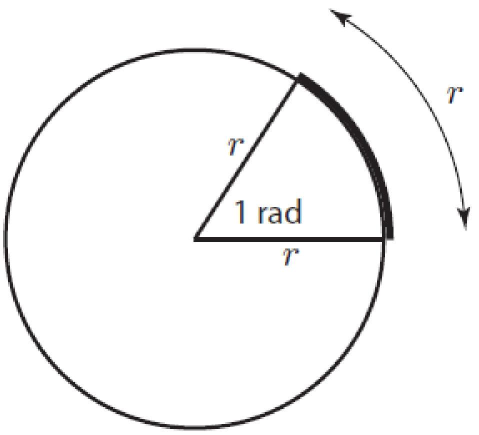
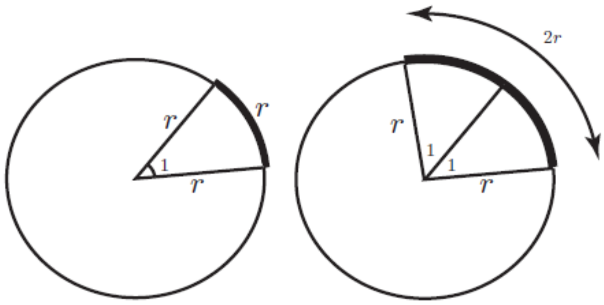
Radian Measure

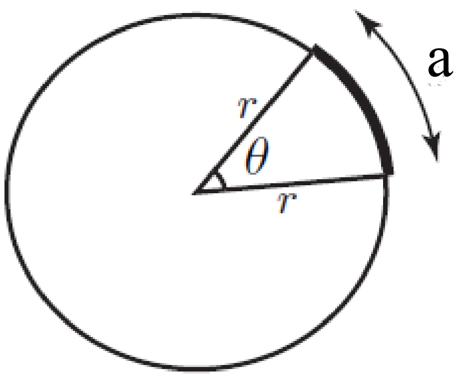
A radian is the angle subtended by an arc of length r (radius)





An angle of 2 rad subtends an arc of 2r...
7 rads subtends an arc of??? 7r

Use the above information to develop a formula to connect arc length, radius and the measure of an angle in radian measure...



has to be in radians

$$a = \theta r$$

Check-Up...

Arrange the following angles in descending order:

$$340^\circ \qquad 4.28\,rad \qquad \frac{9\pi}{5} \qquad (10\pi)^\circ$$

Find the angles co-terminal to θ on the given domain

$$\theta = \frac{5\pi}{6}\ ,\ -2\pi \le \theta \le 8\pi$$

Homework

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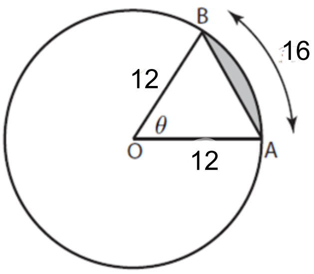
Find the area of the shaded region



Questions from Homework

14. A rotating water sprinkler makes one revolution every 15 s. The water reaches a distance of 5 m from the sprinkler.

a) What is the arc length of the sector watered when the sprinkler rotates through $\frac{5\pi}{3}$? Give your answer as both an exact value and an approximate measure, to the nearest hundredth.

b) Show how you could find the area of the sector watered in part a).

c) What angle does the sprinkler rotate through in 2 min? Express your answer in radians and degrees.

Given:
$r = 5m$
$\theta = \frac{5\pi}{3}$

a) $a = \theta r$

$a = \left(\frac{5\pi}{3}\right)(5) = \frac{25\pi}{3} m$

b) $\frac{A_{sector}}{A_{circle}} = \frac{\theta}{2\pi}$

$25\pi \cdot \frac{A_{sector}}{\cancel{25\pi}} = \frac{5\pi}{3} \cdot \frac{1}{2\cancel{\pi}} \cdot 25\cancel{\pi}$

$A_{sector} = \frac{125\pi}{6} m^2$

c) 1 rev : 15 seconds
8 revs in 120 seconds

In Radians:

$8 \cancel{rev} \times 2\pi \frac{rads}{\cancel{rev}} = 16\pi$ rads.

In Degrees

$8 \cancel{rev} \times 360° / \cancel{rev} = 2880°$

15. Angular velocity describes the rate of change in a central angle over time. For example, the change could be expressed in revolutions per minute (rpm), radians per second, degrees per hour, and so on. All that is required is an angle measurement expressed over a unit of time.

a) Earth makes one revolution every 24 h. Express the angular velocity of Earth in three other ways.

b) An electric motor rotates at 1000 rpm. What is this angular velocity expressed in radians per second?

2000π rads

c) A bicycle wheel completes 10 revolutions every 4 s. Express this angular velocity in degrees per minute.

3600°

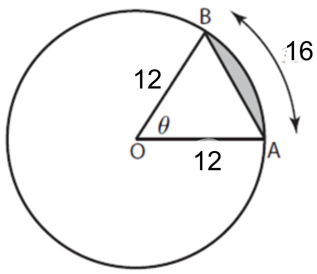
a) $V_a = \frac{\theta}{t} = \frac{360°}{24h} = \frac{2\pi \text{ rads}}{24h} = \frac{\pi \text{ rad}}{12h} = 15°/h$

b) $V_a = \frac{\theta}{t} = \frac{2000\pi \text{ rads}}{\cancel{min}} \cdot \frac{1\cancel{min}}{60sec}$

$= \frac{100\pi}{3}$ rads/sec

$= 104.\overline{6}$ rads/sec

c) $V_a = \frac{\theta}{t} = \frac{3600°}{4\cancel{s}} \cdot \frac{60\cancel{s}}{1min} = 54000°/min$



① Find θ

$\theta = \frac{a}{r}$

$\theta = \frac{16}{12}$

$\theta = \frac{4}{3}$ rads

② $\frac{\text{Sector Area}}{\text{Area of Circle}} = \frac{\text{Central Angle}}{\text{Complete Rev}}$

$\frac{x}{\pi(12)^2} = \frac{4/3}{2\pi}$

$\boxed{x = 96\text{cm}^2}$

③ $A_\Delta = \frac{1}{2}r^2\sin\theta$

$A_\Delta = \frac{1}{2}(12)^2\sin\left(\frac{4}{3}\right)$

$A_\Delta = \frac{1}{2}(144)(0.972)$

$\boxed{A_\Delta = 70\text{ cm}^2}$

④ $A_{seg} = A_{sec} - A_\Delta$

$A_{seg} = 96\text{cm}^2 - 70\text{cm}^2$

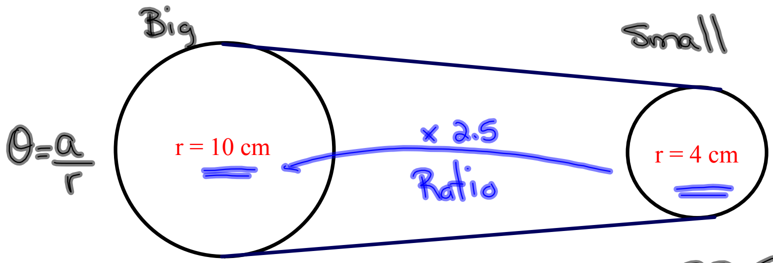
$\boxed{A_{seg} = 26\text{cm}^2}$

Applying our knowledge of rotations and radians...

Ex. (a) If the large wheel rotates $2\pi/3$ radians, how many radians does the smaller wheel rotate? Find θ

(b) If the large wheel completes three revolutions, how much does the small wheel rotate in radians?

(c) If the small wheel rotates $-15\pi/4$ radians, how many radians does the larger wheel rotate?



a)

Big

$\theta = \frac{a}{r}$

$(10)\ \frac{2\pi}{3} = \frac{a}{\cancel{10}}\ (\cancel{10})$

$\boxed{\frac{20\pi}{3}\text{ cm} = a}$

Small

$\theta = \frac{a}{r}$

$\theta = \frac{\frac{20\pi}{3}\ \cancel{cm}}{4\ \cancel{cm}}$

$\theta = \frac{20\pi}{3} \times \frac{1}{4}$

$\theta = \frac{20\pi}{12} = \boxed{\frac{5\pi}{3}\text{ rads}}$

Using Ratio:

$\frac{2\pi}{3} \cdot 2.5 = \boxed{\frac{5\pi}{3}}$

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b) 3 revs = 6π rads

Big

$\theta = \frac{a}{r}$

$(10)\ 6\pi = \frac{a}{\cancel{10}}\ (\cancel{10})$

$\boxed{60\pi\text{ cm} = a}$

Small

$\theta = \frac{a}{r}$

$\theta = \frac{60\pi\ \cancel{cm}}{4\ \cancel{cm}}$

$\boxed{\theta = 15\pi\text{ rads}}$

Using Ratio:

$6\pi \cdot 2.5 = \boxed{15\pi}$

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c) Small

$\theta = \frac{a}{r}$

$\frac{-15\pi}{4} = \frac{a}{4}$

$\boxed{-15\pi\text{ cm} = a}$

Big

$\theta = \frac{a}{r}$

$\theta = \frac{-15\pi\ \cancel{cm}}{10\ \cancel{cm}}$

$\boxed{\theta = -\frac{3\pi}{2} \text{ or } -1.5\pi\text{ rads}}$

Using Ratio

$-\frac{15\pi}{4} \div 2.5 = \boxed{-1.5\pi}$

Angular Velocity

Angular velocity - amount of rotation around a central point per unit of time

$$v_a = \frac{\theta}{t} \qquad \theta = \frac{a}{r}$$

θ = angle (radians)

v_a = angular velocity

a = arc length

t = time

r = radius

Ex. The roller on a computer printer makes 2200 rpm (revolution per minute). Find the roller's angular velocity.

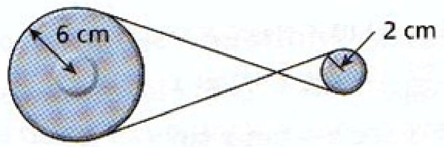
$$2200\ \frac{\cancel{\text{revs}}}{\text{min}} \times 2\pi\ \frac{\text{rads}}{\cancel{\text{rev}}} = \boxed{4400\pi\ \frac{\text{rads}}{\text{min}}}$$

angular velocity (V_a)

$$= \frac{4400\pi\ \text{rads}}{\text{min}}$$

convert to rads/sec $\rightarrow \frac{4400\pi\ \text{rads}}{60\text{s}} = \boxed{73.\overline{3}\ \frac{\text{rads}}{\text{sec}}}$

Two flywheels are connected by a belt, as shown in the diagram below. The larger one has a radius of 6 cm and the smaller one has a radius of 2 cm.



Ratio: 3:1

(a) If the small wheel rotates –300°, then through how many radians does the large wheel rotate?

$\rightarrow -\frac{5\pi}{3}$

(b) If the large wheel rotates $\frac{7\pi}{6}$ radians, ~~what distance~~ would a point on the circumference of the small wheel rotate?

Find a

Little Wheel

$a = \theta r$

$a = \left(-\frac{5\pi}{3}\right)(2)$

$a = -\frac{10\pi}{3}$ cm

Big Wheel

$\theta = \frac{a}{r}$

$\theta = \frac{-\frac{10\pi}{3}}{6}$

$\theta = -\frac{10\pi}{3} \cdot \frac{1}{6} = -\frac{10\pi}{18} = \boxed{+\frac{5\pi}{9}}$

Using Ratios:

$-\frac{5\pi}{3} \div 3$

$-\frac{5\pi}{3} \times \frac{1}{3} = -\frac{5\pi}{9} = \boxed{+\frac{5\pi}{9}}$

When the little wheel rotates in a negative direction the large wheel rotates in a positive direction

b) Given:

$\theta = \frac{7\pi}{6}$

$r = 6$

$a = \theta r$

$a = \left(\frac{7\pi}{6}\right)(6) = \boxed{7\pi \text{ cm}}$

would be the same for both wheels

Ex. A small electrical motor turns at 2200 rpm.

(a) Express the angular velocity in rad/s.
(b) Find the distance a point 0.8cm from the center of rotation travels in 0.008 s.

radius

a) $2200 \text{ rpm} \times 2\pi = \frac{4400\pi \text{ rads}}{\text{min}}$

$$V_a = \frac{\theta}{t} = \frac{4400\pi \text{ rads}}{\cancel{\text{min}}} \cdot \frac{1 \cancel{\text{min}}}{60 \text{sec}} = \frac{4400\pi \text{ rads}}{60 \text{ sec}}$$

$$= 73.\overline{3}\pi \text{ rads/sec}$$

$$\approx 230.3 \text{ rads/sec}$$

b) $\theta = (V_a)(t)$

$$\theta = \left(230.3 \frac{\text{rads}}{\cancel{\text{sec}}}\right)(0.008 \cancel{\text{sec}})$$

$$\theta = 1.8424 \text{ rads}$$

$$a = \theta r$$

$$a = (1.8424 \text{ rads})(0.8 \text{ cm})$$

$$a = \boxed{1.47 \text{ cm}}$$

Homework

Ex. A Ferris Wheel rotates 3 times each minute. The passengers sit in seats that are 5 m from the center of the wheel. What is the angular velocity of the wheel in radians per second? What distance do the passengers travel in 6.5 seconds?

Answer: a) $V_a = 0.314$ rads/sec
b) $a = 10.2$ m

Ex. A bicycle wheel has a radius of 36 cm and is turning at 4.8m/s. Determine the ang velocity of this wheel?

Answer: $V_a = 13.\overline{3}$ rads/sec