Questions from Quiz

① $f(x)=\sqrt{3x+1}$ $f(x+h)=\sqrt{3(x+h)+1}=\sqrt{3x+3h+1}$

$$f'(x)=\lim_{h\to 0}\frac{\sqrt{3x+3h+1}-\sqrt{3x+1}}{h}\cdot\frac{\left(\sqrt{3x+3h+1}+\sqrt{3x+1}\right)}{\left(\sqrt{3x+3h+1}+\sqrt{3x+1}\right)}$$

$$=\lim_{h\to 0}\frac{3x+3h+1-(3x+1)}{h\left(\sqrt{3x+3h+1}+\sqrt{3x+1}\right)}$$

$$=\lim_{h\to 0}\frac{3\cancel{h}}{\cancel{h}\left(\sqrt{3x+3h+1}+\sqrt{3x+1}\right)}=\frac{3}{2\sqrt{3x+1}}$$

② b) $f'(x)=\frac{1}{2x^{1/2}}+\frac{1}{3x^{2/3}}+\frac{1}{4x^{3/4}}$

$(2x^2+5x)^3($

④ $f(x)=\frac{x^2}{\sqrt{x}-3}$

$$f'(x)=\frac{2x(\sqrt{x}-3)-x^2\left(\frac{1}{2\sqrt{x}}\right)}{(\sqrt{x}-3)^2}$$

$$f'(x)=\frac{2x^{3/2}\cdot 2\sqrt{x}-6x\cdot 2\sqrt{x}-\frac{x^2}{2\sqrt{x}}\cdot 2\sqrt{x}}{2\sqrt{x}(\sqrt{x}-3)^2}$$

$$f'(x)=\frac{4x^2-12x^{3/2}-x^2}{2\sqrt{x}(\sqrt{x}-3)^2}$$

$$f'(x)=\frac{3x^2-12x^{3/2}}{2\sqrt{x}(\sqrt{x}-3)^2}=\frac{3x^{3/2}(x^{1/2}-4)}{2x^{1/2}(x^{1/2}-3)^2}=\frac{3x(\sqrt{x}-4)}{2(\sqrt{x}-3)^2}$$

⑤ $y=\quad 3x+4)^2$ Find equation of tangent when $x=-1$

(i) when $x=-1$
$y=(2-5)^3(-3+4)^2$
$y=(-27)(1)$
$y=-27$
Point $\leftarrow$
$(-1,-27)$

(ii) $y=(2x^2+5x)^3(3x+4)^2$

$y'=3(2x^2+5x)^2(4x+5)(3x+4)^2+(2x^2+5x)^3(2)(3x+4)(3)$

(iii) $y'(-1)$

$y'(-1)=3(-3)^2(1)(1)^2+(-3)^3(2)(1)(3)$

$m=y'(-1)=27+(-162)=\underline{\underline{-135}}$ ← Slope

(iv) $y-y_1=m(x-x_1)$

$y+27=-135(x+1)$

$y+27=-135x-135$

$\boxed{135x+y+162=0}$

Questions from Homework

④ Find $\left.\frac{dy}{dt}\right]_{t=1}$ if $y=\sqrt{1+r^2}$ and $r=\frac{t+1}{2t+1}$

when $t=1$

$r=\frac{2}{3}$

$y=(1+r^2)^{1/2}$

$\frac{dy}{dr}=\frac{1}{2}(1+r^2)^{-1/2}(2r)$

$\frac{dy}{dr}=\frac{\cancel{2}r}{\cancel{2}(1+r^2)^{1/2}}$

$\frac{dy}{dr}=\frac{r}{\sqrt{1+r^2}}$

$\frac{dr}{dt}=\frac{1(2t+1)-2(t+1)}{(2t+1)^2}$

$\frac{dr}{dt}=\frac{2t+1-2t-2}{(2t+1)^2}$

$\frac{dr}{dt}=\frac{-1}{(2t+1)^2}$

$\left.\frac{dy}{dt}\right]_{t=1}=\left[\frac{r}{\sqrt{1+r^2}}\right]\left[\frac{-1}{(2t+1)^2}\right]$

$=\left[\frac{(2/3)}{\sqrt{1+(2/3)^2}}\right]\left[\frac{-1}{9}\right]$

$=\left[\frac{2/3}{\sqrt{13/9}}\right]\left[\frac{-1}{9}\right]$

$=\left[\frac{2/3}{\sqrt{13}/3}\right]\left[\frac{-1}{9}\right]$

$=\left[\frac{2}{\cancel{3}}\cdot\frac{\cancel{3}}{\sqrt{13}}\right]\left[-\frac{1}{9}\right]$

$=\frac{-2}{9\sqrt{13}}$

⑥b) $f(x)=(2x+1)(4x-1)^5$

$f'(x)=2(4x-1)^5+(2x+1)(5)(4x-1)^4(4)$

$f'(x)=2(4x-1)^5+20(2x+1)(4x-1)^4$

$f'(x)=2(4x-1)^4[4x-1+10(2x+1)]$

$f'(x)=2(4x-1)^4(4x-1+20x+10)$

$f'(x)=2(4x-1)^4(24x+9)$

$f'(x)=6(4x-1)^4(8x+3)$

Differentiation Rules

Product Rule:

The Product Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

Express the product rule verbally if you are considering a function of the form...

$f(x)$= (First) X (Second)

" The derivative of the product of two functions is the the first multiplied by the derivative of second, plus the derivative of first multiplied by the second"

Get in the habit of verbalizing the rule as you differentiate...it will help when the functions get more complicated.

Quotient Rule:

The Quotient Rule If f and g are differentiable, then

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\dfrac{d}{dx}[f(x)] - f(x)\dfrac{d}{dx}[g(x)]}{[g(x)]^2}$$

Express the quotient rule verbally ...

" The denominator multiplied by the derivative of the numerator, minus the numerator multiplied by the derivative of the denominator, all over the denominator squared"

Combining the Chain Rule With the Product and Quotient Rule:

The Chain Rule If f and g are both differentiable and $F = f \circ g$ is the composite function defined by $F(x) = f(g(x))$, then F is differentiable and F' is given by the product

$$F'(x) = f'(g(x))g'(x)$$

Differentiate the following function and simplify your answer:

$$y = (x^2+1)^3(2-3x)^4$$

$$y' = (x^2+1)^3(4)(2-3x)^3(-3) + 3(x^2+1)^2(2x)(2-3x)^4$$

$$= -12(x^2+1)^3(2-3x)^3 + 6x(x^2+1)^2(2-3x)^4$$

$$= -6(x^2+1)^2(2-3x)^3\left[2(x^2+1) - x(2-3x)\right]$$

$$= -6(x^2+1)^2(2-3x)^3\left[2x^2+2-2x+3x^2\right]$$

$$= \boxed{-6(x^2+1)^2(2-3x)^3(5x^2-2x+2)}$$

$$g(x) = \frac{(3x+2)^2}{2x}$$

$$g'(x) = \frac{2x(2)(3x+2)(3) - (3x+2)^2(2)}{(2x)^2}$$

$$= \frac{12x(3x+2) - 2(3x+2)^2}{4x^2}$$

$$= \frac{2(3x+2)\left[6x - (3x+2)\right]}{4x^2}$$

$$= \frac{\cancel{2}(3x+2)(3x-2)}{\cancel{4}x^2}$$

$$= \boxed{\frac{(3x+2)(3x-2)}{2x^2}} \quad \text{or} \quad \frac{9x^2-4}{2x^2}$$

Differentiate the following functions and simplify your answers:

$$s=\left(\frac{2t-1}{t+2}\right)^6$$

$$2t+4-2t+1$$

$$\frac{ds}{dt}=6\left[\frac{2t-1}{t+2}\right]^5\left[\frac{(t+2)(2)-(2t-1)(1)}{(t+2)^2}\right]$$

$$\frac{ds}{dt}=6\left[\frac{(2t-1)^5}{(t+2)^5}\right]\left[\frac{5}{(t+2)^2}\right]$$

$$\frac{ds}{dt}=\frac{30(2t-1)^5}{(t+2)^7}$$

$$g(x)=(9x^{-3})(5x^3-1)^6$$

$$g'(x)=(9x^{-3})[6(5x^3-1)^5(15x^2)]-27x^{-4}(5x^3-1)^6$$

$$g'(x)=810x^{-1}(5x^3-1)^5-27x^{-4}(5x^3-1)^6$$

$$30x^3-5x^3+1$$

$$g'(x)=27x^{-4}(5x^3-1)^5[30x^3-(5x^3-1)]$$

$$g'(x)=27x^{-4}(5x^3-1)^5(25x^3+1)$$

$$g'(x)=\frac{27(5x^3-1)^5(25x^3+1)}{x^4}$$

Example 1

Let $F(x) = f(g(x))$ $F'(x) = f'(g(x)) \cdot g'(x)$

If $f(2) = 3$, $f'(2) = \underline{5}$, $g(1) = \underline{2}$ and $g'(1) = \underline{4}$ find $F'(1)$.

$$F'(1) = f'(\underline{g(1)}) \cdot g'(1)$$
$$= \underline{f'(2)} \cdot \underline{g'(1)}$$
$$= 5 \cdot 4$$
$$= 20$$

Question #6 ex. 2.4

Given
$f(2) = 3$
$f'(2) = 5$
$g(2) = -1$
$g'(2) = -4$

find $(fg)'(2) = f'(2)\,g(2) + f(2)\,g'(2)$
$$= (5)(-1) + (3)(-4)$$
$$= -5 - 12$$
$$= -17$$

find $\left(\frac{f}{g}\right)'(2) = \frac{f'(2)\,g(2) - f(2)\,g'(2)}{(g(2))^2}$
$$= \frac{(5)(-1) - (3)(-4)}{(-1)^2}$$
$$= \frac{-5 + 12}{1}$$
$$= 7$$

Example 2

If $y = u^{10} + u^5 + 2$, where $u = 1 - 3x^2$, find $\left.\frac{dy}{dx}\right]_{x=1}$

$\frac{dy}{du} = 10u^9 + 5u^4$

$\frac{du}{dx} = -6x$

when $x = 1$
$u = 1 - 3(1)^2 = -2$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$\left.\frac{dy}{dx}\right]_{x=1} = (10u^9 + 5u^4)(-6x)$

$= (10(-2)^9 + 5(-2)^4)(-6(1))$

$= (-5120 + 80)(-6)$

$= (-5040)(-6)$

$= 30240$

If $y = u^{10} + u^5 + 2$, where $u = 1 - 3x^2$, find $\left.\frac{dy}{dx}\right]_{x=1}$

$y = (1-3x^2)^{10} + (1-3x^2)^5 + 2$

$\frac{dy}{dx} = 10(1-3x^2)^9(-6x) + 5(1-3x^2)^4(-6x)$

$\left.\frac{dy}{dx}\right]_{x=1} = 10(-2)^9(-6) + 5(-2)^4(-6)$

$= 30720 - 480$

$= 30240$

Homework

③ $y = u^2 - 2u^5$ and $u = x - x^{1/2}$ Find $\left.\frac{dy}{dx}\right|_{x=4}$

$y = (x - x^{1/2})^2 - 2(x - x^{1/2})^5$

$\frac{dy}{dx} = 2(x-\sqrt{x})\left(1 - \frac{1}{2\sqrt{x}}\right) - 10(x-\sqrt{x})^4\left(1 - \frac{1}{2\sqrt{x}}\right)$

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#3-10

3 Find $\left.\frac{dy}{dx}\right]_{x=4}$ if $y = u^2 - 2u^5$ and $u = x - \sqrt{x}$

$\frac{dy}{du} = 2u - 10u^4$

$\frac{du}{dx} = 1 - \frac{1}{2}x^{-1/2}$

$\frac{du}{dx} = 1 - \frac{1}{2\sqrt{x}}$

when $x = 4$ $u = 2$

$$\left.\frac{dy}{dx}\right]_{x=4} = \left[\frac{dy}{du}\right]\left[\frac{du}{dx}\right]$$

$$= \left[2u - 10u^4\right]\left[1 - \frac{1}{2\sqrt{x}}\right]$$

$$= \left[2(2) - 10(2)^4\right]\left[1 - \frac{1}{2\sqrt{4}}\right]$$

$$= (-\cancel{156}^{\,-39})\left(\frac{3}{\cancel{4}_{\,1}}\right)$$

$$= \boxed{-117}$$

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#3-10

④ $\left.\frac{dy}{dt}\right]_{t=1}$

$y=\sqrt{1+r^2}$

$\frac{dy}{dr}=\frac{1}{2}(1+r^2)^{-1/2}(2r)$

$\frac{dy}{dr}=\frac{r}{\sqrt{1+r^2}}$

$r=\frac{t+1}{2t+1}$

$\frac{dr}{dt}=\frac{1(2t+1)-2(t+1)}{(2t+1)^2}$

$\frac{dr}{dt}=\frac{-1}{(2t+1)^2}$

when $t=1$ $r=\frac{2}{3}$

$$\left.\frac{dy}{dt}\right]_{t=1}=\left[\frac{dy}{dr}\right]\left[\frac{dr}{dt}\right]$$

$$=\left[\frac{r}{\sqrt{1+r^2}}\right]\left[\frac{-1}{(2t+1)^2}\right]$$

$$=\left[\frac{2/3}{\sqrt{1+(2/3)^2}}\right]\left[\frac{-1}{(2(1)+1)^2}\right]$$

$$=\left[\frac{2/3}{\frac{\sqrt{13}}{\sqrt{9}}}\right]\left[-\frac{1}{9}\right]$$

$$=\left[\frac{2}{\cancel{3}}\cdot\frac{\cancel{3}}{\sqrt{13}}\right]\left[-\frac{1}{9}\right]$$

$$=\boxed{\frac{-2}{9\sqrt{13}}}$$