

Transformations of Exponential Functions

Focus on...

- applying translations, stretches, and reflections to the graphs of exponential functions
- representing these transformations in the equations of exponential functions
- solving problems that involve exponential growth or decay

Link the Ideas

The graph of a function of the form $f(x) = a(c)^{b(x-h)} + k$ is obtained by applying transformations to the graph of the base function $y = c^x$, where $c > 0$.

Parameter	Transformation	Example
a	<ul style="list-style-type: none"> Vertical stretch about the x-axis by a factor of a For $a < 0$, reflection in the x-axis $(x, y) \rightarrow (x, ay)$ 	
b	<ul style="list-style-type: none"> Horizontal stretch about the y-axis by a factor of $\frac{1}{ b }$ For $b < 0$, reflection in the y-axis $(x, y) \rightarrow (\frac{x}{b}, y)$ 	
k	<ul style="list-style-type: none"> Vertical translation up or down $(x, y) \rightarrow (x, y + k)$ 	
h	<ul style="list-style-type: none"> Horizontal translation left or right $(x, y) \rightarrow (x + h, y)$ 	

Example 1

Apply Transformations to Sketch a Graph

Consider the base function $y = 3^x$. For each transformed function,

- state the parameters and describe the corresponding transformations
- create a table to show what happens to the given points under each transformation

$y = 3^x$
$(-1, \frac{1}{3})$
$(0, 1)$
$(1, 3)$
$(2, 9)$
$(3, 27)$

- sketch the graph of the base function and the transformed function
 - describe the effects on the domain, range, equation of the horizontal asymptote, and intercepts
- $y = 2(3)^{x-4}$
 - $y = -\frac{1}{2}(3)^{\frac{1}{5}x} - 5$

Solution

a) i) Compare the function $y = 2(3)^{x-4}$ to $y = a(c)^{b(x-h)} + k$ to determine the values of the parameters.

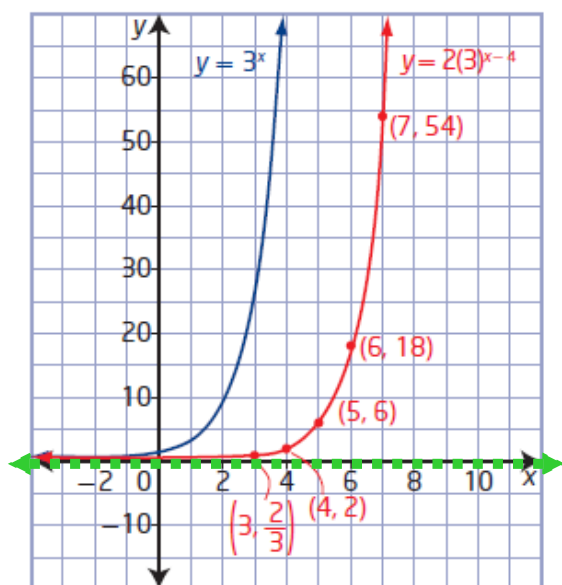
- $b = 1$ corresponds to no horizontal stretch.
- $a = 2$ corresponds to a vertical stretch of factor 2. Multiply the y -coordinates of the points in column 1 by 2.
- $h = 4$ corresponds to a translation of 4 units to the right. Add 4 to the x -coordinates of the points in column 2.
- $k = 0$ corresponds to no vertical translation.

ii) Add columns to the table representing the transformations.

$y = 3^x$	$y = 2(3)^{x-4}$
$(-1, \frac{1}{3})$	$(3, \frac{2}{3})$
$(0, 1)$	$(4, 2)$
$(1, 3)$	$(5, 6)$
$(2, 9)$	$(6, 18)$
$(3, 27)$	$(7, 54)$

$$(x, y) \rightarrow [x+4, 2y+0]$$

iii) To sketch the graph, plot the points from column 3 and draw a smooth curve through them.



y -int: $(x=0)$

$$y = 2(3)^{x-4}$$

$$y = 2(3)^{0-4}$$

$$y = 2(3)^{-4}$$

$$y = 2(3)^{-4}$$

$$y = 2\left(\frac{1}{3}\right)^4$$

$$y = 2\left(\frac{1}{81}\right)$$

$$y = \frac{2}{81} \rightarrow \left(0, \frac{2}{81}\right)$$

iv) The domain remains the same: $\{x \mid x \in \mathbb{R}\}$.

The range also remains unchanged: $\{y \mid y > 0, y \in \mathbb{R}\}$.

The equation of the asymptote remains as $y = 0$.

There is still no x -intercept, but the y -intercept changes to $\frac{2}{81}$ or approximately 0.025.

$$b) y = -\frac{1}{2}(3)^{\frac{1}{5}x} - 5 \rightarrow y = a(c)^{b(x-h)} + k$$

- i) state the parameters and describe the corresponding transformations
- ii) create a table to show what happens to the given points under each transformation
- iii) sketch the graph of the base function and the transformed function
- iv) describe the effects on the domain, range, equation of the horizontal asymptote, and intercepts

i) $a = -\frac{1}{2}$ → vertical stretch by a factor of $\frac{1}{2}$ and a reflection in the x-axis

$b = \frac{1}{5}$ → horizontal stretch by a factor of 5.

$h = 0$ → no horizontal translation

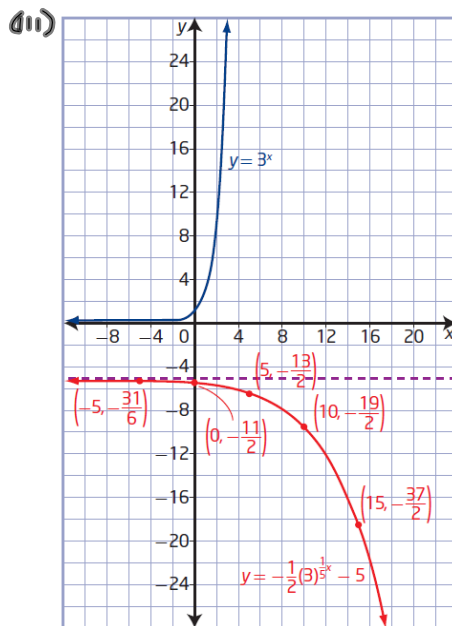
$k = -5$ → translated 5 units down

ii) $y = 3^x \quad (x, y) \rightarrow [5x+0, \frac{1}{5}y - 5]$

x	y
-1	$\frac{1}{3}$
0	1
1	3
2	9
3	27

$$y = -\frac{1}{2}(3)^{\frac{1}{5}x} - 5$$

x	y
-5	$-\frac{31}{6} = -5.16$
0	$-\frac{11}{2} = -5.5$
5	$-\frac{19}{2} = -9.5$
10	$-\frac{37}{2} = -18.5$
15	$-\frac{71}{2} = -35.5$



(iv) D: $\{x | x \in \mathbb{R}\}$

R: $\{y | y < -5, y \in \mathbb{R}\}$

HA: $y = -5$

x int: none

y int: $(x=0)$

$$y = \frac{1}{2}(3)^{\frac{1}{5}(0)} - 5$$

$$y = -\frac{1}{2}(3)^0 - 5$$

$$y = -\frac{1}{2}(1) - 5$$

$$y = -\frac{1}{2} - 5$$

$$y = -\frac{1}{2} - \frac{10}{2} = \left(-\frac{11}{2}\right)$$

From chart

$$(0, -\frac{11}{2})$$

Your Turn

Transform the graph of $y = 4^x$ to sketch the graph of $y = 4^{-2(x+5)} - 3$. Describe the effects on the domain, range, equation of the horizontal asymptote, and intercepts.

Homework

#1-7 and #10 on page 354