Warm Up

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1. Simplify: $\dfrac{\frac{1}{x^2}-\frac{1}{9}}{x-3}$

$$\dfrac{\frac{9}{9x^2}-\frac{x^2}{9x^2}}{x-3}$$

Factor out a -1 → $\dfrac{9-x^2}{9x^2} \times \dfrac{1}{x-3}$

Difference of Squares → $\dfrac{-1(x^2-9)}{9x^2} \times \dfrac{1}{x-3}$

$$\dfrac{-(x+3)\cancel{(x-3)}}{9x^2} \times \dfrac{1}{\cancel{(x-3)}}$$

$$\boxed{\dfrac{-(x+3)}{9x^2}}$$

3. Rationalize the denominator:

$$\dfrac{x+2}{[\sqrt{x-4}-\sqrt{x-6}]} \cdot \dfrac{[\sqrt{x-4}+\sqrt{x-6}]}{[\sqrt{x-4}+\sqrt{x-6}]}$$

$$\dfrac{(x+2)(\sqrt{x-4}+\sqrt{x-6})}{x-4-(x-6)}$$

$$\dfrac{(x+2)(\sqrt{x-4}+\sqrt{x-6})}{x-4-x+6}$$

$$\boxed{\dfrac{(x+2)(\sqrt{x-4}+\sqrt{x-6})}{2}}$$

Conjugates:

① $2+\sqrt{5} \longrightarrow 2-\sqrt{5}$

② $3-3\sqrt{6} \longrightarrow 3+3\sqrt{6}$

③ $\sqrt{x+4}-\sqrt{x-3} \longrightarrow \sqrt{x+4}+\sqrt{x-3}$

④ $-5\sqrt{3}+\sqrt{x+3} \longrightarrow -5\sqrt{3}-\sqrt{x+3}$

The common sense definition of a limit...

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What *is* a limit?

the limit of a function is the intended height of that function

A formal definition of a limit...

We write $\lim_{x \to a} f(x) = L$ if we can make the values of $f(x)$ <u>arbitrarily close to L</u>

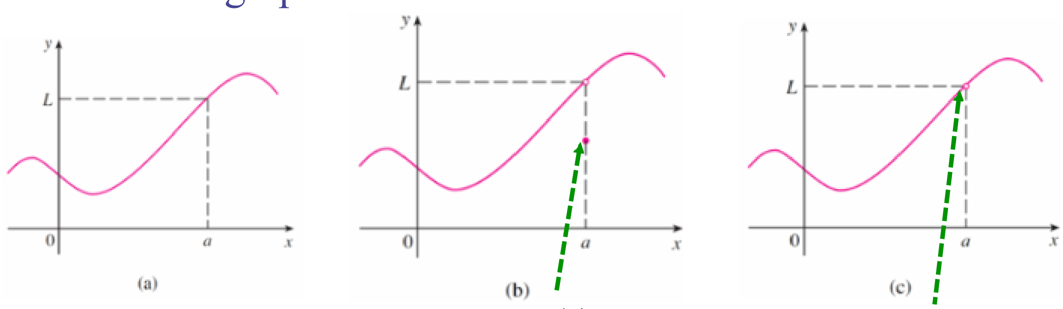
- (as close to L as we like)

by taking x to be <u>sufficiently close to a</u>

- (on either side of a)

but <u>not</u> equal to a.

Look at the graphs of these three functions...



Notice $f(a) \neq L$

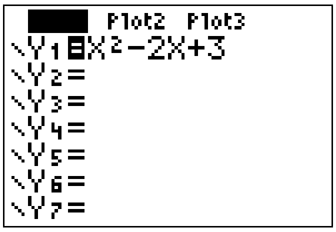
Notice $f(a)$ is undefined

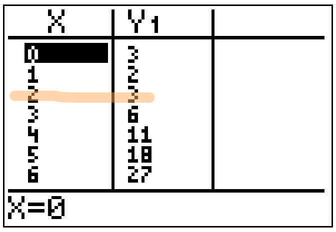
But in each case, regardless of what happens at a, it is true that

$$\lim_{x \to a} f(x) = L$$

Limit of a Function

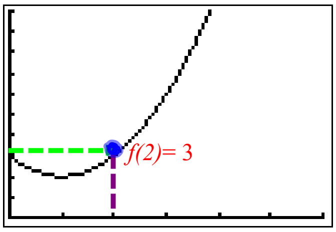
Let's examine the function $f(x) = x^2 - 2x + 3$ Parabola



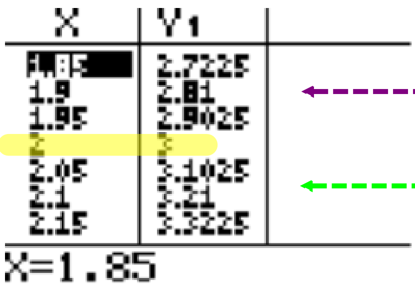


X	Y1
0	3
1	2
2	3
3	6
4	11
5	18
6	27

X=0



We can see that *f(2)*= 3...let's check the behaviour of *f* as we get closer and closer to $x = 2$.



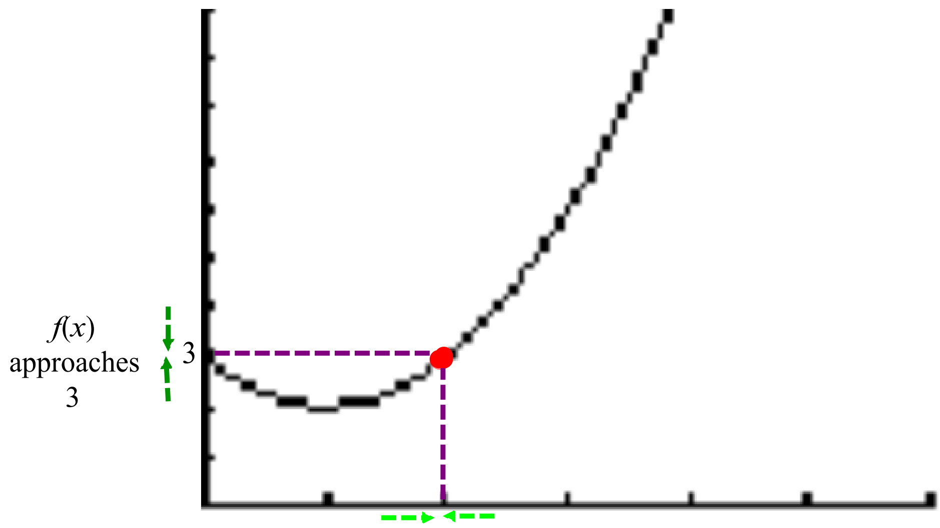
X	Y1
1.85	2.7225
1.9	2.81
1.95	2.9025
2	3
2.05	3.1025
2.1	3.21
2.15	3.3225

X=1.85

As x gets closer to 2 from the left y is getting closer to 3.

As x gets closer to 2 from the right y is getting closer to 3.

From the above, the notion of the limit of a function arises...



As x approaches 2

Notation: $\lim_{x \to 2} f(x) = 3$ or $\lim_{x \to 2} x^2 - 2x + 3 = 3$

"The limit of the function *f*(*x*) as *x* approaches 2 is equal to 3."

Evaluating Limits

I. Using a Graph:

- We looked at this in the previous two examples

II. Algebraically:

- Direct Substitution...

Examples:

$$\lim_{x\to -2}\frac{x^2-2x+1}{x+3}$$

$$\lim_{x\to -2}\frac{(-2)^2-2(-2)+1}{(-2)+3}$$

$$\lim_{x\to -2}\frac{4+4+1}{1}=9$$

$$\lim_{x\to 3}\left(16-x^2\right)$$

$$\lim_{x\to 3}\left(16-(3)^2\right)$$

$$\lim_{x\to 3}\left(16-9\right)=7$$

- Indeterminate limits... $\Rightarrow$ Direct substitution leads to $\frac{0}{0}$

$\Rightarrow$ Factor
$\Rightarrow$ Rationalize
$\Rightarrow$ Expand
$\Rightarrow$ Find Common Denominators

Examples:

$$\lim_{x\to 4}\frac{x^2-16}{x-4}$$

$$\lim_{x\to 4}\frac{(x+4)\cancel{(x-4)}}{\cancel{(x-4)}}$$

$$\lim_{x\to 4}(4+4)=\boxed{8}$$

$$\lim_{h\to 0}\frac{\sqrt{4+h}-2}{h}\cdot\frac{(\sqrt{4+h}+2)}{(\sqrt{4+h}+2)}$$

$$\lim_{h\to 0}\frac{4+h-4}{h(\sqrt{4+h}+2)}$$

$$\lim_{h\to 0}\frac{\cancel{h}}{\cancel{h}(\sqrt{4+h}+2)}$$

$$\lim_{h\to 0}\frac{1}{\sqrt{4+0}+2}=\boxed{\frac{1}{4}}$$

Try these...remember to use your algebra skills to try and eliminate the **indeterminate form**.

$$\lim_{x \to 0} \frac{x^2 + 3x}{(x+2)^2 - (x-2)^2}$$

$$\lim_{x \to -2} \frac{x^4 - 16}{x^3 + 8}$$

$$\lim_{x \to 2} \frac{(x+2)^2 - 16}{x^2 - 4}$$

$$\lim_{x \to 2} \frac{\frac{1}{x} - \frac{1}{2}}{x - 2}$$

Homework