# Questions from Homework

8. If seafood is not kept frozen (below 0 °C), it will spoil due to bacterial growth. The relative rate of spoilage increases with temperature according to the model

$R = 100(2.7)^{\frac{T}{8}}$, where T is the temperature, in degrees Celsius, and R is the relative spoilage rate.

a) Sketch a graph of the relative spoilage rate R versus the temperature T from 0 °C to 25 °C.

b) Use your graph to predict the temperature at which the relative spoilage rate doubles to 200.

c) What is the relative spoilage rate at 15 °C?

d) If the maximum acceptable relative spoilage rate is 500, what is the maximum storage temperature?

a)
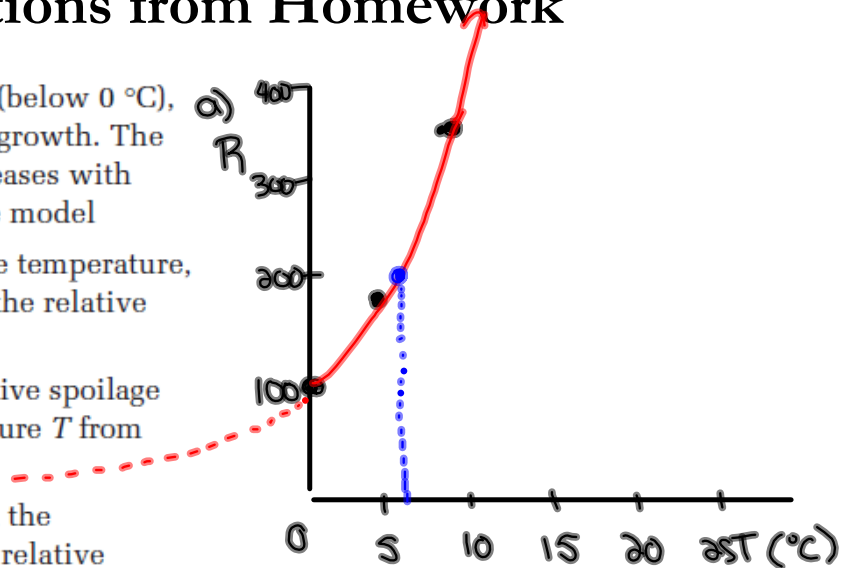
b) $R = 100(2.7)^{\frac{T}{8}}$

$200 = 100(2.7)^{\frac{T}{8}}$ (Divide by 100)

$2 = (2.7)^{\frac{T}{8}}$ (Get a common base) $\frac{\log 2}{\log 2.7} =$

$(2.7)^{0.7} = (2.7)^{\frac{T}{8}}$ (Set exponents equal)

$8 \times \quad 0.7 = \frac{T}{8} \times 8$

$\boxed{5.6°C = T}$

9. A bacterial culture starts with 2000 bacteria
and doubles every 0.75 h. After how many
hours will the bacteria count be 32 000? ← Final Amount

Given:

Initial Amount = 2000

Base: 2 (Doubles)

Exp: $\dfrac{x}{0.75}$

$$A = 2000(2)^{\frac{x}{0.75}}$$

$$32000 = 2000(2)^{\frac{x}{0.75}}$$
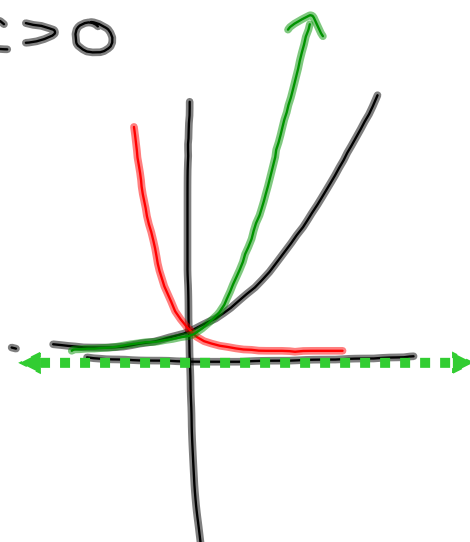
$$16 = 2^{\frac{x}{0.75}}$$

$$2^4 = 2^{\frac{x}{0.75}}$$

$$4 = \frac{x}{0.75}$$

$$\boxed{3 = x}$$

For any exponential function of the form $y = c^x \rightarrow c > 0$

$y = c^x$

D: $\{x | x \in R\}$

R: $\{y | y > 0\}$

HA: $y = 0$

x int: none

y int: $(0, 1)$

Given:

$$y = -3(2)^{2x+2} + 4$$

$$y = -3(2)^{2(x+1)} + 4$$

i) state the parameters and describe the corresponding transformations
ii) create a table to show what happens to the given points under each transformation
iii) sketch the graph of the base function and the transformed function
iv) describe the effects on the domain, range, equation of the horizontal asymptote, and intercepts
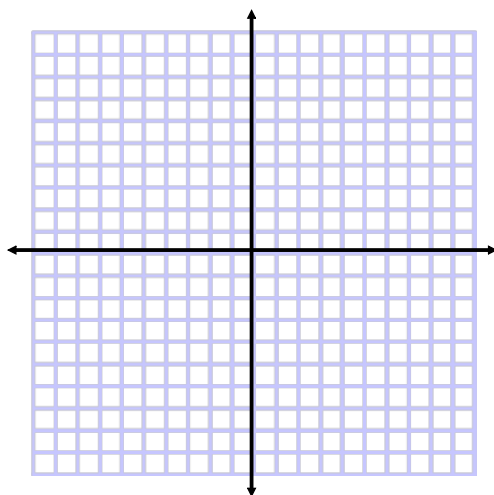
Understanding Logarithms

Focus on...

- demonstrating that a logarithmic function is the inverse of an exponential function
- sketching the graph of $y = \log_c x$, $c > 0$, $c \neq 1$
- determining the characteristics of the graph of $y = \log_c x$, $c > 0$, $c \neq 1$
- explaining the relationship between logarithms and exponents
- expressing a logarithmic function as an exponential function and vice versa
- evaluating logarithms using a variety of methods

For the exponential function $y = c^x$, the inverse is $x = c^y$. This inverse is also a function and is called a **logarithmic function**. It is written as $y = \log_c x$, where c is a positive number other than 1.

Logarithmic Form   **Exponential Form**

exponent

$$\log_c x = y \qquad\qquad c^y = x$$
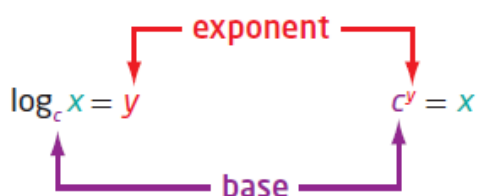
base

Since our number system is based on powers of 10, **logarithms** with base 10 are widely used and are called **common logarithms**. When you write a common logarithm, you do not need to write the base. For example, log 3 means $\log_{10} 3$.

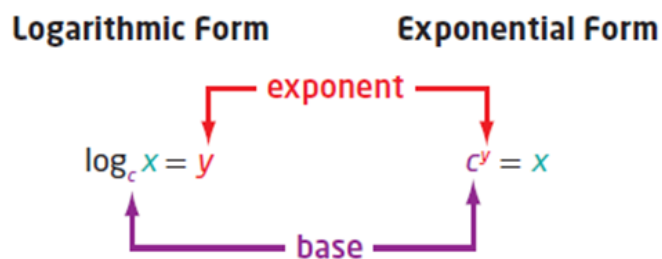| **logarithmic function**	**logarithm**	**common logarithm**
• a function of the form $y = \log_c x$, where $c > 0$ and $c \neq 1$, that is the inverse of the exponential function $y = c^x$	• an exponent • in $x = c^y$, y is called the logarithm to base c of x	• a logarithm with base 10

Write each of the following in logarithmic form

a) $32 = 2^5$ b) $2^{-5} = \dfrac{1}{32}$ c) $x = 10^y$

Logarithmic Form   **Exponential Form**

exponent

$\log_c x = y$ $c^y = x$

base

Write each of the following in exponential form

a) $\log_4 16 = 2$ b) $\log_2 \left(\dfrac{1}{32}\right) = -5$ c) $\log 65 = 1.8129$

Example 1

Evaluating a Logarithm

Evaluate.

a) $\log_7 49$ **b)** $\log_6 1$ **c)** $\log 0.001$ **d)** $\log_2 \sqrt{8}$

Example 2

Determine an Unknown in an Expression in Logarithmic Form

Determine the value of x.

a) $\log_5 x = -3$

b) $\log_x 36 = 2$

c) $\log_{64} x = \dfrac{2}{3}$

Example 3

Graph the Inverse of an Exponential Function

a) State the inverse of $f(x) = 3^x$.

b) Sketch the graph of the inverse. Identify the following characteristics of the inverse graph:
- the domain and range
- the x-intercept, if it exists
- the y-intercept, if it exists
- the equations of any asymptotes

Solution

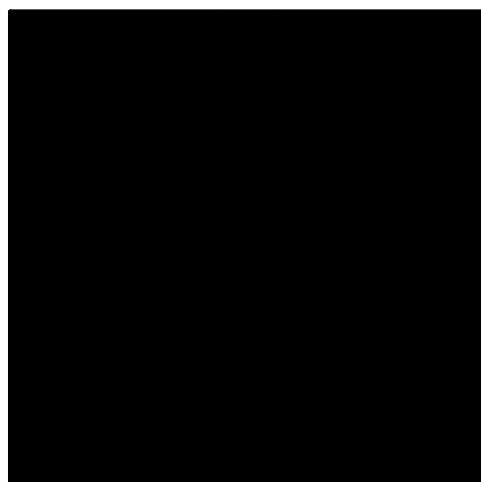**a)** The inverse of $y = f(x) = 3^x$ is _____ or,

expressed in logarithmic form, _____ . Since the

inverse is a function, it can be written in function

notation as _____

> How do you know that $y = \log_3 x$ is a function?

b) Set up tables of values for both the exponential function, $f(x)$, and its inverse, $f^{-1}(x)$. Plot the points and join them with a smooth curve.

$f(x) = 3^x$	
x	y
−3	
−2	
−1	
0	
1	
2	
3	

$f^{-1}(x) = \log_3 x$	
x	y

The graph of the inverse, $f^{-1}(x) = \log_3 x$, is a reflection of the graph

of $f(x) = 3^x$ about the line $y = x$. For $f^{-1}(x) = \log_3 x$,

- the domain is _____ and the range is _____
- the x-intercept is _____
- there is no y-intercept
- the vertical asymptote, the ___ axis, has equation _____ there is no ___ asymptote

> How do the characteristics of $f^{-1}(x) = \log_3 x$ compare to the characteristics of $f(x) = 3^x$?

Key Ideas

- A logarithm is an exponent.

- Equations in exponential form can be written in logarithmic form and vice versa.

Exponential Form	Logarithmic Form
$x = c^y$	$y = \log_c x$

- The inverse of the exponential function $y = c^x$, $c > 0$, $c \neq 1$, is $x = c^y$ or, in logarithmic form, $y = \log_c x$. Conversely, the inverse of the logarithmic function $y = \log_c x$, $c > 0$, $c \neq 1$, is $x = \log_c y$ or, in exponential form, $y = c^x$.

- The graphs of an exponential function and its inverse logarithmic function are reflections of each other in the line $y = x$, as shown.

- For the logarithmic function $y = \log_c x$, $c > 0$, $c \neq 1$,

  - the domain is $\{x \mid x > 0, x \in R\}$

  - the range is $\{y \mid y \in R\}$

  - the x-intercept is 1

  - the vertical asymptote is $x = 0$, or the y-axis

- A common logarithm has base 10. It is not necessary to write the base for common logarithms:
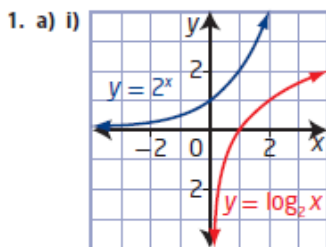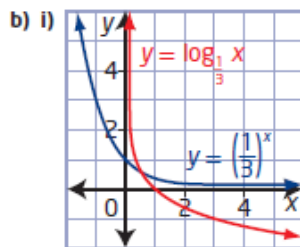
$$\log_{10} x = \log x$$
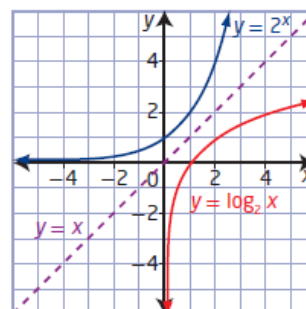
Homework

#1-5, 8, 10, 12, 13, 17 on page 380
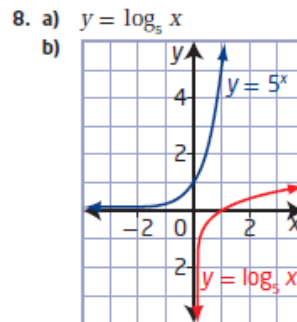
8.1 Understanding Logarithms, pages 380 to 382

1. a) i)

ii) $y = \log_2 x$

iii) domain $\{x \mid x > 0, x \in R\}$, range $\{y \mid y \in R\}$, x-intercept 1, no y-intercept, vertical asymptote $x = 0$

b) i)

ii) $y = \log_{\frac{1}{3}} x$

iii) domain $\{x \mid x > 0, x \in R\}$, range $\{y \mid y \in R\}$, x-intercept 1, no y-intercept, vertical asymptote $x = 0$

2. a) $\log_{12} 144 = 2$ **b)** $\log_8 2 = \dfrac{1}{3}$

c) $\log_{10} 0.000\,01 = -5$ **d)** $\log_7 (y + 3) = 2x$

3. a) $5^2 = 25$ **b)** $8^{\frac{2}{3}} = 4$

c) $10^6 = 1\,000\,000$ **d)** $11^y = x + 3$

4. a) 3   **b)** 0   **c)** $\dfrac{1}{3}$ **d)** -3

5. $a = 4; b = 5$

8. a) $y = \log_5 x$

b)

domain $\{x \mid x > 0, x \in R\}$, range $\{y \mid y \in R\}$, x-intercept 1, no y-intercept, vertical asymptote $x = 0$

10. They are reflections of each other in the line $y = x$.

11. a) They have the exact same shape.

    b) One of them is increasing and the other is decreasing.

12. a) 216   **b)** 81   **c)** 64   **d)** 8

13. a) 7   **b)** 6

14. a) 0   **b)** 1

15. -1

16. 16

17. a) $t = \log_{1.1} N$ **b)** 145 days

18. The larger asteroid had a relative risk that was 1479 times as dangerous.

19. 1000 times as great

20. 5

21. $m = 14, n = 13$

22. $4n$

23. $y = 3^{2^x}$