

Example 4

Solve a Problem Involving Exponential Growth and Decay

When an animal dies, the amount of radioactive carbon-14 (C-14) in its bones decreases. Archaeologists use this fact to determine the age of a fossil based on the amount of C-14 remaining.

- The half-life of C-14 is 5730 years.

Head-Smashed-In Buffalo Jump in southwestern Alberta is recognized as the best example of a buffalo jump in North America. The oldest bones unearthed at the site had 49.5% of the C-14 left.

How old were the bones when they were found?
(Solve for t)



Buffalo skull display, Head-Smashed-In buffalo Jump Visitor Centre, near Fort McLeod, Alberta

Solution

Carbon-14 decays by one half for each 5730-year interval. The mass, m , remaining at time t can be found using the relationship $m(t) = m_0 \left(\frac{1}{2}\right)^{\frac{t}{5730}}$, where m_0 is the original mass.

Since 49.5% of the C-14 remains after t years, substitute $0.495m_0$ for $m(t)$ in the formula $m(t) = m_0 \left(\frac{1}{2}\right)^{\frac{t}{5730}}$.

$$0.495m_0 = m_0 \left(\frac{1}{2}\right)^{\frac{t}{5730}}$$

$$0.495 = 0.5^{\frac{t}{5730}}$$

$$\log 0.495 = \log 0.5^{\frac{t}{5730}}$$

$$\log 0.495 = \frac{t}{5730} \log 0.5$$

$$\frac{5730 \log 0.495}{\log 0.5} = t$$

$$5813 \approx t$$

Instead of taking the common logarithm of both sides, you could have converted from exponential form to logarithmic form. Try this. Which approach do you prefer? Why?

The oldest buffalo bones found at Head-Smashed-In Buffalo Jump date to about 5813 years ago. The site has been used for at least 6000 years.

Homework

Chapter 7 Review pg. 366-367 (Do all questions)

Chapter 7: Exponential Functions

For $y = c^x$

D: $\{x | x \in \mathbb{R}\}$

R: $\{y | y > \underline{0}, y \in \mathbb{R}\}$

x int: none

y int: $(0, 1)$

HA: $y = \underline{0}$

For $y = a c^{b(x-h)} + k$

D: $\{x | x \in \mathbb{R}\}$

R: $\{y | y > k, y \in \mathbb{R}\}$ *if $a < 0$, switch inequality*

x int: sub 0 in for y

y int: sub 0 in for x

HA: $y = k$

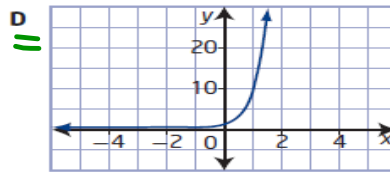
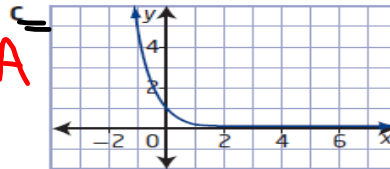
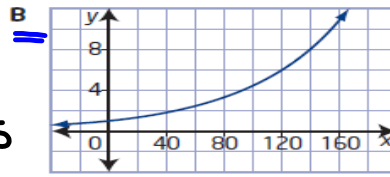
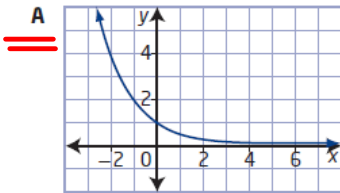
Page 366

1. Match each item in set A with its graph from set B.

Set A

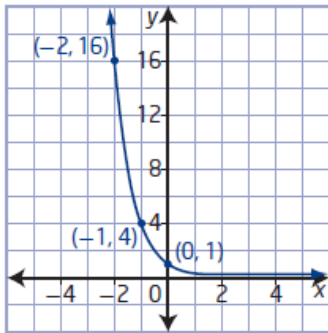
- Inc** a) The population of a country, in millions, grows at a rate of 1.5% per year. $c = 1.015$ **B**
- Inc** b) $y = 10^x$ $c = 10$ **D**
- Dec** c) Tungsten-187 is a radioactive isotope that has a half-life of 1 day. $c = 0.5$ **A**
- Dec** d) $y = 0.2^x$ $c = 0.2$ **C**

Set B



3. What exponential function in the form $y = c^x$ is represented by the graph shown?

What is your base ($c = ?$)



x	y
-2	16
-1	4
0	1

$$y = \left(\frac{1}{4}\right)^x$$

9) c) $(\sqrt[3]{216})^5$

$(216^{1/3})^5$

$(216)^{5/3}$

$(6^3)^{5/3}$

6^5

