Understanding Logarithms

Focus on...

$(x,y) \rightarrow (y,x)$
reflection in the line $y=x$

- demonstrating that a logarithmic function is the inverse of an exponential function
- sketching the graph of $y = \log_c x$, $c > 0$, $c \neq 1$
- determining the characteristics of the graph of $y = \log_c x$, $c > 0$, $c \neq 1$
- explaining the relationship between logarithms and exponents
- expressing a logarithmic function as an exponential function and vice versa
- evaluating logarithms using a variety of methods

Questions from Homework

5. Identify the following characteristics of the graph of each function.

i) the equation of the asymptote

ii) the domain and range

iii) the y-intercept, to one decimal place if necessary

iv) the x-intercept, to one decimal place if necessary

a) $y = -5 \log_3 (x + 3)$

b) $y = \log_6 (4(x + 9))$

c) $y = \log_5 (x + 3) - 2$

d) $y = -3 \log_2 (x + 1) - 6$

d) $y = -3\log_2(x+1) - 6$

$a = -3$ $b = 1$ $h = -1$ $k = -6$

(i) VA: $x = -1$

(ii) D: $\{x \mid x > -1, x \in R\}$

R: $\{y \mid y \in R\}$

(iii) y int $(x=0)$

$y = -3\log_2(x+1) - 6$

$y = -3\log_2(0+1) - 6$

$y = -3\log_2 1 - 6$

$y = -3(0) - 6$

$y = -6$

$(0, -6)$

(iv) x-int $(y=0)$

$y = -3\log_2(x+1) - 6$

$0 = -3\log_2(x+1) - 6$

$6 = -3\log_2(x+1)$

$-2 = \log_2(x+1)$

$2^{-2} = x+1$

$\frac{1}{4} = x+1$

$-\frac{3}{4} = x$

Questions from Homework

11. Explain how the graph of $\frac{1}{3}(y+2) = \log_6(x-4)$ can be generated by transforming the graph of $y = \log_6 x$.

$y+2 = 3\log_6(x-4)$ Divide a+k by $\frac{1}{3}$

$y = 3\log_6(x-4) - 2$ Subtract 2 from both sides

$y = 3\log_6 1(x-4) - 2$

$a=3 \rightarrow$ A vertical stretch by a factor of 3

$b=1 \rightarrow$ No horizontal stretch.

$h=4 \rightarrow$ Translated 4 units right.

$k=-2 \rightarrow$ " 2 " down.

5. Identify the following characteristics of the graph of each function.

i) the equation of the asymptote

ii) the domain and range

iii) the y-intercept, to one decimal place if necessary

iv) the x-intercept, to one decimal place if necessary

a) $y = -5\log_3(x+3)$

b) $y = 1\log_6(4(x+9))$

c) $y = \log_5(x+3) - 2$

d) $y = -3\log_2(x+1) - 6$

b) $a=1$ $b=4$ $h=-9$ $k=0$

$(x,y) \longrightarrow \left[\frac{1}{4}x - 9, 1y + 0\right]$

d) $y = -3\log_2(x+1) - 6$

Base: $y = \log_6 x$

D: $\{x \mid x > 0, x \in R\}$

R: $\{y \mid y \in R\}$

x int: $(1, 0)$

y int: none

VA: $x = 0$

For: $y = \log_6(4(x+9))$

(i) VA: $x = -9$

(ii) D: $\{x \mid x > -9, x \in R\}$

R: $\{y \mid y \in R\}$

(iii) y intercept (Let $x=0$)

$y = \log_6(4(x+9))$

$y = \log_6(4(0+9))$

$y = \log_6 36$

$\boxed{y=2}$ or $(0, 2)$

(iv) x-intercept (Let $y=0$)

$y = \log_6(4(x+9))$

$0 = \log_6(4x+36)$ ← ans.

↑ exp ↑ base

$6^0 = 4x+36$

$1 = 4x+36$

$-35 = 4x$

$\boxed{-\frac{35}{4} = x}$ or $\left(-\frac{35}{4}, 0\right)$ or $(-8.75, 0)$

General Properties of Logarithms:

If $c > 0$ and $c \neq 1$, then...

(i) $\log_c 1 = 0$

(ii) $\log_c c^x = x$

(iii) $c^{\log_c x} = x$

Did You Know?

The input value for a logarithm is called an argument. For example, in the expression $\log_6 1$, the argument is 1.

(i) $\log_5 1 = 0$ (ii) $\log_2 2^3 = 3$ (iii) $7^{\log_7 49} = 49$

$5^{\log_5 10} = 10$

Product Law of Logarithms

The logarithm of a product of numbers can be expressed as the sum of the logarithms of the numbers.

$$\log_c MN = \log_c M + \log_c N$$

Proof

Let $\log_c M = x$ and $\log_c N = y$, where M, N, and c are positive real numbers with $c \neq 1$.

Write the equations in exponential form as $M = c^x$ and $N = c^y$:

$MN = (c^x)(c^y)$

$MN = c^{x+y}$ Apply the product law of powers.

$\log_c MN = x + y$ Write in logarithmic form.

$\log_c MN = \log_c M + \log_c N$ Substitute for x and y.

① $\log_2 xyz = \log_2 x + \log_2 y + \log_2 z$

② $\log_2 6 + \log_2 5 = \log_2 (6 \times 5) = \log_2 30 \approx 4.907$

Quotient Law of Logarithms

The logarithm of a quotient of numbers can be expressed as the difference of the logarithms of the dividend and the divisor.

$$\log_c \frac{M}{N} = \log_c M - \log_c N$$

Proof

Let $\log_c M = x$ and $\log_c N = y$, where M, N, and c are positive real numbers with $c \neq 1$.

Write the equations in exponential form as $M = c^x$ and $N = c^y$:

$$\frac{M}{N} = \frac{c^x}{c^y}$$

$$\frac{M}{N} = c^{x-y} \qquad \text{Apply the quotient law of powers.}$$

$$\log_c \frac{M}{N} = x - y \qquad \text{Write in logarithmic form.}$$

$$\log_c \frac{M}{N} = \log_c M - \log_c N \qquad \text{Substitute for } x \text{ and } y.$$

① $\log_2 \frac{xy}{z} = \log_2 xy - \log_2 z = \log_2 x + \log_2 y - \log_2 z$

② $\log 400 - \log 4 = \log\left(\frac{400}{4}\right) = \log 100 = 2$

Power Law of Logarithms

The logarithm of a power of a number can be expressed as the exponent times the logarithm of the number.

$$\log_c M^P = P \log_c M$$

How could you prove the quotient law using the product law and the power law?

Proof

Let $\log_c M = x$, where M and c are positive real numbers with $c \neq 1$.

Write the equation in exponential form as $M = c^x$.

Let P be a real number.

$$M = c^x$$
$$M^P = (c^x)^P$$
$$M^P = c^{xP} \quad \text{Simplify the exponents.}$$
$$\log_c M^P = xP \quad \text{Write in logarithmic form.}$$
$$\log_c M^P = (\log_c M)P \quad \text{Substitute for } x.$$
$$\log_c M^P = P \log_c M$$

The laws of logarithms can be applied to logarithmic functions, expressions, and equations.

① $\log_2 \sqrt{8}$

$= \log_2 (8)^{1/2}$

$= \frac{1}{2} \log_2 8$

$= \frac{1}{2}(3)$

$= \frac{3}{2}$

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The logarithm of a product of numbers can be expressed as the sum of the logarithms of the numbers.

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Quotient Law of Logarithms

The logarithm of a quotient of numbers can be expressed as the difference of the logarithms of the dividend and the divisor.

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Power Law of Logarithms

The logarithm of a power of a number can be expressed as the exponen times the logarithm of the number.

$$\log_c M^P = P \log_c M$$

How could you prove the quotient law using the product law and the power law?

Homework

Finish Exercise 2

Example 1

Use the Laws of Logarithms to Expand Expressions

Write each expression in terms of individual logarithms of x, y, and z.

a) $\log_5 \frac{xy}{z}$

b) $\log_7 \sqrt[3]{x}$

c) $\log_6 \frac{1}{x^2}$

d) $\log \frac{x^3}{y\sqrt{z}}$

Example 2

Use the Laws of Logarithms to Evaluate Expressions

Use the laws of logarithms to simplify and evaluate each expression.

a) $\log_6 8 + \log_6 9 - \log_6 2$

b) $\log_7 7\sqrt{7}$

c) $2 \log_2 12 - \left(\log_2 6 + \frac{1}{3} \log_2 27\right)$

Example 3

Use the Laws of Logarithms to Simplify Expressions

Write each expression as a single logarithm in simplest form. State the restrictions on the variable.

a) $\log_7 x^2 + \log_7 x - \dfrac{5 \log_7 x}{2}$

b) $\log_5 (2x - 2) - \log_5 (x^2 + 2x - 3)$

Key Ideas

- Let P be any real number, and M, N, and c be positive real numbers with $c \neq 1$. Then, the following laws of logarithms are valid.

Name	Law	Description
Product	$\log_c MN = \log_c M + \log_c N$	The logarithm of a product of numbers is the sum of the logarithms of the numbers.
Quotient	$\log_c \frac{M}{N} = \log_c M - \log_c N$	The logarithm of a quotient of numbers is the difference of the logarithms of the dividend and divisor.
Power	$\log_c M^P = P \log_c M$	The logarithm of a power of a number is the exponent times the logarithm of the number.

- Many quantities in science are measured using a logarithmic scale. Two commonly used logarithmic scales are the decibel scale and the pH scale.

Homework

Do I really understand??...

a) Express the following as a single logarithm... $2\log_2 3^2 + \log_2 6 - 3\log_2 3$

b) Evaluate the following... $\log_2 (32)^{\frac{1}{3}}$

c) Express the following as a single logarithm... $\frac{1}{2}\left[(\log_5 a + 2\log_5 b) - 3\log_5 c\right]$

d) Express as a single logarithm in simplest form...

$$\frac{3}{4}\left[12(\log_b x^2 - 2\log_b x) + 8\log_b \sqrt{x} - 4\log_b \frac{1}{x^7}\right]$$