

Questions from Homework

$$\textcircled{3} \text{ f) } \underline{3x-2}, 3x-5, 3x-8, \underline{3x-11}, \underline{3x-14}$$

$$a = 3x-2 \quad t_n = 3x-2 + (n-1)(-3)$$

$$d = -3 \quad t_n = 3x-2-3n+3$$

$$\boxed{t_n = 3x-3n+1}$$

$$\text{g) } \frac{2}{x}, \frac{4}{x}, \frac{6}{x}, \frac{8}{x}, \frac{10}{x}$$

$$a = \frac{2}{x} \quad t_n = \frac{2}{x} + (n-1)\left(\frac{2}{x}\right)$$

$$d = \frac{2}{x} \quad t_n = \frac{2}{x} + \frac{2n}{x} - \frac{2}{x}$$

$$\boxed{t_n = \frac{2n}{x}}$$

$$\textcircled{4} \text{ b) } a = \frac{1}{2} \quad \frac{1}{2}, \frac{1}{6}, -\frac{1}{6}$$

$$d = -\frac{1}{3}$$

$$t_2 = \frac{1}{2} + (2-1)\left(-\frac{1}{3}\right) \quad \left| \quad t_3 = \frac{1}{2} + (3-1)\left(-\frac{1}{3}\right)\right.$$

$$= \frac{1}{2} + (1)\left(-\frac{1}{3}\right) \quad \left| \quad = \frac{1}{2} + 2\left(-\frac{1}{3}\right)\right.$$

$$= \frac{1}{2} - \frac{1}{3} \quad \left| \quad = \frac{1}{2} - \frac{2}{3}\right.$$

$$= \frac{3}{6} - \frac{2}{6} \quad \left| \quad = \frac{3}{6} - \frac{4}{6}\right.$$

$$= \boxed{\frac{1}{6}}$$

$$= \boxed{-\frac{1}{6}}$$

Geometric Sequences

Ex: 2, 4, 8, 16, 32

Sequences of numbers that follow a pattern of multiplying a fixed number from one term to the next are called geometric sequences.

- To find the next term, multiply the previous term by a common ratio.
- In the sequence 2, 4, 8, 16, 32 we are multiplying by 2.
- This common ratio is called "r" ($r = t_2/t_1$).
- The first term is still called "a" or " t_1 ".
- The second term is called " t_2 ".
- The last term or an indicated term is called " t_n ". (general term)
- The position of a term or the number of terms is called "n".

Geometric Sequences

Remember $r = t_2/t_1$

Find "r" and the next term!

1, 2, 4, 8, ..., 16

$$r = \frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_4}{t_3} = \boxed{2}$$

16, -8, 4, -2, 1, ... $-\frac{1}{2}$

$$r = \frac{t_2}{t_1} = \frac{-8}{16} = \boxed{-\frac{1}{2}}$$

0.01, 0.06, 0.36, 2.16, ... 12.96

$$r = \frac{t_2}{t_1} = \frac{0.06}{0.01} = 6$$

Geometric Sequences

To find any given term in a geometric sequence we use the following formula:

$$t_n = ar^{n-1}$$

Examples

Find the indicated term

1. 3, 6, 12... a = 3 r = 2

$$\begin{aligned} t_7 &= (3)(2)^{7-1} \\ &= (3)(2)^6 \\ &= 3(64) \\ &= \boxed{192} \end{aligned}$$

2. 2, -1, $\frac{1}{2}$, $\frac{-1}{4}$... a = 2 r = $-\frac{1}{2}$

$$\begin{aligned} t_9 &= (2)\left(-\frac{1}{2}\right)^{9-1} \\ &= (2)\left(-\frac{1}{2}\right)^8 \\ &= 2\left(\frac{1}{256}\right) \xrightarrow{*} \left(-\frac{1}{2}\right)^8 = \frac{(-1)^8}{(2)^8} = \frac{1}{256} \\ &= \frac{2}{256} \\ &= \boxed{\frac{1}{128}} \end{aligned}$$

We can also determine the number of terms in the sequence.

$$t_n = ar^{n-1}$$

How many terms are in the following sequences?
(Solve for "n")

$$9, 27, 81, \dots, 2187 \quad a=9 \quad r=3 \quad t_n=2187$$

$$\frac{2187}{9} = \frac{(9)(3)^{n-1}}{9}$$

$$243 = 3^{n-1}$$

$$3^5 = 3^{n-1}$$

$$5 = n-1$$

$$6 = n$$

$$* \frac{\log 243}{\log 3} = 5$$

$$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots, \frac{1}{1024}$$

$$a = \frac{1}{2} \quad r = \frac{1}{2} \quad t_n = \frac{1}{1024}$$

$$\frac{\frac{1}{1024}}{\frac{1}{2}} = \frac{(\frac{1}{2})(\frac{1}{2})^{n-1}}{\frac{1}{2}}$$

$$\frac{1}{512} = \left(\frac{1}{2}\right)^{n-1}$$

$$\left(\frac{1}{2}\right)^9 = \left(\frac{1}{2}\right)^{n-1}$$

$$9 = n-1$$

$$10 = n$$

$$* \frac{\log \left(\frac{1}{512}\right)}{\log \left(\frac{1}{2}\right)} = 9$$

Find "a", "r", and " t_n " for the following sequences!
 → Geometric

$$t_2 = 12, t_5 = 768$$

$$t_2 = ar^{2-1}$$

$$t_5 = ar^{5-1}$$

$$t_2 = ar$$

$$t_5 = ar^4$$

$$ar = 12$$

$$ar^4 = 768$$

2x2 system

$$\frac{ar^4 = 768}{ar = 12}$$

$$r^3 = 64$$

$$r = 4$$

$$ar = 12$$

$$a(4) = 12$$

$$4a = 12$$

$$a = 3$$

$$t_n = ar^{n-1}$$

$$t_n = (3)(4)^{n-1}$$

$$t_3 = 64, t_7 = 4$$

Homework

#1- #6

Ex: of when you can simplify

$$t_n = 2(8)^{n-1}$$

$$t_n = (2)(2^3)^{n-1}$$

Express 8 with base 2

$$t_n = (2)(2)^{3n-3}$$

Multiply 3 by (n-1)

$$t_n = 2^{1+3n-3}$$

Since multiplying powers with the same base add your exponents.

$$t_n = 2^{3n-2}$$