Questions from Homework

Ex 10.4

④ c) $36, 18, 9, \ldots, \frac{9}{128}$

$a = 36$

$r = \frac{18}{36} = \frac{1}{2}$

$t_n = \frac{9}{128}$

$t_n = ar^{n-1}$

$\frac{9}{\cancel{128}_{36}} = \frac{\cancel{(36)}\left(\frac{1}{2}\right)^{n-1}}{\cancel{36}}$

$\frac{1}{512} = \left(\frac{1}{2}\right)^{n-1}$

$* \left(\frac{1}{2}\right)^{9} = \left(\frac{1}{2}\right)^{n-1}$

$* \frac{\log(1/512)}{\log(1/2)}$

$9 = n-1$

$10 = n$

② d) $2^{50}, 2^{48}, 2^{46}, \ldots$

$a = 2^{50}$

$r = \frac{2^{48}}{2^{50}} = 2^{48-50} = 2^{-2}$

$t_{15} = (2^{50})(2^{-2})^{14}$

$t_{15} = (2^{50})(2^{-28})$

$t_{15} = 2^{22}$

⑥

$t_3 = \frac{1}{9}$	$t_7 = 9$
$t_3 = ar^{3-1}$	$t_7 = ar^{7-1}$
$t_3 = ar^2$	$t_7 = ar^6$
$\boxed{ar^2 = \frac{1}{9}}$	$\boxed{ar^6 = 9}$

$\frac{ar^6 = 9}{ar^2 = 1/9}$

$r^4 = 81$

$\boxed{r = \pm 3}$

$ar^2 = \frac{1}{9}$

$a(3)^2 = \frac{1}{9}$

$9a = \frac{1}{9}$

$\boxed{a = \frac{1}{81}}$

$\frac{1}{81}, \pm\frac{1}{27}, \frac{1}{9}, \pm\frac{1}{3}$

$\boxed{t_4 = \pm\frac{1}{3}}$

④ b) $16, -8, 4, \ldots, \frac{1}{4}$

$n = ?$

$a = 16$

$r = \frac{-8}{16} = \frac{-1}{2}$

$t_n = \frac{1}{4}$

$t_n = ar^{n-1}$

$\frac{1}{4} = (16)\left(\frac{-1}{2}\right)^{n-1}$

$\frac{1}{64} = \left(\frac{-1}{2}\right)^{n-1}$

Get common base.

$* \left(\frac{-1}{\cancel{2}}\right)^{6} = \left(\frac{-1}{\cancel{2}}\right)^{n-1}$

$\frac{\log(1/64)}{\log(1/2)} = 6$

$6 = n-1$

$\boxed{7 = n}$

⑥

$t_{10} = 2560$	$t_5 = 80$
$t_{10} = ar^{10-1}$	$t_5 = ar^{5-1}$
$t_{10} = ar^9$	$t_5 = ar^4$
$2560 = ar^9$	$80 = ar^4$
$\boxed{ar^9 = 2560}$	$\boxed{ar^4 = 80}$

$\frac{ar^9 = 2560}{ar^4 = 80}$

$r^5 = 32$

$\boxed{r = 2}$

$ar^4 = 80$

$a(2)^4 = 80$

$16a = 80$

$\boxed{a = 5}$

$t_{12} = (5)(2)^{12-1}$

$t_{12} = (5)(2)^{11}$

$t_{12} = (5)(2048) = \boxed{10240}$

Arithmetic Series

Series: The sum of the terms of a sequence. The sum is usually finite: 1+2+3+4+5. However it could be infinite: 2+4+8+16+... You can find the sum of many finite series and certain types of infinite series by using formulas.

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_n = \frac{n}{2}(a + t_n)$$

Ex: $2+5+8+11+\underline{\underline{14}}$

$t_5 = \underline{\underline{15}}$ $S_5 = 40$

↑

Sum of the first five terms

$t_4 = 11$ $S_4 = 26$

⋮

Find the sum of the first 100 terms of the arithmetic series 1+4+7+10+...

$a = 1$
$d = t_2 - t_1 = 3$
$n = 100$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_{100} = \frac{100}{2}[2(1) + (100-1)3]$$

$$S_{100} = 50(2 + 297)$$

$$S_{100} = 50(299)$$

$$S_{100} = 14950$$

Find the sum of the following series

$$\frac{1}{2}+1+\frac{3}{2}+2,\ldots+20$$

Hint: How many terms are there?

$a = ½$

$d = ½$

$t_{n = 20}$

$n =$

$$t_n = a+(n-1)d$$

$$20 = \frac{1}{2}+(n-1)\left(\frac{1}{2}\right)$$

$$20 = \cancel{\frac{1}{2}}+\frac{n}{2}-\cancel{\frac{1}{2}}$$

$$20 = \frac{n}{2}$$

$$40 = n$$

$$S_n = \frac{n}{2}(a+t_n)$$

$$S_{40} = \frac{40}{2}\left(\frac{1}{2}+\frac{20}{1}\right)$$

$$S_{40} = \overset{10}{\cancel{20}}\left(\frac{41}{\cancel{2}}\right)$$

$$\boxed{S_{40} = 410}$$

How many terms are in the series: 3+8+13+...+248 if its sum is 6275?

$a = 3$
$d = 5$
$S_n = 6275$
$t_n = 248$

$$S_n = \frac{n}{2}(a + t_n)$$

$$6275 = \frac{n}{2}(3 + 248)$$

$$6275 = \frac{251n}{2}$$

$$12550 = 251n$$

$$\boxed{50 = n}$$

Find the indicated sums of the following series:

S_{15} of 2+6+10....

$a=2$
$d=4$
$n=15$

$$S_{15} = \frac{15}{2}[2(2)+(15-1)4]$$
$$= \frac{15}{2}(4+56)$$
$$= \frac{15}{2}(60)$$
$$= 450$$

S_{20} of -15-10-5+...

$a=-15$
$d=5$
$n=20$

$$S_{20} = \frac{20}{2}[2(-15)+(20-1)5]$$
$$= 10(-30+95)$$
$$= 10(65)$$
$$= 650$$

Homework

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