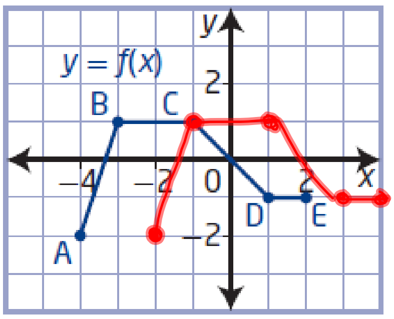
Warm-Up

8. Copy and complete the table.

Translation	Transformed Function	Transformation of Points	
vertical	$y = f(x) + 5$	$(x, y) \rightarrow (x, y + 5)$	
H	$y = f(x + 7)$	$(x, y) \rightarrow (x - 7, y)$	$h = -7$
H	$y = f(x - 3)$	$(x,y) \rightarrow (x+3, y)$	$h = 3$
V	$y = f(x) - 6$	$(x,y) \rightarrow (x, y-6)$	$K = -6$
horizontal and vertical	$y + 9 = f(x + 4)$	$(x,y) \rightarrow (x-4, y-9)$	$h = -4$ $K = -9$
horizontal and vertical	$y = f(x-4) - 6$	$(x, y) \rightarrow (x + 4, y - 6)$	$h = 4$ $K = -6$
H+V	$y = f(x+2) + 3$	$(x, y) \rightarrow (x - 2, y + 3)$	$h = -2$ $K = 3$
horizontal and vertical	$y = f(x - h) + k$	$(x,y) \rightarrow (x+h, y+K)$	

Questions from Homework

2

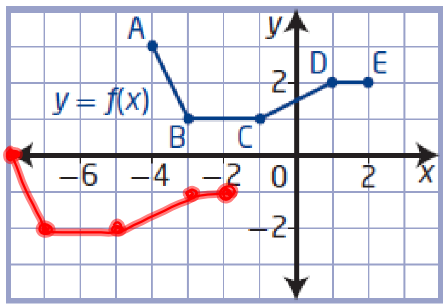


2 b) $h(x) = f(x-2)$ $h=2$

$(x, y) \longrightarrow (x+2, y)$

A	(−4, −2)	A'	(−2, −2)
B	(−3, 1)	B'	(−1, 1)
C	(−1, 1)	C'	(1, 1)
D	(1, −1)	D'	(3, −1)
E	(2, −1)	E'	(4, −1)

4



$h=-4$ $K=-3$

a) $s(x) = f(x+4) - 3$

$(x, y) \longrightarrow (x-4, y-3)$

A(−4, 3)	A' (−8, 0)
B(−3, 1)	B' (−7, −2)
C(−1, 1)	C' (−5, −2)
D(1, 2)	D' (−3, −1)
E(2, 2)	E' (−2, −1)

Transformations:

New Functions From Old Functions

Translations

Stretches

Reflections

Reflections and Stretches

Focus on...

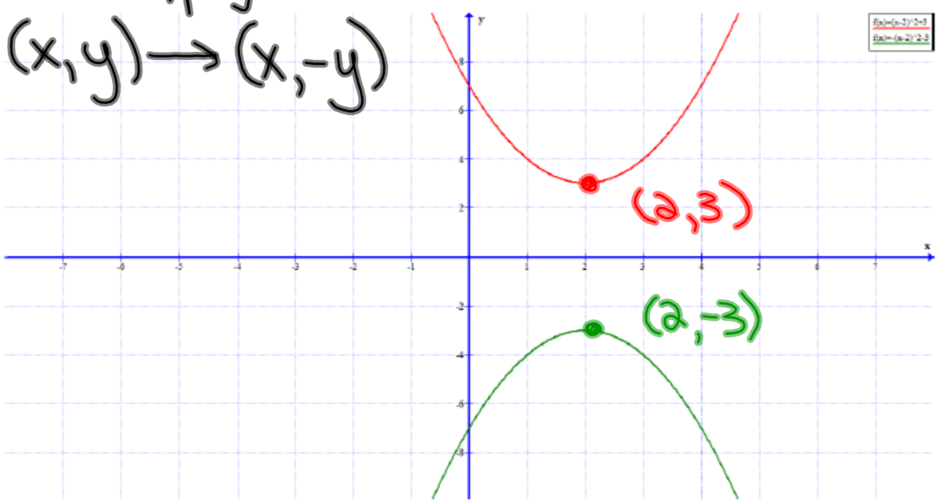
- developing an understanding of the effects of reflections on the graphs of functions and their related equations
- developing an understanding of the effects of vertical and horizontal stretches on the graphs of functions and their related equations

A **reflection** of a graph creates a mirror image in a line called the line of reflection. Reflections, like translations, do not change the shape of the graph. However, unlike translations, reflections may change the orientation of the graph.

- When the output of a function $y = f(x)$ is multiplied by -1, the result, $y = -f(x)$, is a reflection of the graph in the x-axis. (vertical reflection)

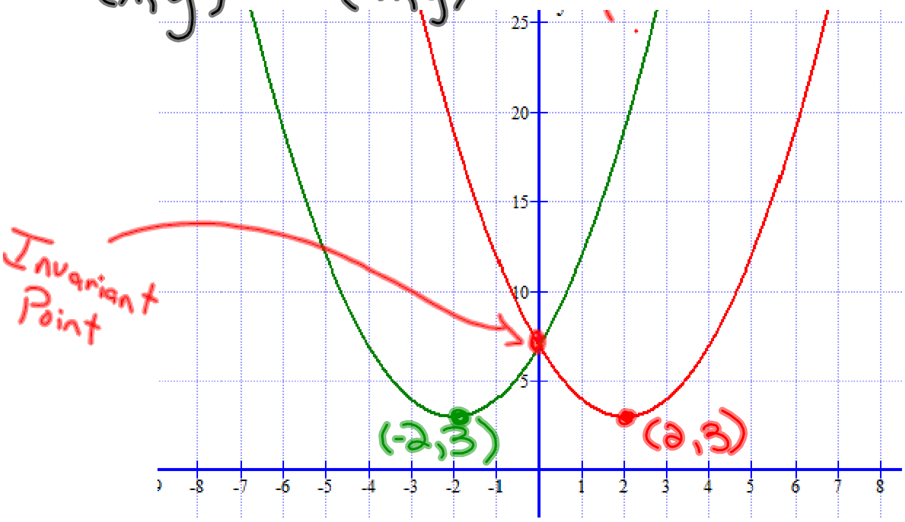
Mapping

$(x, y) \rightarrow (x, -y)$



- When the input of a function $y = f(x)$ is multiplied by -1, the result, $y = f(-x)$, is a reflection of the graph in the y-axis. (horizontal reflection)

Mapping:
$(x, y) \rightarrow (-x, y)$



invariant point

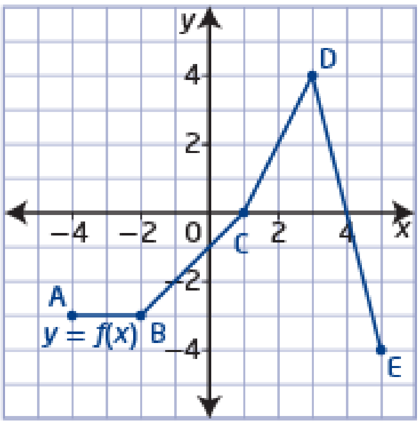
- a point on a graph that remains unchanged after a transformation is applied to it
- any point on a curve that lies on the line of reflection is an invariant point

Remember...

- When the output of a function $y = f(x)$ is multiplied by -1, the result, $y = -f(x)$, is a reflection of the graph in the x-axis.

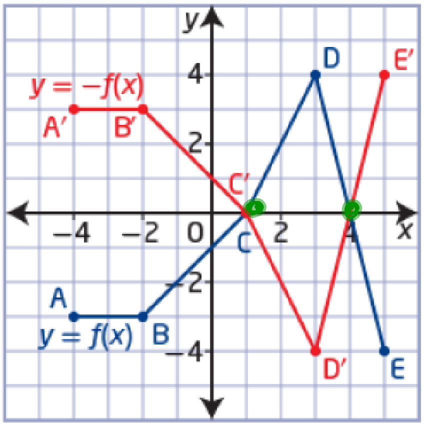
- Sketch $y = -f(x)$ on the axis below

- Invariant Points



$(x,y) \rightarrow (x,-y)$

A(-4,-3)	A'(-4,3)
B(-2,-3)	B'(-2,3)
C(1,0)	C'(1,0)
D(3,4)	D'(3,-4)
E(5,-4)	E'(5,4)



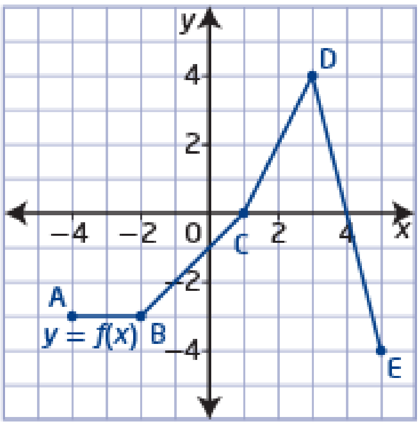
Remember...

- When the input of a function $y = f(x)$ is multiplied by -1, the result, $y = f(-x)$, is a reflection of the graph in the y-axis.

- Sketch $y = f(-x)$ on the axis below

• Invariant Point

$(x,y) \rightarrow (-x,y)$



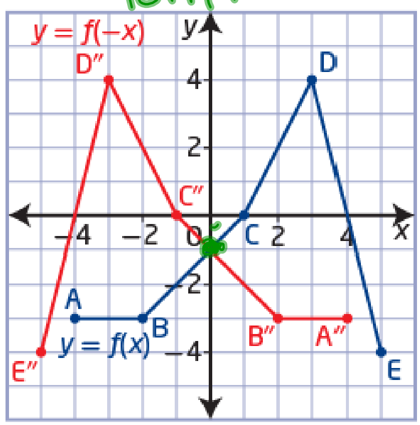
A(-4,-3) A'(4,-3)

B(-2,-3) B'(2,-3)

C(1,0) C'(-1,0)

D(3,4) D'(-3,4)

E(5,-4) E'(-5,-4)



Questions from Homework

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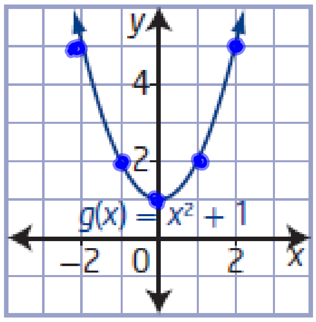
3. Consider each graph of a function.

- Copy the graph of the function and sketch its reflection in the x-axis on the same set of axes. (Vertical Reflection)
- State the equation of the reflected function in simplified form.
- State the domain and range of each function.

b)



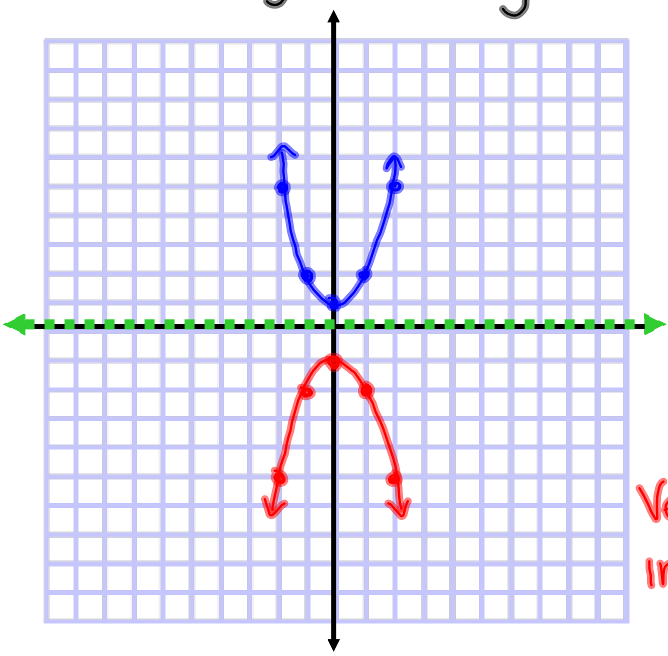
$g(x) = x^2 + 1$

x	y
-2	5
-1	2
0	1
1	2
2	5

$(x, y) \longrightarrow (x, -y)$

$h(x) = -(x^2 + 1)$

x	y
-2	-5
-1	-2
0	-1
1	-2
2	-5



Vertical reflection in the x-axis

$g(x) = x^2 + 1$

Domain: $\{x \mid x \in R\}$

Range: $\{y \mid y \geq 1, y \in R\}$

$h(x) = -(x^2 + 1) = -x^2 - 1$

Domain: $\{x \mid x \in R\}$

Range: $\{y \mid y \leq -1, y \in R\}$

Vertical and Horizontal Stretches

A **stretch**, unlike a translation or a reflection, changes the shape of the graph. However, like translations, stretches do not change the orientation of the graph.

- When the output of a function $y = f(x)$ is multiplied by a non-zero constant a, the result, $y = af(x)$ or $\frac{y}{a} = f(x)$, is a vertical stretch of the graph about the x-axis by a factor of $|a|$. If $a < 0$, then the graph is also reflected in the x-axis.
- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b, the result, $y = f(bx)$, is a horizontal stretch of the graph about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

stretch

- a transformation in which the distance of each x-coordinate or y-coordinate from the line of reflection is multiplied by some scale factor
- scale factors between 0 and 1 result in the point moving closer to the line of reflection; scale factors greater than 1 result in the point moving farther away from the line of reflection

ex: $\frac{1}{2}, \frac{2}{5}, 0.\overline{3}$
compression

ex: 2, 5, 16
stretch

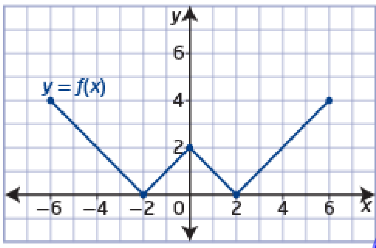
Vertical Stretch or Compression...

- When the output of a function $y = f(x)$ is multiplied by a non-zero constant a, the result, $y = af(x)$ or $\frac{y}{a} = f(x)$, is a vertical stretch of the graph about the x-axis by a factor of $|a|$. If $a < 0$, then the graph is also reflected in the x-axis. (vertically) "a" is negative

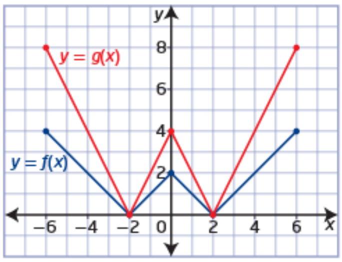
a) $g(x) = 2f(x)$ Vertical stretch by a factor of 2

$a = 2$

$(x, y) \rightarrow (x, 2y)$



(x, y)	$(x, 2y)$
(-6,4)	(-6,8)
(-2,0)	(-2,0) *
(0,2)	(0,4)
(2,0)	(2,0) *
(6,4)	(6,8)



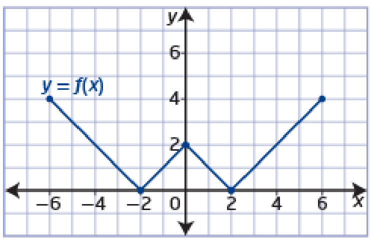
The invariant points are $(-2, 0)$ and $(2, 0)$.

For $f(x)$, the domain is $\{x \mid -6 \leq x \leq 6, x \in \mathrm{R}\}$, or $[-6, 6]$, and the range is $\{y \mid 0 \leq y \leq 4, y \in \mathrm{R}\}$, or $[0, 4]$.

For $g(x)$, the domain is $\{x \mid -6 \leq x \leq 6, x \in \mathrm{R}\}$, or $[-6, 6]$, and the range is $\{y \mid 0 \leq y \leq 8, y \in \mathrm{R}\}$, or $[0, 8]$.

b) $g(x) = \frac{1}{2}f(x)$

$a = \frac{1}{2}$



The invariant points are and

For $f(x)$, the domain is

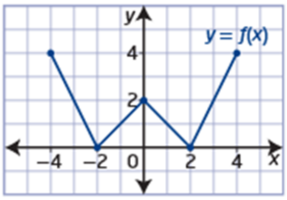
and the range is

For $g(x)$, the domain is
and the range is

Horizontal Stretch or Compression...

- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b, the result, $y = f(bx)$, is a horizontal stretch of the graph about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

a) $g(x) = f(2x)$

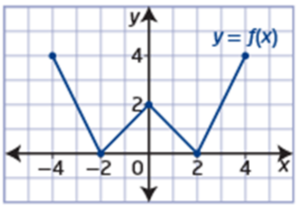


The invariant point is

For $f(x)$, the domain is
or and the range is
or

For $g(x)$, the domain is
or and the range is
or

b) $g(x) = f\left(\frac{1}{2}x\right)$



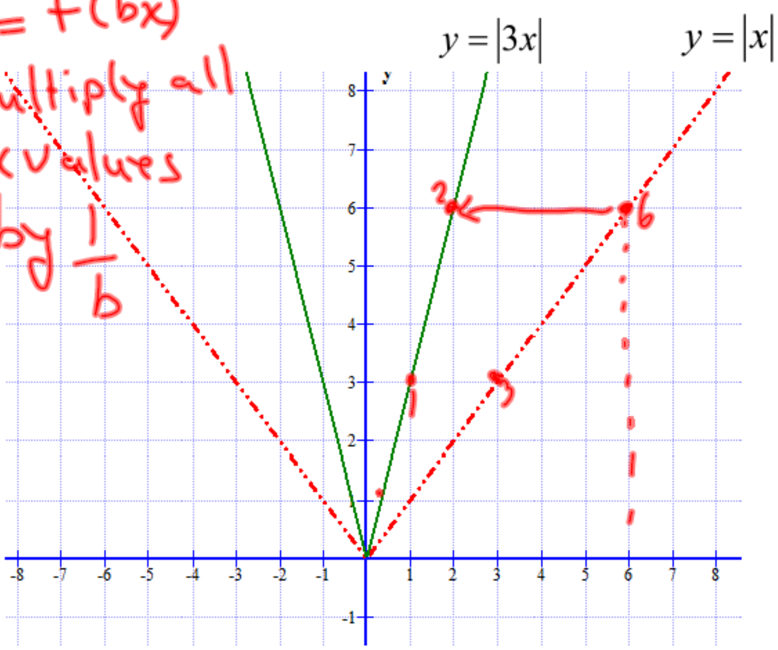
The invariant point is

For $f(x)$, the domain is
and the range is

For $g(x)$, the domain is
and the range is

Horizontal Stretch or Compression...

$y = f(bx)$
multiply all
x values
by $\frac{1}{b}$



Horizontal Stretch or Compression...

- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b, the result, $y = f(bx)$, is a horizontal stretch of the graph about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

$$y = -3f(-2x) + 7$$

Homework

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