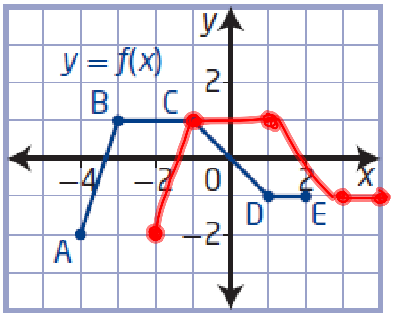
Warm-Up

8. Copy and complete the table.

Translation	Transformed Function	Transformation of Points		
vertical	$y = f(x) + 5$	$(x, y) \to (x, y + 5)$		
H	$y = f(x + 7)$	$(x, y) \to (x - 7, y)$	$h = -7$	
H	$y = f(x - 3)$	$(x,y) \to (x+3, y)$	$h = 3$	
V	$y = f(x) - 6$	$(x,y) \to (x, y-6)$	$K = -6$	
horizontal and vertical	$y + 9 = f(x + 4)$	$(x,y) \to (x-4, y-9)$	$h = -4$	$K = -9$
horizontal and vertical	$y = f(x-4) - 6$	$(x, y) \to (x + 4, y - 6)$	$h = 4$	$K = -6$
H+V	$y = f(x+2) + 3$	$(x, y) \to (x - 2, y + 3)$	$h = -2$	$K = 3$
horizontal and vertical	$y = f(x - h) + k$	$(x,y) \to (x+h, y+K)$		

Questions from Homework

②



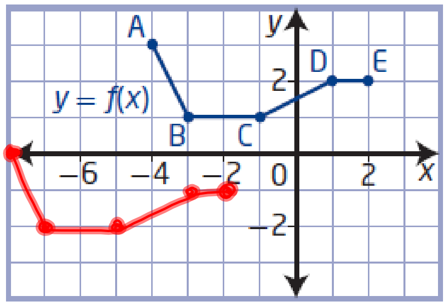
②b) $h(x) = f(x-2)$ $h=2$

$(x,y) \longrightarrow (x+2, y)$

A $(-4,-2)$	A' $(-2,-2)$
B $(-3,1)$	B' $(-1,1)$
C $(-1,1)$	C' $(1,1)$
D $(1,-1)$	D' $(3,-1)$
E $(2,-1)$	E' $(4,-1)$

$h=-4$ $k=-3$

④



a) $g(x) = f(x+4) - 3$

$(x,y) \longrightarrow (x-4, y-3)$

A$(-4,3)$	A'$(-8,0)$
B$(-3,1)$	B'$(-7,-2)$
C$(-1,1)$	C'$(-5,-2)$
D$(1,2)$	D'$(-3,-1)$
E$(2,2)$	E'$(-2,-1)$

Transformations:

New Functions From Old Functions

Translations

Stretches

Reflections

Reflections and Stretches

Focus on…

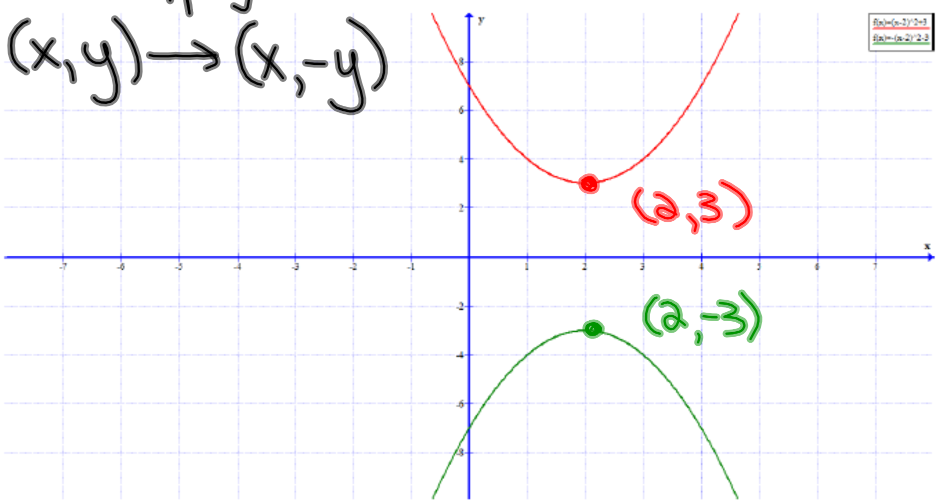
- developing an understanding of the effects of reflections on the graphs of functions and their related equations
- developing an understanding of the effects of vertical and horizontal stretches on the graphs of functions and their related equations

A **reflection** of a graph creates a mirror image in a line called the line of reflection. Reflections, like translations, do not change the shape of the graph. However, unlike translations, reflections may change the orientation of the graph.

- When the output of a function $y = f(x)$ is multiplied by -1, the result, $y = -f(x)$, is a reflection of the graph in the x-axis. (vertical reflection)

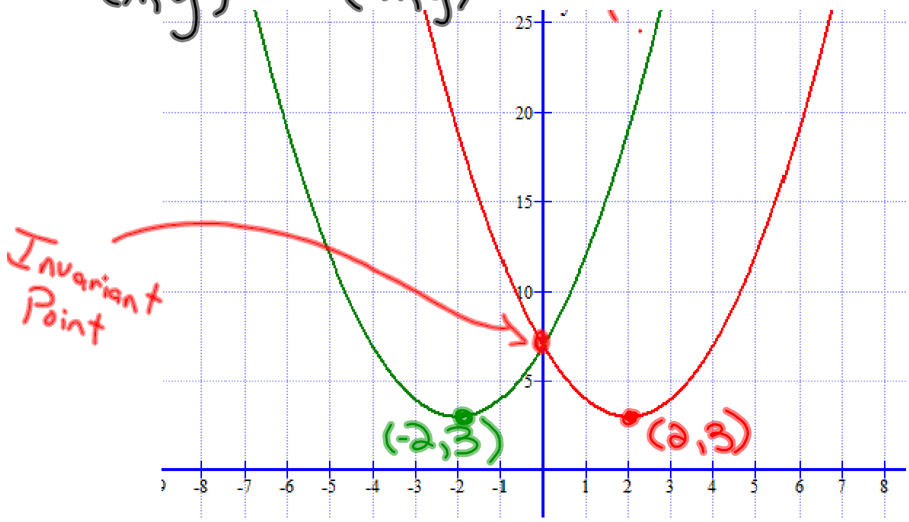
Mapping

$(x, y) \rightarrow (x, -y)$



- When the input of a function $y = f(x)$ is multiplied by -1, the result, $y = f(-x)$, is a reflection of the graph in the y-axis. (horizontal reflection)

Mapping:
$(x,y) \rightarrow (-x,y)$



invariant point

- a point on a graph that remains unchanged after a transformation is applied to it
- any point on a curve that lies on the line of reflection is an invariant point

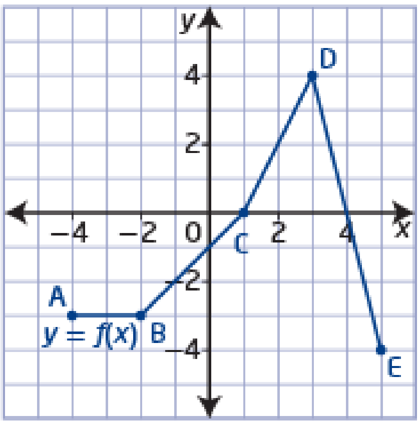
Remember...

- When the output of a function $y = f(x)$ is multiplied by -1, the result, $y = -f(x)$, is a reflection of the graph in the x-axis.

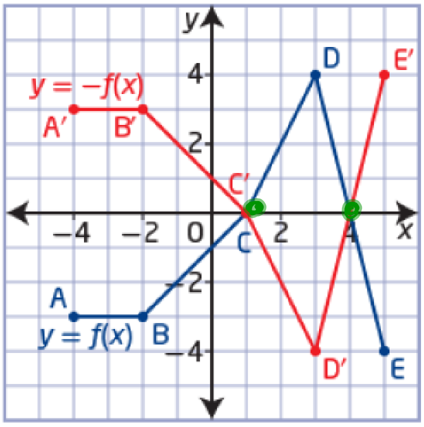
- Sketch $y = -f(x)$ on the axis below

- Invariant Points

$(x,y) \longrightarrow (x,-y)$



A(-4,-3)	A'(-4,3)
B(-2,-3)	B'(-2,3)
C(1,0)	C'(1,0)
D(3,4)	D'(3,-4)
E(5,-4)	E'(5,4)



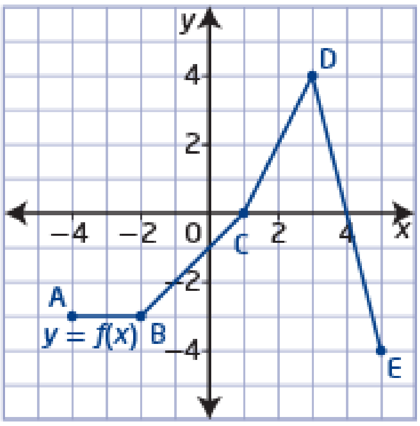
Remember...

- When the input of a function $y = f(x)$ is multiplied by -1, the result, $y = f(-x)$, is a reflection of the graph in the y-axis.

- Sketch $y = f(-x)$ on the axis below

- Invariant Point

$(x,y) \rightarrow (-x,y)$



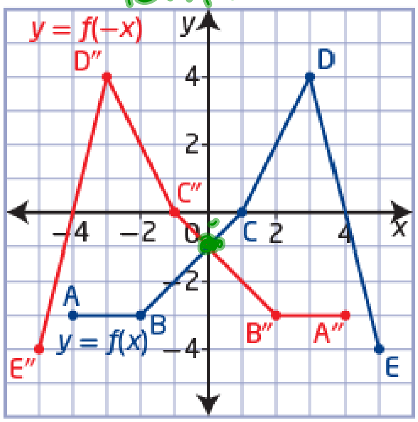
A(-4,-3) A'(4,-3)

B(-2,-3) B'(2,-3)

C(1,0) C'(-1,0)

D(3,4) D'(-3,4)

E(5,-4) E'(-5,-4)



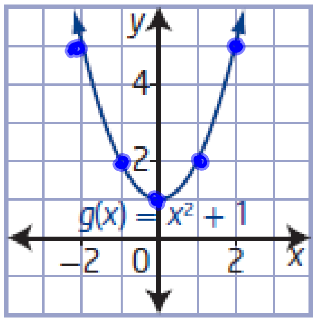
Questions from Homework

Page 28 #1, 3, and 4

3. Consider each graph of a function.

- Copy the graph of the function and sketch its reflection in the x-axis on the same set of axes. (Vertical Reflection)
- State the equation of the reflected function in simplified form.
- State the domain and range of each function.

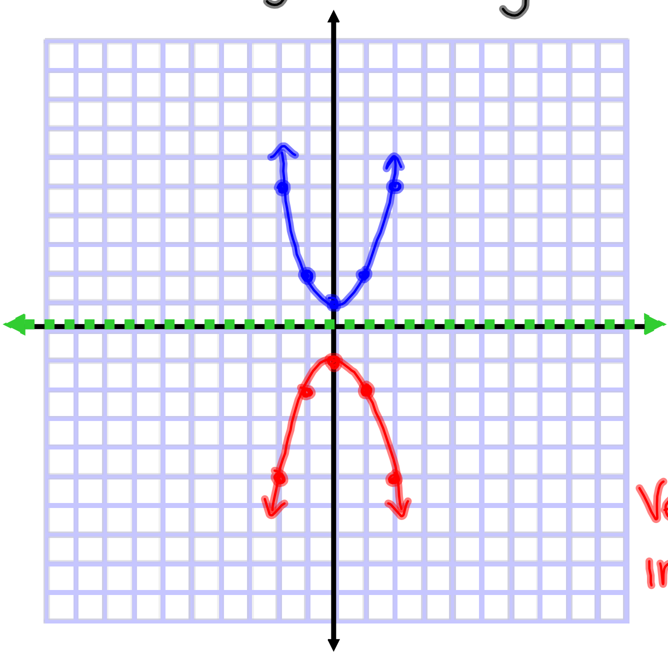
b)



$g(x)=x^2+1$

x	y
-2	5
-1	2
0	1
1	2
2	5

$(x,y)\longrightarrow(x,-y)$



$h(x)=-(x^2+1)$

x	y
-2	-5
-1	-2
0	-1
1	-2
2	-5

Vertical reflection in the x-axis

$g(x)=x^2+1$

Domain: $\{x \mid x\in R\}$

Range: $\{y \mid y\geq 1, y\in R\}$

$h(x)=-(x^2+1)=-x^2-1$

Domain: $\{x \mid x\in R\}$

Range: $\{y \mid y\leq -1, y\in R\}$

Vertical and Horizontal Stretches

A **stretch**, unlike a translation or a reflection, changes the shape of the graph. However, like translations, stretches do not change the orientation of the graph.

- When the output of a function $y = f(x)$ is multiplied by a non-zero constant a, the result, $y = af(x)$ or $\frac{y}{a} = f(x)$, is a vertical stretch of the graph about the x-axis by a factor of $|a|$. If $a < 0$, then the graph is also reflected in the x-axis.
- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b, the result, $y = f(bx)$, is a horizontal stretch of the graph about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

stretch

- a transformation in which the distance of each x-coordinate or y-coordinate from the line of reflection is multiplied by some scale factor
- scale factors between 0 and 1 result in the point moving closer to the line of reflection; scale factors greater than 1 result in the point moving farther away from the line of reflection

ex: $\frac{1}{2}, \frac{2}{5}, 0.\overline{3}$
compression

ex: 2, 5, 16
stretch

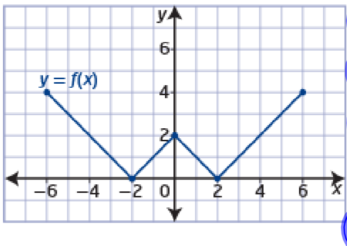
Vertical Stretch or Compression...

- When the output of a function $y = f(x)$ is multiplied by a non-zero constant a, the result, $y = af(x)$ or $\frac{y}{a} = f(x)$, is a vertical stretch of the graph about the x-axis by a factor of $|a|$. If $a < 0$, then the graph is also reflected in the x-axis. (vertically) "a" is negative

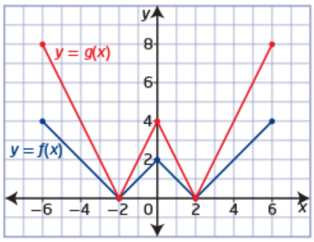
a) $g(x) = 2f(x)$ Vertical stretch by a factor of 2

$a = 2$

$(x, y) \rightarrow (x, 2y)$



(-6,4)	(-6,8)
(-2,0)	(-2,0) *
(0,2)	(0,4)
(2,0)	(2,0) *
(6,4)	(6,8)



The invariant points are (−2, 0) and (2, 0).

For $f(x)$, the domain is
$\{x \mid -6 \leq x \leq 6, x \in R\}$, or $[-6, 6]$,
and the range is
$\{y \mid 0 \leq y \leq 4, y \in R\}$, or $[0, 4]$.

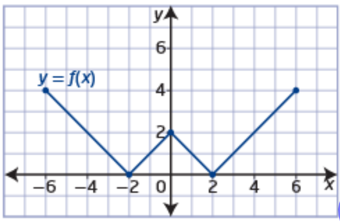
For $g(x)$, the domain is $\{x \mid -6 \leq x \leq 6, x \in R\}$, or $[-6, 6]$,
and the range is $\{y \mid 0 \leq y \leq 8, y \in R\}$, or $[0, 8]$.

"compression"

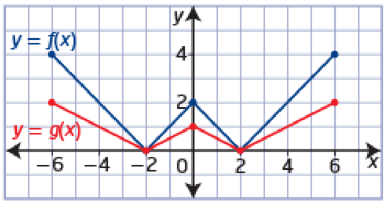
b) $g(x) = \frac{1}{2}f(x)$ Vertical stretch by a factor of $\frac{1}{2}$

$a = \frac{1}{2}$

$(x, y) \rightarrow (x, \frac{1}{2}y)$



(-6,4)	(-6,2)
(-2,0)	(-2,0) *
(0,2)	(0,1)
(2,0)	(2,0) *
(6,4)	(6,2)



The invariant points are (−2, 0) and (2, 0).

For $f(x)$, the domain is
$\{x \mid -6 \leq x \leq 6, x \in R\}$, or $[-6, 6]$,
and the range is
$\{y \mid 0 \leq y \leq 4, y \in R\}$, or $[0, 4]$.

For $g(x)$, the domain is $\{x \mid -6 \leq x \leq 6, x \in R\}$, or $[-6, 6]$,
and the range is $\{y \mid 0 \leq y \leq 2, y \in R\}$, or $[0, 2]$.

Vertical stretch factors change the range (y values)

Horizontal Stretch or Compression...

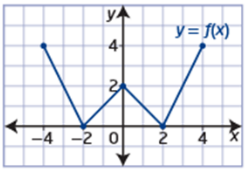
- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b, the result, $y = f(bx)$, is a horizontal stretch of the graph about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

(reciprocals)

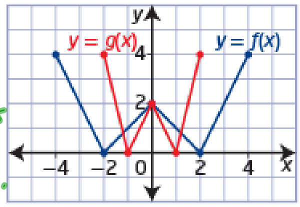
a) $g(x) = f(2x)$ Horizontal "compression" Stretch by a factor $\frac{1}{2}$

$b=2$

$(x,y) \to (\frac{1}{2}x, y)$



(-4,4)	(-2,4)
(-2,0)	(-1,0)
(0,2)	(0,2) *
(2,0)	(1,0)
(4,4)	(2,4)



The invariant point is (0, 2).

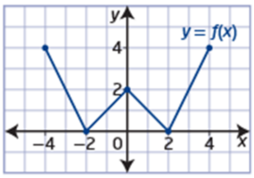
For $f(x)$, the domain is $\{x \mid -4 \le x \le 4, x \in \mathbb{R}\}$, or $[-4, 4]$, and the range is $\{y \mid 0 \le y \le 4, y \in$ or $[0, 4]$.

For $g(x)$, the domain is $\{x \mid -2 \le x \le 2, x \in \mathbb{R}\}$, or $[-2, 2]$, and the range is $\{y \mid 0 \le y \le 4, y \in \mathbb{R}\}$, or $[0, 4]$.

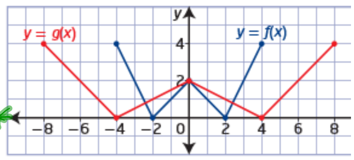
b) $g(x) = f\left(\frac{1}{2}x\right)$ Horizontal stretch by a factor $\frac{1}{1/2} = 2$

$b = \frac{1}{2}$

$(x,y) \to (2x, y)$



(-4,4)	(-8,4)
(-2,0)	(-4,0)
(0,2)	(0,2) *
(2,0)	(4,0)
(4,4)	(8,4)



The invariant point is (0, 2).

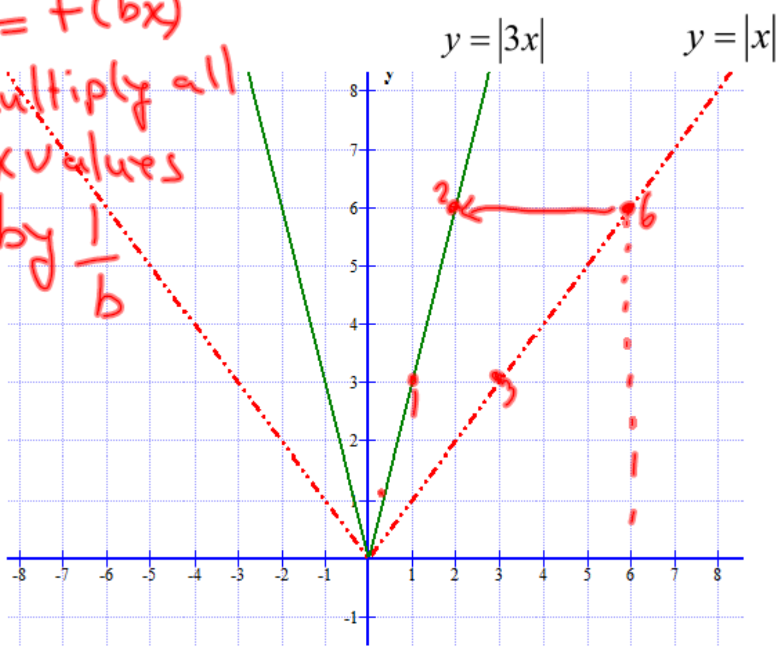
For $f(x)$, the domain is $\{x \mid -4 \le x \le 4, x \in \mathbb{R}\}$, or $[-4, 4]$, and the range is $\{y \mid 0 \le y \le 4, y \in \mathbb{R}\}$, or $[0, 4]$.

For $g(x)$, the domain is $\{x \mid -8 \le x \le 8, x \in \mathbb{R}\}$, or $[-8, 8]$, and the range is $\{y \mid 0 \le y \le 4, y \in \mathbb{R}\}$, or $[0, 4]$.

Horizontal stretch factors change the domain (x values)

Horizontal Stretch or Compression...

$y = f(bx)$
Multiply all x values by $\frac{1}{b}$



State the parameters a, b, h, and k and then describe the transformations

$y = -3f(-2x) + 7$

Homework

Page 28 # 2, 5, 6, 7