Questions from Homework
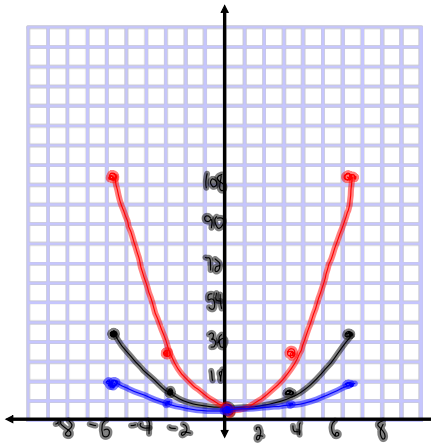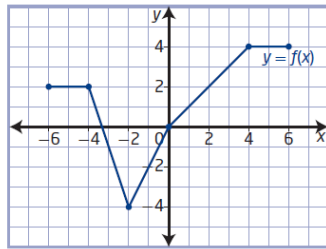
2. a) Copy and complete the table of values for the given functions.

$(x, 3y)$ $(x, \frac{1}{3}y)$

x	$f(x) = x^2$	$g(x) = 3f(x)$	$h(x) = \frac{1}{3}f(x)$
-6	36	(-6, 108)	(-6, 12)
-3	9	(-3, 27)	(-3, 3)
0	0	(0, 0)	(0, 0)
3	9	(3, 27)	(3, 3)
6	36	(6, 108)	(6, 12)

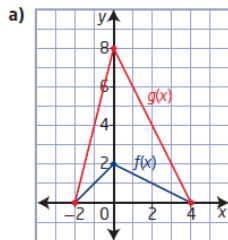**6.** The graph of the function $y = f(x)$ is vertically stretched about the x-axis by a factor of 2. $a = 2$

Original   Transformed

$y = f(x)$ $g(x) = 2f(x)$

$(x, y) \longrightarrow (x, ay)$

a) D: $[-6, 6]$ D: $[-6, 6]$
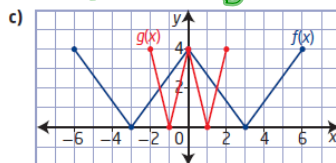
R: $[-4, 4]$ R: $[-8, 8]$

b) Vertical stretch only changes the range.

7. Describe the transformation that must be applied to the graph of $f(x)$ to obtain the graph of $g(x)$. Then, determine the equation of $g(x)$ in the form $y = af(bx)$.

a)

Vertical Stretch Factor of 4

$(x, y) \longrightarrow (x, 4y)$

$y = f(x)$ $g(x) = 4f(x)$

$a = 4$

(-2, 0)	(-2, 0)
(0, 2)	(0, 8)
(4, 0)	(4, 0)

horizontal compression by a factor of $\frac{1}{3}$

$b = 3$

c)

$(x, y) \longrightarrow (\frac{1}{3}x, y)$

$y = f(x)$ $g(x) = f(3x)$

(-6, 4)	(-2, 4)
(-3, 0)	(-1, 0)
(0, 4)	(0, 4)
(3, 0)	(1, 0)
(6, 4)	(2, 4)

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Warm-Up...

Given that (-2 , 5) is a point on the graph of $y = f(x)$, determine the coordinates of this point once the following transformations are applied...

(1) $y = 3f(x)$

$a = 3$ $(x,y) \longrightarrow (x, 3y)$

$(-2,5) \longrightarrow (-2,15)$

(2) $y = f\left(\dfrac{1}{3}x\right)$

$b = \dfrac{1}{3}$ $(x,y) \longrightarrow (-3x, y)$

$(-2,5) \longrightarrow (6,5)$

(3) $y = 4f\left[\dfrac{1}{2}(x + 5)\right] - 3$

$a = 4$ $(x,y) \longrightarrow (2x - 5, 4y - 3)$

$b = \dfrac{1}{2}$ $(-2,5) \longrightarrow (-9, 17)$

$h = -5$

$K = -3$

(4) $y - 5 = -2f(-2x + 6)$

factor

$y = -2f(-2x + 6) + 5$

$y = -2f[-2(x - 3)] + 5$

$a = -2$ $(x,y) \longrightarrow \left(-\dfrac{1}{2}x + 3, -2y + 5\right)$

$b = -2$

$h = 3$ $(-2,5) \longrightarrow (4, -5)$

$K = 5$

Transformations:

2. The function $y = f(x)$ is transformed to the function $g(x) = -3f(4x - 16) - 10$. Copy ~~factor~~ *factor* and complete the following statements by filling in the blanks.

The function $f(x)$ is transformed to the function $g(x)$ by a horizontal stretch about the [a] by a factor of [b]. It is vertically stretched about the [c] by a factor of [d]. It is reflected in the [e], and then translated [f] units to the right and [g] units down.

Summary of Transformations...

Transformations of the graphs of functions	
$f(x) + k$	shift $f(x)$ up k units
$f(x) - k$	shift $f(x)$ down k units
$f(x + h)$	shift $f(x)$ left h units
$f(x - h)$	shift $f(x)$ right h units
$f(-x)$	reflect $f(x)$ about the y-axis
$-f(x)$	reflect $f(x)$ about the x-axis
$af(x)$	When $0 < a < 1$ – vertical shrinking of $f(x)$ When $a > 1$ – vertical stretching of $f(x)$ **Multiply the y values by a**
$f(bx)$	When $0 < b < 1$ – horizontal stretching of $f(x)$ When $b > 1$ – horizontal shrinking of $f(x)$ **Divide the x values by b**

Transformations:

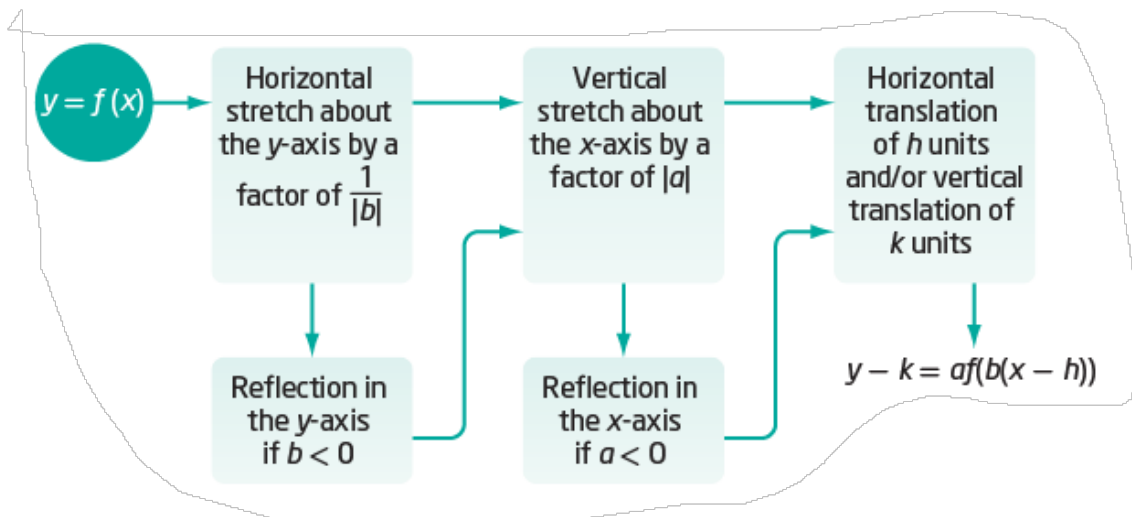$$y = f(x) \xrightarrow{\hspace{4cm}} y = af\big(b(x-h)\big) + k$$

Mapping Rule: $(x, y) \rightarrow \left(\dfrac{1}{b}x + h, ay + k \right)$

Important note for sketching...

   Transformations should be applied in following order:
1. Reflections
2. Stretches
3. Translations

Remember....RST

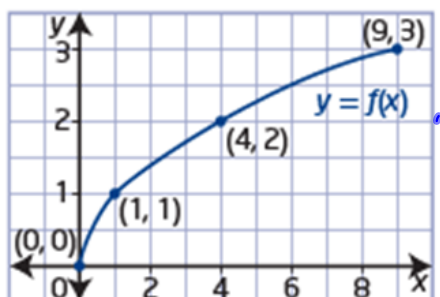| $y = f(x)$ →	Horizontal stretch about the y-axis by a factor of $\dfrac{1}{|b|}$	→	Vertical stretch about the x-axis by a factor of $|a|$	→	Horizontal translation of h units and/or vertical translation of k units
	↓		↓		↓
	Reflection in the y-axis if $b < 0$		Reflection in the x-axis if $a < 0$		$y - k = af(b(x-h))$

Example 1

Graph a Transformed Function

Describe the combination of transformations that must be applied to the function $y = f(x)$ to obtain the transformed function. Sketch the graph, showing each step of the transformation.
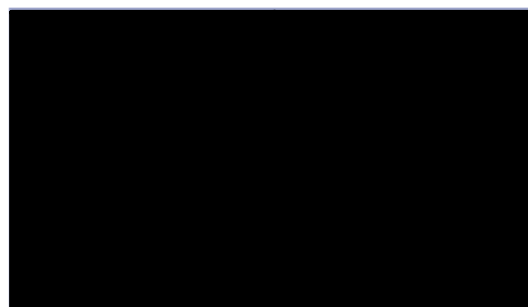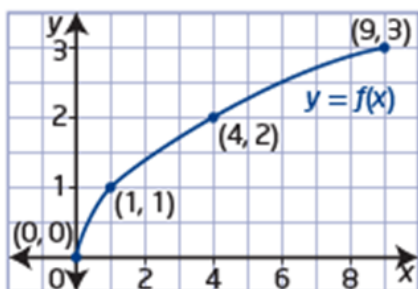
a) $y = 3f(2x)$

b) $y = f(3x + 6)$
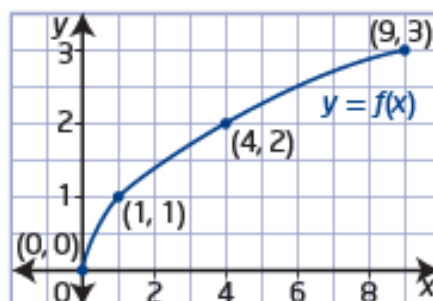
a) $y = 3f(2x)$

The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{2}$ and then vertically stretched about the x-axis by a factor of 3.

b) $y = f(3x + 6)$

The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{3}$ and then horizontally translated 2 units to the left.
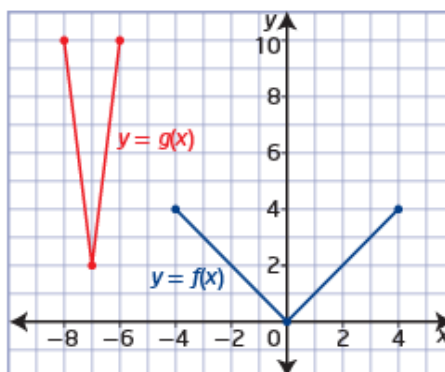
Homework

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Example 3

Write the Equation of a Transformed Function Graph

The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$. Explain your answer.
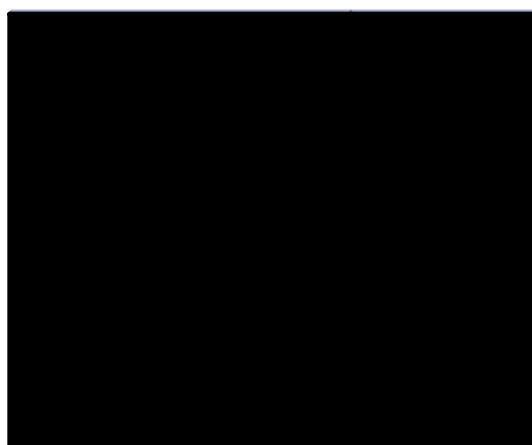
Solution

Locate key points on the graph of $f(x)$ and their image points on the graph of $g(x)$.

$(-4, 4) \rightarrow (-8, 10)$

$(0, 0) \rightarrow (-7, 2)$

$(4, 4) \rightarrow (-6, 10)$

The equation of the transformed function is ███████████████

███████████████

How could you use the mapping

$$(x, y) \rightarrow \left(\frac{1}{b}x + h,\ ay + k\right) \text{ to}$$

verify this equation?

17. The graph of the function $y = 2x^2 + x + 1$ is stretched vertically about the x-axis by a factor of 2, stretched horizontally about the y-axis by a factor of $\frac{1}{3}$, and translated 2 units to the right and 4 units down. Write the equation of the transformed function.