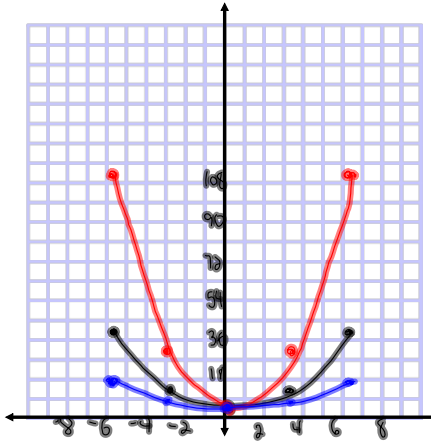


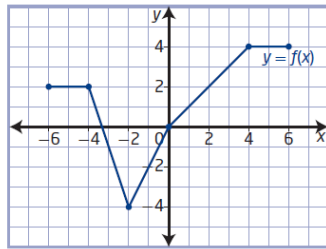
Questions from Homework

2. a) Copy and complete the table of values for the given functions.

x	$f(x) = x^2$	$g(x) = 3f(x)$	$h(x) = \frac{1}{3}f(x)$
-6	36	(-6, 108)	(-6, 12)
-3	9	(-3, 27)	(-3, 3)
0	0	(0, 0)	(0, 0)
3	9	(3, 27)	(3, 3)
6	36	(6, 108)	(6, 12)



6. The graph of the function $y = f(x)$ is vertically stretched about the x -axis by a factor of 2.

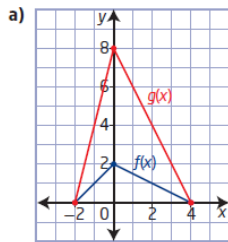


Original $y = f(x)$
 $(x, y) \rightarrow (x, 2y)$
 D: $[-6, 6]$
 R: $[-4, 4]$

Transformed $g(x) = 2f(x)$
 $(x, 2y)$
 D: $[-6, 6]$
 R: $[-8, 8]$

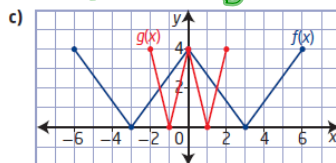
b) Vertical stretch only changes the range.

7. Describe the transformation that must be applied to the graph of $f(x)$ to obtain the graph of $g(x)$. Then, determine the equation of $g(x)$ in the form $y = af(bx)$.



Vertical Stretch factor of 4
 $\alpha = 4$
 $(x, y) \rightarrow (x, 4y)$
 $y = f(x)$
 $g(x) = 4f(x)$
 (-2, 0) (-2, 0)
 (0, 2) (0, 8)
 (4, 0) (4, 0)

horizontal compression by a factor of 1/3
 $b = 3$



$(x, y) \rightarrow (\frac{1}{3}x, y)$
 $y = f(x)$
 $g(x) = f(3x)$
 (-6, 4) (-2, 4)
 (-3, 0) (-1, 0)
 (0, 4) (0, 4)
 (3, 0) (1, 0)
 (6, 4) (2, 4)

Warm-Up...

Given that $(-2, 5)$ is a point on the graph of $y = f(x)$, determine the coordinates of this point once the following transformations are applied...

$$(1) y = \underline{3}f(x)$$

$$a = 3 \quad (x, y) \rightarrow (x, 3y)$$

$$(-2, 5) \rightarrow (-2, 15)$$

$$(2) y = f\left(\frac{1}{3}x\right)$$

$$b = \frac{1}{3} \quad (x, y) \rightarrow (3x, y)$$

$$(-2, 5) \rightarrow (6, 5)$$

$$(3) y = \underline{4}f\left[\frac{1}{2}(x \underline{+5})\right] \underline{-3}$$

$$a = 4 \quad (x, y) \rightarrow (2x - 5, 4y - 3)$$

$$b = \frac{1}{2} \quad (-2, 5) \rightarrow (-9, 17)$$

$$h = -5$$

$$k = -3$$

$$(4) y - 5 = -2f(-2x + 6)$$

Factor

$$y = -2f(-2x + 6) + 5$$

$$y = \underline{-2}f[\underline{-2}(x - 3)] + \underline{5}$$

$$a = -2 \quad (x, y) \rightarrow \left(\frac{1}{2}x + 3, -2y + 5\right)$$

$$b = -2 \quad (-2, 5) \rightarrow (4, -5)$$

$$h = 3$$

$$k = 5$$

Transformations:

2. The function $y = f(x)$ is transformed to the function $g(x) = -3f(4x - 16) - 10$. Copy and complete the following statements by filling in the blanks. $g(x) = \underline{-3}f[\underline{4}(x - \underline{4})] - \underline{\underline{10}}$ *Factor*

The function $f(x)$ is transformed to the function $g(x)$ by a horizontal stretch about the **a** by a factor of **b**. It is vertically stretched about the **c** by a factor of **d**. It is reflected in the **e**, and then translated **f** units to the right and **g** units down.

$$\begin{aligned} a &= -3 \\ b &= 4 \\ h &= 4 \\ k &= -10 \end{aligned}$$

- a) y-axis
 b) $\frac{1}{4}$
 c) x-axis
 d) 3
 e) x-axis
 f) 4
 g) 10

Summary of Transformations...

Transformations of the graphs of functions	
$f(x) + k$	shift $f(x)$ up k units
$f(x) - k$	shift $f(x)$ down k units
$f(x + h)$	shift $f(x)$ left h units
$f(x - h)$	shift $f(x)$ right h units
$f(-x)$	reflect $f(x)$ about the y-axis
$-f(x)$	reflect $f(x)$ about the x-axis
$af(x)$	When $0 < a < 1$ – vertical shrinking of $f(x)$
	When $a > 1$ – vertical stretching of $f(x)$ Multiply the y values by a
$f(bx)$	When $0 < b < 1$ – horizontal stretching of $f(x)$
	When $b > 1$ – horizontal shrinking of $f(x)$ Divide the x values by b

} vertical translation $(x, y) \rightarrow (x, y + k)$
 } horizontal translation $(x, y) \rightarrow (x + h, y)$
 horizontal $(x, y) \rightarrow (-x, y)$
 vertical $(x, y) \rightarrow (x, -y)$
 vertical stretch $(x, y) \rightarrow (x, ay)$
 horizontal stretch $(x, y) \rightarrow (\frac{1}{b}x, y)$

Transformations:

$$y = f(x) \longrightarrow y = af(b(x - h)) + k$$



Mapping Rule:

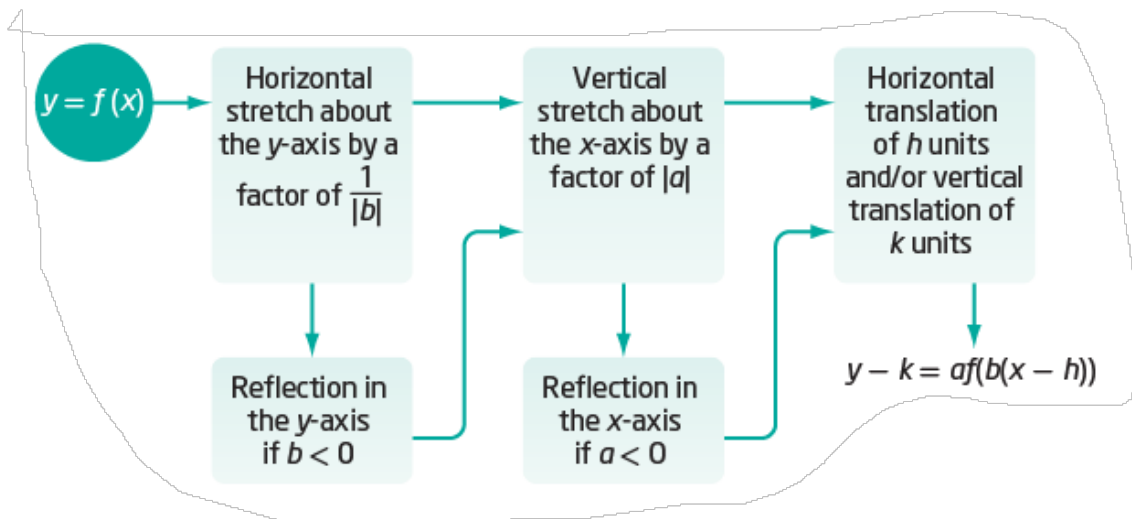
$$(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k \right)$$

Important note for sketching...

Transformations should be applied in following order:

1. Reflections
2. Stretches
3. Translations

Remember...RST



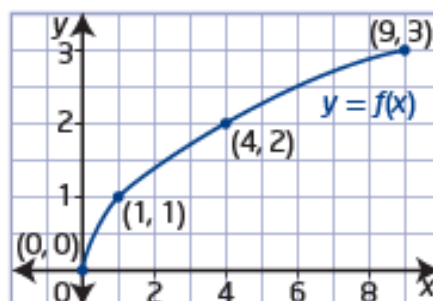
Example 1

Graph a Transformed Function

Describe the combination of transformations that must be applied to the function $y = f(x)$ to obtain the transformed function. Sketch the graph, showing each step of the transformation.

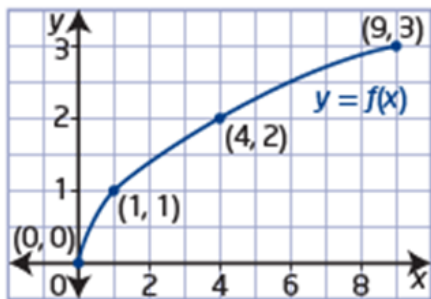
a) $y = 3f(2x)$

b) $y = f(3x + 6)$

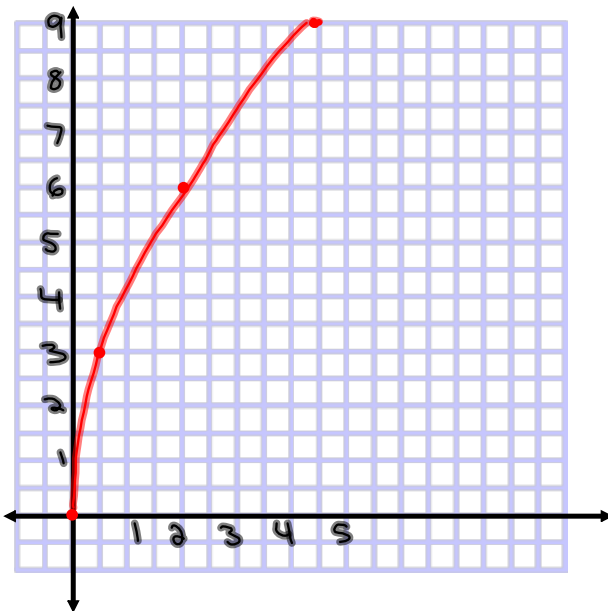


a) $y = 3f(2x)$ $a=3$ $b=2$ $h=0$ $k=0$

The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{2}$ and then vertically stretched about the x-axis by a factor of 3.

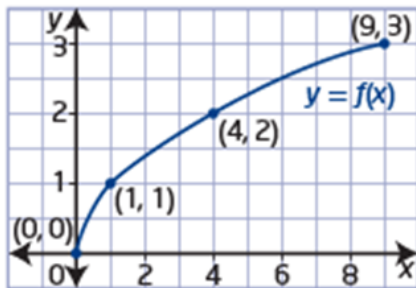


$(x,y) \rightarrow [\frac{1}{2}x, 3y]$
 $y = f(x)$ $y = 3f(2x)$
 $(0, 0)$ $(0, 0)$
 $(1, 1)$ $(\frac{1}{2}, 3)$
 $(4, 2)$ $(2, 6)$
 $(9, 3)$ $(\frac{9}{2}, 9)$



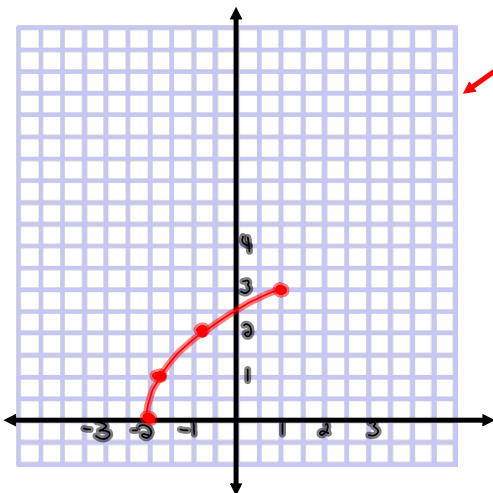
b) $y = f(3x + 6)$ (Factor out a 3)
 $y = f[3(x+2)] + 0$ $a=1$ $b=3$ $h=-2$ $k=0$

The graph of $y = f(x)$ is horizontally stretched about the y -axis by a factor of $\frac{1}{3}$ and then horizontally translated 2 units to the left.



$$(x, y) \rightarrow \left[\frac{1}{3}x - 2, y \right]$$

$y = f(x)$	$y = f[3(x-2)]$
(0, 0)	(-2, 0)
(1, 1)	($-\frac{5}{3}$, 1)
(4, 2)	($-\frac{2}{3}$, 2)
(9, 3)	(1, 3)



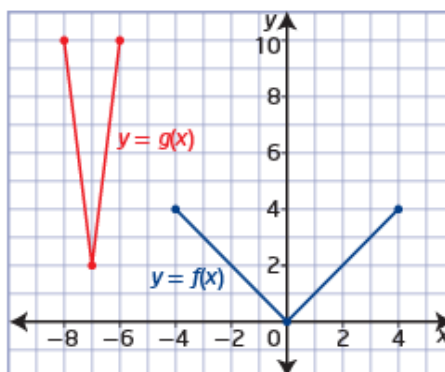
Homework

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Example 3

Write the Equation of a Transformed Function Graph

The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$. Explain your answer.



Solution

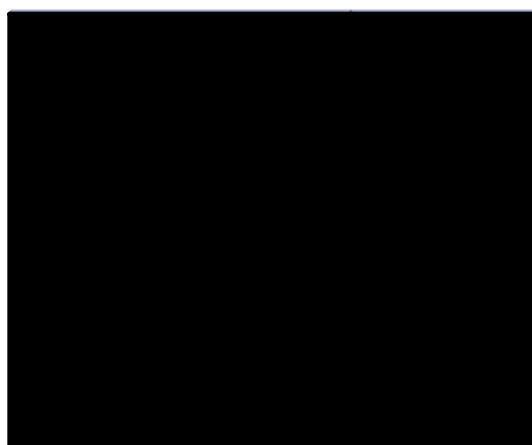
Locate key points on the graph of $f(x)$ and their image points on the graph of $g(x)$.

$$(-4, 4) \rightarrow (-8, 10)$$

$$(0, 0) \rightarrow (-7, 2)$$

$$(4, 4) \rightarrow (-6, 10)$$

The equation of the transformed function is XXXXXXXXXX



How could you use the mapping $(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$ to verify this equation?

17. The graph of the function $y = 2x^2 + x + 1$ is stretched vertically about the x -axis by a factor of 2, stretched horizontally about the y -axis by a factor of $\frac{1}{3}$, and translated 2 units to the right and 4 units down. Write the equation of the transformed function.

is stretched vertically about the x -axis by a factor of 2, stretched horizontally about the y -axis by a factor of $\frac{1}{3}$, and translated 2 units to the right and 4 units down. Write the equation of the transformed function.