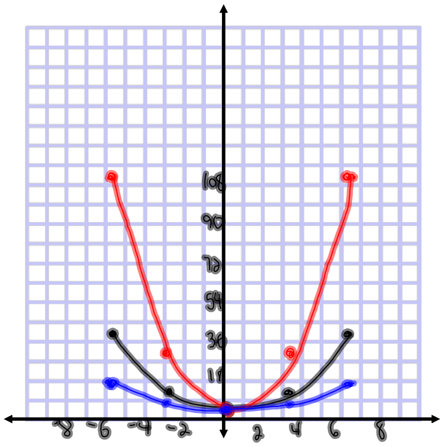
Questions from Homework

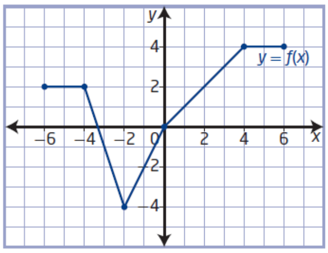
2. a) Copy and complete the table of values for the given functions.

$(x, 3y)$ $\left(x, \frac{1}{3}y\right)$

x	$f(x) = x^2$	$g(x) = 3f(x)$	$h(x) = \frac{1}{3}f(x)$
−6	36)	(-6,108)	(-6,12)
−3	9)	(-3,27)	(-3,3)
0	0)	(0,0)	(0,0)
3	9)	(3,27)	(3,3)
6	36)	(6,108)	(6,12)



6. The graph of the function $y = f(x)$ is ~~vertically stretched~~ about the x-axis by a factor of 2. $a = 2$



Original
$y = f(x)$
$(x, y) \longrightarrow (x, 2y)$

Transformed
$g(x) = 2f(x)$

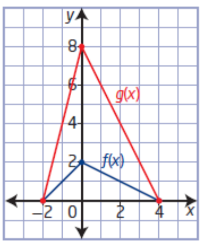
a) D: $[-6, 6]$ D: $[-6, 6]$

R: $[-4, 4]$ R: $[-8, 8]$

b) Vertical stretch only changes the range.

7. Describe the transformation that must be applied to the graph of $f(x)$ to obtain the graph of $g(x)$. Then, determine the equation of $g(x)$ in the form $y = af(bx)$.

a)



Vertical stretch by a factor of 4
$a = 4$

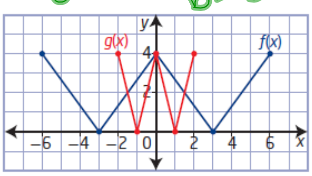
$(x, y) \longrightarrow (x, 4y)$

$y = f(x)$	$g(x) = 4f(x)$
(-2,0)	(-2,0)
(0,2)	(0,8)
(4,0)	(4,0)

horizontal compression by a factor of $\frac{1}{3}$
$b = 3$

$(x, y) \longrightarrow \left(\frac{1}{3}x, y\right)$

c)



$y = f(x)$	$g(x) = f(3x)$
(-6,4)	(-2,4)
(-3,0)	(-1,0)
(0,4)	(0,4)
(3,0)	(1,0)
(6,4)	(2,4)

Warm-Up...

Given that (-2 , 5) is a point on the graph of $y = f(x)$, determine the coordinates of this point once the following transformations are applied...

(1) $y = 3f(x)$

$a=3$ $(x,y) \rightarrow (x, 3y)$

$(-2,5) \rightarrow (-2,15)$

(2) $y = f\left(-\frac{1}{3}x\right)$

$b = -\frac{1}{3}$ $(x,y) \rightarrow (-3x, y)$

$(-2,5) \rightarrow (6,5)$

(3) $y = 4f\left[\frac{1}{2}(x+5)\right] - 3$

$a=4$ $(x,y) \rightarrow (2x-5, 4y-3)$

$b=\frac{1}{2}$ $(-2,5) \rightarrow (-9,17)$

$h=-5$

$K=-3$

(4) $y - 5 = -2f(-2x+6)$

Factor

$y = -2f(-2x+6) + 5$

$y = -2f[-2(x-3)] + 5$

$a=-2$ $(x,y) \rightarrow (-\frac{1}{2}x+3, -2y+5)$

$b=-2$ $(-2,5) \rightarrow (4,-5)$

$h=3$

$K=5$

Transformations:

2. The function $y = f(x)$ is transformed to the function $g(x) = -3f(4x - 16) - 10$. Copy and complete the following statements by filling in the blanks. $g(x) = -3f[4(x-4)] - 10$ factor

$a = -3$
$b = 4$
$h = 4$
$k = -10$

The function $f(x)$ is transformed to the function $g(x)$ by a horizontal stretch about the a by a factor of b. It is vertically stretched about the c by a factor of d. It is reflected in the e, and then translated f units to the right and g units down.

a) y-axis
b) $\frac{1}{4}$
c) x-axis
d) 3
e) x-axis
f) 4
g) 10

Summary of Transformations...

Transformations of the graphs of functions			
$f(x)+k$	shift $f(x)$ up k units	vertical translation	$(x,y) \to (x, y+k)$
$f(x)-k$	shift $f(x)$ down k units		
$f(x+h)$	shift $f(x)$ left h units	horizontal translation	$(x,y) \to (x+h, y)$
$f(x-h)$	shift $f(x)$ right h units		
$f(-x)$	reflect $f(x)$ about the y-axis	horizontal	$(x,y) \to (-x,y)$
$-f(x)$	reflect $f(x)$ about the x-axis	vertical	$(x,y) \to (x,-y)$
$af(x)$	When $0 < a < 1$ – vertical shrinking of $f(x)$ When $a > 1$ – vertical stretching of $f(x)$ **Multiply the y values by a**	vertical stretch	$(x,y) \to (x, ay)$
$f(bx)$	When $0 < b < 1$ – horizontal stretching of $f(x)$ When $b > 1$ – horizontal shrinking of $f(x)$ **Divide the x values by b**	horizontal stretch	$(x,y) \to (\frac{1}{b}x, y)$

Transformations:

$$y = f(x) \longrightarrow y = af\left(b(x-h)\right) + k$$

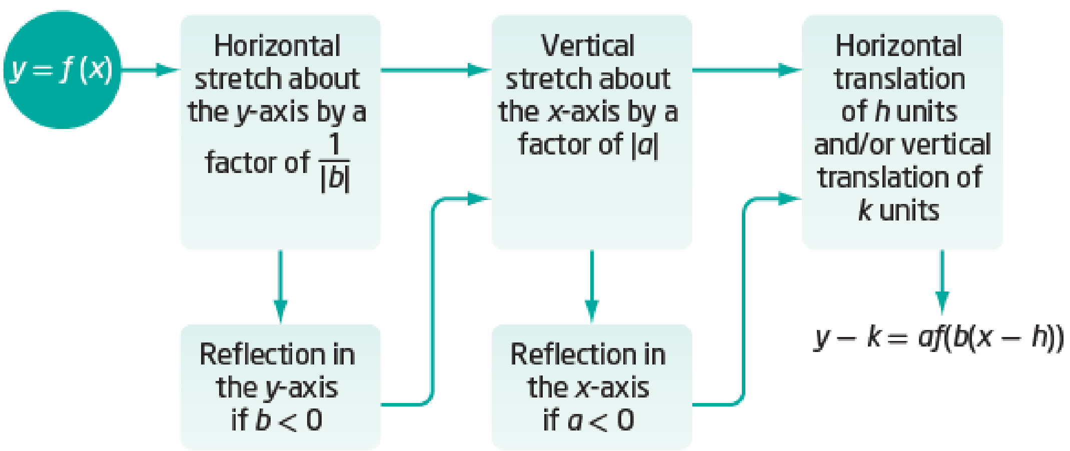
Mapping Rule: $$(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$$

Important note for sketching...

Transformations should be applied in following order:

1. Reflections
2. Stretches
3. Translations

Remember....RST



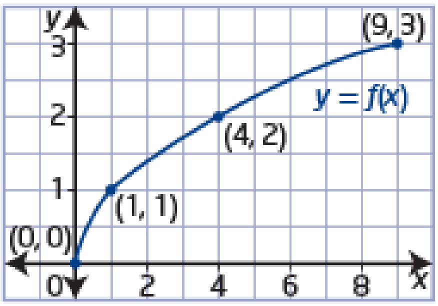
Example 1

Graph a Transformed Function

Describe the combination of transformations that must be applied to the function $y = f(x)$ to obtain the transformed function. Sketch the graph, showing each step of the transformation.

a) $y = 3f(2x)$

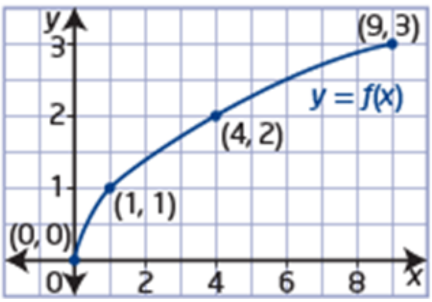
b) $y = f(3x + 6)$



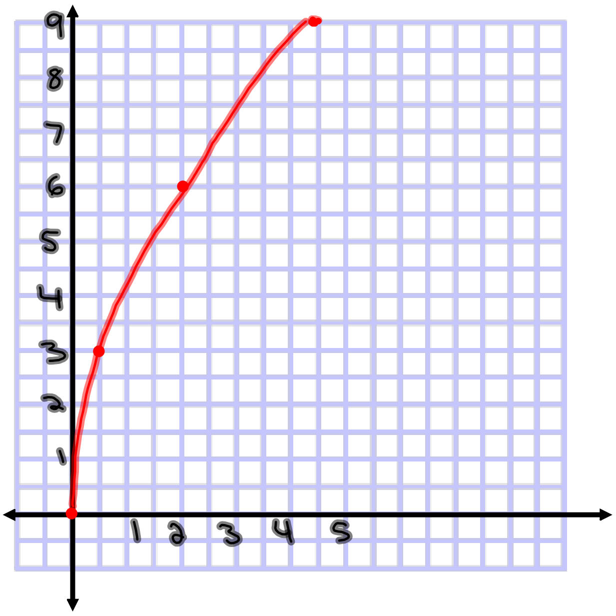
a) $y = 3f(2x)$ $a=3$ $b=2$ $h=0$ $k=0$

The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{2}$ and then vertically stretched about the x-axis by a factor of 3.

$(x,y) \longrightarrow \left[\frac{1}{2}x, 3y\right]$



$y=f(x)$	$y=3f(2x)$
(0, 0)	(0, 0)
(1, 1)	$(\frac{1}{2}, 3)$
(4, 2)	(2, 6)
(9, 3)	$(\frac{9}{2}, 9)$

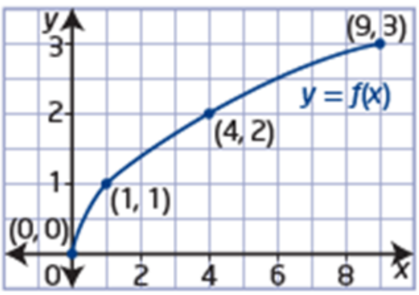


b) $y = f(3x + 6)$ (Factor out a 3)

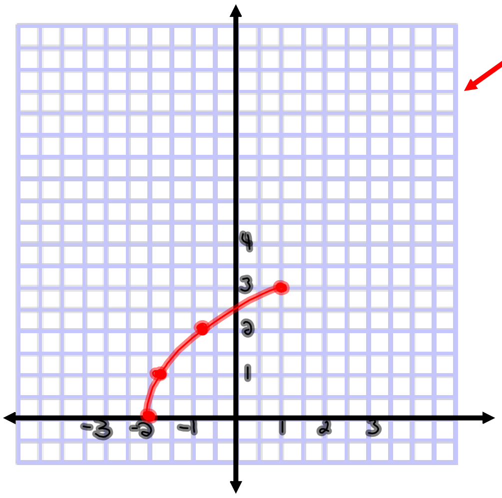
$y = 1f[3(x+2)] + 0$ $a=1$ $b=3$ $h=-2$ $k=0$

The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{3}$ and then horizontally translated 2 units to the left.

$(x, y) \longrightarrow \left[\frac{1}{3}x - 2,\ y\right]$



$y=f(x)$	$y=f[3(x-2)]$
(0, 0)	(-2, 0)
(1, 1)	$\left(-\frac{5}{3}, 1\right)$
(4, 2)	$\left(-\frac{2}{3}, 2\right)$
(9, 3)	(1, 3)



Questions From Homework

3. Copy and complete the table by describing the transformations of the given functions, compared to the function $y = f(x)$.

Function	Reflections	Vertical Stretch Factor	Horizontal Stretch Factor	Vertical Translation	Horizontal Translation
$y - 4 = f(x - 5)$				4	5
$y + 5 = 2f(3x)$		2	$\frac{1}{3}$	-5	
$y = \frac{1}{2}f\left(\frac{1}{2}(x - 4)\right)$		$\frac{1}{2}$	2		4
$y + 2 = -3f(2(x + 2))$	vertical reflection in x-axis	3	$\frac{1}{2}$	-2	-2

$\rightarrow (x,y) \rightarrow \left[\frac{1}{2}x - 2, -3y - 2\right]$

6. The key point $(-12, 18)$ is on the graph of $y = f(x)$. What is its image point under each transformation of the graph of $f(x)$?

d) $y = -2f\left(-\frac{2}{3}x - 6\right) + 4$ ← Factor out a $-\frac{2}{3}$

$y = -2f\left[-\frac{2}{3}(x + 9)\right] + 4$

$a = -2 \quad b = -\frac{2}{3} \quad h = -9 \quad k = 4$

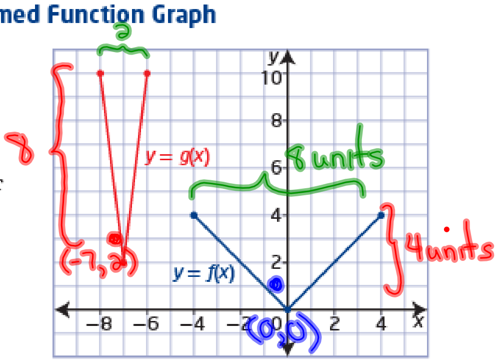
$(x,y) \rightarrow \left[-\frac{3}{2}x - 9, -2y + 4\right]$

$(-12, 18) \rightarrow (9, -32)$

Example 3

Write the Equation of a Transformed Function Graph

The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$. Explain your answer.



Solution

Locate key points on the graph of $f(x)$ and their image points on the graph of $g(x)$.

$(-4, 4) \rightarrow (-8, 10)$
$(0, 0) \rightarrow (-7, 2)$
$(4, 4) \rightarrow (-6, 10)$

The equation of the transformed function is $g(x) = 2f(4(x + 7)) + 2$.

① Reflections → None

② Vertical stretch Factor (Range $\frac{New}{Old}$) $= \frac{8}{4} = 2$ $a=2$

③ Horizontal stretch Factor (Domain $\frac{New}{Old}$) $= \frac{2}{8} = \frac{1}{4}$ $b=4$

④ Horizontal Translation: (Pick a point where x=0) $(0,0) \rightarrow (-7,2)$ $h=-7$

⑤ Vertical Translation (Pick a point where y=0) $(0,0) \rightarrow (-7,2)$ $k=2$

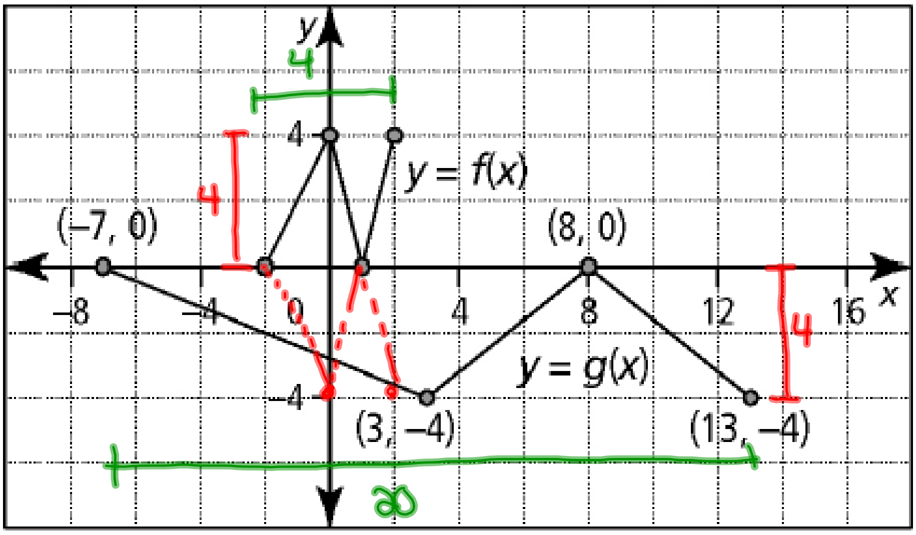
⑥ Equation: $\boxed{y = 2f[4(x+7)] + 2}$

Mapping Rule $(x,y) \rightarrow [\frac{1}{4}x - 7, 2y + 2]$

(-4,4)	(-8,10)
(0,0)	(-7,2)
(4,4)	(-6,10)

The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$.

$y = -f\left(\frac{1}{5}(x-3)\right)$



① Reflection: Vertical (x-axis)

② VSF: $\frac{4}{4} = 1$ (No vertical stretch) $a = -1$

③ HSF: $\frac{20}{4} = 5$ $b = \frac{1}{5}$

④ HT: $(0,4) \rightarrow (3,-4)$ $h = 3$

⑤ VT: $(-2,0) \rightarrow (-7,0)$ $k = 0$

⑥ Equation: $g(x) = -1f\left[\frac{1}{5}(x-3)\right] + 0$

$g(x) = -f\left[\frac{1}{5}(x-3)\right]$

Homework

Page 38 # 3-6
Plus 7, 8, 9 (a, c, e) and 10