Warm Up

Evaluate the following limits, if they exist:

1. $\lim_{x\to 2} \dfrac{x-2}{x^3-8}$

$\lim_{x\to 2} \dfrac{(x-2)}{(x-2)(x^2+2x+4)}$

$\lim_{x\to 2} \dfrac{1}{4+4+4} = \boxed{\dfrac{1}{12}}$

2. $\lim_{x\to 7} \dfrac{(\sqrt{x+2}-3)}{x-7}$ $\dfrac{(\sqrt{x+2}+3)}{(\sqrt{x+2}+3)}$

$\lim_{x\to 7} \dfrac{x+2-9}{(x-7)(\sqrt{x+2}+3)}$

$\lim_{x\to 7} \dfrac{(x-7)}{(x-7)(\sqrt{x+2}+3)}$

$\lim_{x\to 7} \dfrac{1}{3+3} = \boxed{\dfrac{1}{6}}$

3. $\lim_{h\to 0} \dfrac{(a+h)^2-a^2}{h}$

$\lim_{h\to 0} \dfrac{(a+h-a)(a+h+a)}{h}$

$\lim_{h\to 0} \dfrac{h(2a+h)}{h} = \boxed{2a}$

Questions from Homework

(6b) $\lim_{x \to -8} \dfrac{x^2 + 16x + 64}{x + 8}$

$\lim_{x \to -8} \dfrac{(x+8)(x+8)}{(x+8)} = \dfrac{0}{1} = \boxed{0}$

(6a) $\lim_{h \to 0} \dfrac{(4+h)^3 - 64}{h}$

$\lim_{h \to 0} \dfrac{((4+h)-4)((4+h)^2 + 4(4+h) + 16)}{h}$

$\lim_{h \to 0} \dfrac{h((4+h)^2 + 4(4+h) + 16)}{h} = 16 + 16 + 16$

$= \boxed{48}$

(6f) $\lim_{h \to 0} \dfrac{\frac{4(2+h)^2}{1} - \frac{1}{4}\cdot\frac{4(2+h)^2}{1}}{h(4(2+h)^2)}$

$\lim_{h \to 0} \dfrac{4 - (2+h)^2}{4h(2+h)^2}$ ← difference of squares.

$\lim_{h \to 0} \dfrac{(2 - (2+h))(2 + (2+h))}{4h(2+h)^2}$

$\lim_{h \to 0} \dfrac{-h(4+h)}{4h(2+h)^2}$

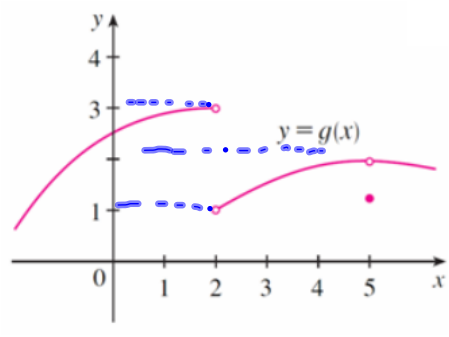
$\lim_{h \to 0} \dfrac{-1(4+0)}{4(2+0)^2} = \dfrac{-4}{16} = \boxed{\dfrac{-1}{4}}$

The common sense definition of a limit...

When does a limit exist?

One-sided limits

Use the graph shown below to evaluate the following limits:



1. $\lim_{x\to 2^-} g(x) =$ 3 ("as x approaches 2 from the left")

2. $\lim_{x\to 2^+} g(x) =$ 1 ("as x approaches 2 from the right")

3. $\lim_{x\to 2} g(x) =$ DNE

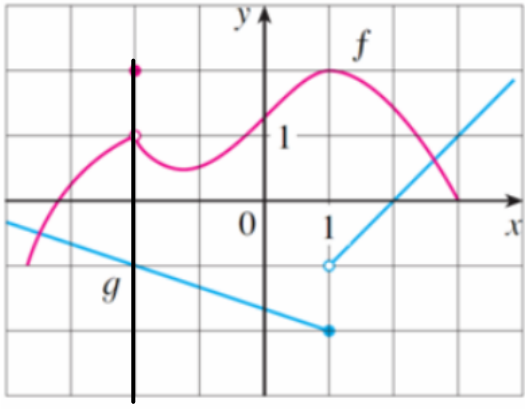
4. $\lim_{x\to 5^-} g(x) =$ 2

5. $\lim_{x\to 5^+} g(x) =$ 2

6. $\lim_{x\to 5} g(x) =$ 2

Notice... $g(5) =$ 1.2 Pick the closed dot.

Example:



Evaluate each of the following:

$f(-2) =$ 2 $\quad \lim_{x\to 1^-} g(x) =$ -2 $\quad g(1) =$ -2

$\lim_{x\to 1^+} g(x) =$ -1 $\quad \lim_{x\to 1} g(x) =$ DNE $\quad \lim_{x\to 1} f(x) =$ 2

$\lim_{x\to -2} f(x) =$ 1

Homework

Page 27

1, 2, 3

③ a) $\lim_{x \to 3} \dfrac{\frac{1}{3} - \frac{1}{x}}{(x-3)}$ $\cdot \dfrac{3x}{3x}$ CD: $= 3x$

$$\lim_{x \to 3} \frac{x-3}{3x(x-3)} = \frac{1}{3x} = \frac{1}{3(3)} = \boxed{\frac{1}{9}}$$

b) $\lim_{h \to 0} \dfrac{(2+h)^3 - 8}{h}$

$$\lim_{h \to 0} \frac{((2+h)-2)\left[(2+h)^2 + 2(2+h) + 4\right]}{h}$$

$$\lim_{h \to 0} \frac{h\left[(2+h)^2 + 2(2+h) + 4\right]}{h} = \boxed{12}$$