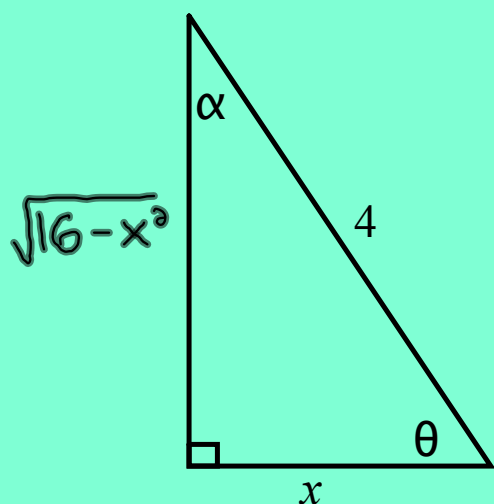


Warm Up

Using the diagram shown determine an expression for each of the following:



$$\sin \theta = \frac{\sqrt{16-x^2}}{4}$$

$$\sec \alpha = \frac{4}{\sqrt{16-x^2}}$$

$$\tan \alpha = \frac{x}{\sqrt{16-x^2}}$$

$$\tan \theta = \frac{\sqrt{16-x^2}}{x}$$

$$\cos^{-1}\left(\frac{x}{4}\right) = \theta$$

$$\sec^{-1}\left(\frac{4}{\sqrt{16-x^2}}\right) = \alpha$$

$$\cos \theta = \frac{x}{4}$$

$$\sec \alpha = \frac{4}{\sqrt{16-x^2}}$$

Derivatives of Transcendental Functions

transcendental functions

(mathematics) Functions which cannot be given by any algebraic expression involving only their variables and constants.

Examples include the functions $\log x$, $\sin x$, $\cos x$, e^x and any functions containing them.

Inverse Trigonometric Functions

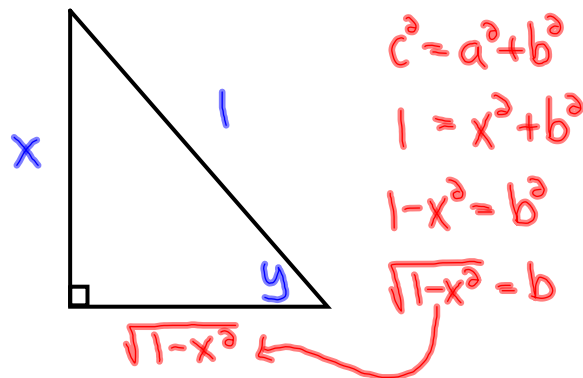
Let's review the definition of an inverse trigonometric function:

$$y = \sin^{-1} x \quad \text{or} \quad y = \text{Arc sin } x$$

What do the above statements mean verbally?

"y represents the angle whose sine ratio is equal to x." smy = x

Express this visually:



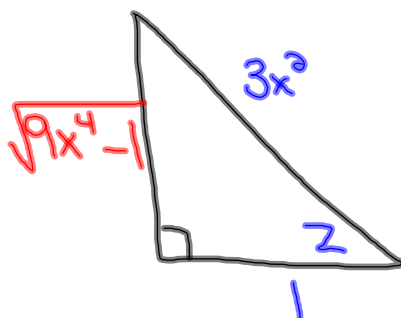
Example:

- ① Represent the inverse trigonometric function

$z = \sec^{-1}(3x^2)$ with a diagram.

- ② Evaluate: $y = \tan[\sec^{-1}(3x^2)]$

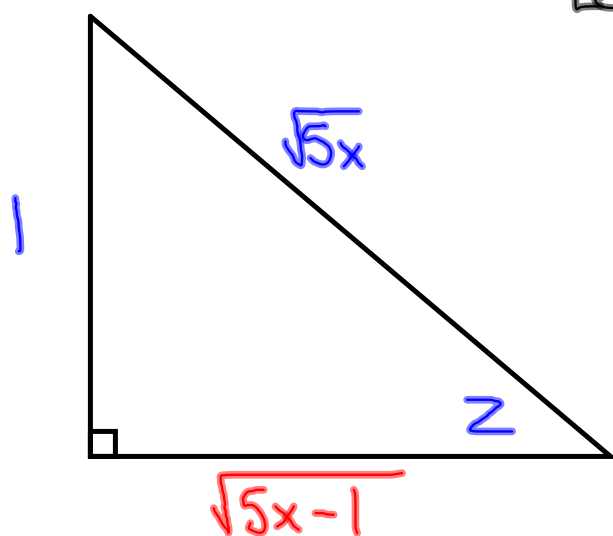
① $\sec z = \frac{3x^2}{1}$ ↙ hyp
← adj



② $y = \tan[\sec^{-1}(3x^2)]$
 $y = \tan z$
 $y = \sqrt{9x^4 - 1}$

Example:

Evaluate the following: $y = \cos\left[\underbrace{\csc^{-1}\sqrt{5x}}^z\right]$



Let: $\underline{z} = \underline{\csc^{-1}\sqrt{5x}}$

$$\csc z = \sqrt{5x} = \frac{\sqrt{5x}}{1}$$

← hyp
← opp

$$y = \cos \underline{z} = \frac{\sqrt{5x-1}}{\sqrt{5x}}$$

Review of Trig Derivatives from last month
In a nutshell...we have

$$\frac{d}{du}(\sin u) = \cos u \cdot du$$

$$\frac{d}{du}(\csc u) = -\csc u \cot u \cdot du$$

$$\frac{d}{du}(\cos u) = -\sin u \cdot du$$

$$\frac{d}{du}(\sec u) = \sec u \tan u \cdot du$$

$$\frac{d}{du}(\tan u) = \sec^2 u \cdot du$$

$$\frac{d}{du}(\cot u) = -\csc^2 u \cdot du$$

Ex: $y = \sin(3x^2)$ $u = 3x^2$ $u' = 6x$

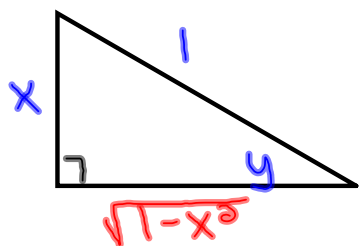
$$y' = \cos 3x^2 \cdot 6x$$

$$y' = 6x \cos 3x^2$$

Differentiating Inverse Trigonometric Functions

Let's start with finding the derivative of $y = \arcsin x$
 ($y = \sin^{-1} x$) which is... $\sin y = x$

Implicit Diff \rightarrow $\boxed{\sin y = x}$



$$(\cos y) \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{\cos y}$$

$$\frac{dy}{dx} = \frac{1}{\boxed{\cos y}} = \frac{1}{\sqrt{1 - \sin^2 y}} = \boxed{\frac{1}{\sqrt{1 - x^2}}}$$

Once again like before, if $y = \sin^{-1} u$, where u is a differentiable function of x , then application of the chain rule gives us:

$$\boxed{\frac{dy}{dx} (\sin^{-1} u) = \frac{1}{\sqrt{1 - u^2}} \frac{du}{dx}}$$

In a similar manner, the derivative of $y = \arccos u$
 ($y = \cos^{-1} u$) can be shown to be:

$$\boxed{\frac{dy}{dx} (\cos^{-1} u) = \frac{-1}{\sqrt{1 - u^2}} \frac{du}{dx}}$$

We list the derivatives of the **six inverse trigonometric functions**...

(arcsin, arccos, arctan, arccsc, arcsec, arccot)

In each case u denotes a differentiable function of x .

$$\frac{d}{dx}(\arcsin u) = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx}$$

$$\frac{d}{dx}(\text{arc csc } u) = \frac{-1}{|u|\sqrt{u^2-1}} \frac{du}{dx}$$

$$\frac{d}{dx}(\arccos u) = \frac{-1}{\sqrt{1-u^2}} \frac{du}{dx}$$

$$\frac{d}{dx}(\text{arc sec } u) = \frac{1}{|u|\sqrt{u^2-1}} \frac{du}{dx}$$

$$\frac{d}{dx}(\arctan u) = \frac{1}{1+u^2} \frac{du}{dx}$$

$$\frac{d}{dx}(\text{arc cot } u) = \frac{-1}{1+u^2} \frac{du}{dx}$$

There is no common agreement on the definition of arcsec x (or arccsc x) for negative values of x . When we defined the range of arcsec x , we chose to preserve the identity $\text{arcsec } x = \arccos(1/x)$. One of the consequences of this choice is that the slope of the graph of the inverse secant function is always positive, which accounts for the absolute value sign in the formula for the derivative of arcsec x .

Examples:

Differentiate each of the following and simplify your answers

$$u = 2x^2 \quad du = 4x$$

$$y = \cos^{-1}(2x^2)$$

$$y' = \frac{-1}{\sqrt{1-(2x^2)^2}} \cdot 4x$$

$$y' = \frac{-4x}{\sqrt{1-4x^4}}$$

$$u = 3x \quad du = 3$$

$$y = \arctan 3x$$

$$y' = \frac{1}{1+(3x)^2} \cdot 3$$

$$y' = \frac{3}{1+9x^2}$$

$$u = x^{1/2} \quad u' = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

$$y = \sin^{-1} \sqrt{x}$$

$$y' = \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}\sqrt{1-x}} = \frac{1}{2\sqrt{x-x^2}}$$

$$y = 2\arccos 2x + \sqrt{1-4x^2} \quad = 2\cos^{-1} 2x + (1-4x^2)^{1/2}$$

$$y' = 2 \left[\frac{-1}{\sqrt{1-4x^2}} \cdot 2 \right] + (0)\cos^{-1} 2x + \frac{1}{2}(1-4x^2)^{-1/2} (-8x)$$

$$y' = \frac{-4}{\sqrt{1-4x^2}} - \frac{4x}{\sqrt{1-4x^2}} = \frac{-4-4x}{\sqrt{1-4x^2}}$$

$$f(x) = x \tan^{-1} \sqrt{x} \quad u = \sqrt{x} \quad du = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

$$f'(x) = x \left[\frac{1}{1+x} \cdot \frac{1}{2\sqrt{x}} \right] + \tan^{-1} \sqrt{x}$$

$$= \frac{x}{2\sqrt{x}(1+x)} + \tan^{-1} \sqrt{x}$$

$$= \frac{\sqrt{x}}{2(1+x)} + \tan^{-1} \sqrt{x}$$

Examples:

Differentiate each of the following...

$$u = 3x^2 \quad du = 6x$$

$$f(x) = x^3 \sin^{-1}(3x^2)$$

$$f'(x) = x^3 \left[\frac{1}{\sqrt{1-9x^4}} \cdot 6x \right] + 3x^2 \sin^{-1}(3x^2)$$

$$f(x) = \sqrt{3x - \tan^{-1} \sqrt{x}} = (3x - \tan^{-1} \sqrt{x})^{\frac{1}{2}} \quad u = \sqrt{x} \quad du = \frac{1}{2\sqrt{x}}$$

$$f'(x) = \frac{1}{2} (3x - \tan^{-1} \sqrt{x})^{-\frac{1}{2}} \left[3 - \left(\frac{1}{1+x} \cdot \frac{1}{2\sqrt{x}} \right) \right]$$

Homework:

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#1, 2, 4, 6



Questions from Homework

$$\textcircled{2} f(x) = x \tan^{-1} x$$

$$f'(x) = 1 \tan^{-1} x + x \left(\frac{1}{1+x^2} \right) (1)$$

$$f'(x) = \tan^{-1} x + \frac{x}{1+x^2}$$

$$f'(1) = \tan^{-1}(1) + \frac{(1)}{1+(1)^2}$$

what angle has a tangent value equal to 1

$$f'(1) = \frac{\pi}{4} + \frac{1}{2}$$

$$f'(1) = \frac{\pi}{4} + \frac{2}{4} = \boxed{\frac{\pi+2}{4}}$$

$$\textcircled{4} f(x) = (3 \tan^{-1} x)^4$$

$$f'(x) = 4(3 \tan^{-1} x)^3 \left[\cancel{0}(\tan^{-1} x) + 3 \left(\frac{1}{1+x^2} \right) (1) \right]$$

$$f'(x) = 4(3 \tan^{-1} x)^3 \left(\frac{3}{1+x^2} \right) = \frac{12(3 \tan^{-1} x)^3}{1+x^2}$$

$$f'(\sqrt{3}) = \frac{12(3 \tan^{-1}(\sqrt{3}))^3}{1+(\sqrt{3})^2}$$

what angle has a tangent value equal to $\sqrt{3}$

$$f'(\sqrt{3}) = \frac{12(3(\frac{\pi}{3}))^3}{1+3}$$

$$f'(\sqrt{3}) = \frac{12\pi^3}{4} = \boxed{3\pi^3}$$

$$\textcircled{6} f(x) = (x-3)(6x-x^2)^{1/2} + 9\sin^{-1}\left(\frac{x-3}{3}\right)$$

$$f'(x) = 1(6x-x^2)^{1/2} + (x-3)\left(\frac{1}{2}\right)(6x-x^2)^{-1/2}(6-2x) + 9\left(\frac{1}{\sqrt{1-\left(\frac{x-3}{3}\right)^2}}\right)\left(\frac{1}{3}\right)$$

$$f'(x) = \sqrt{6x-x^2} + \frac{(x-3)(3-x)}{\sqrt{6x-x^2}} + \frac{3}{\sqrt{1-\left(\frac{x-3}{3}\right)^2}}$$

$$f'(3) = 3 + 0 + 3 = \boxed{6}$$

$$f(x) = \frac{\cot^3 5x}{\cot^{-1}(5x)}$$

$$f(x) = \tan[\text{arc csc}(x^5)]$$