Understanding Logarithms

- Focus on...

 demonstrating that a logarithmic function is the inverse of an the line q = xexponential function
- sketching the graph of $y = \log_c x$, c > 0, $c \ne 1$
- determining the characteristics of the graph of $y = \log_c x$, c > 0, $c \ne 1$
- · explaining the relationship between logarithms and exponents
- expressing a logarithmic function as an exponential function and vice versa
- · evaluating logarithms using a variety of methods

General Properties of Logarithms: an exponent!

If C > 0 and $C \ne 1$, then... (i) $\log_C 1 = 0$ (ii) $\log_C c^x = x$ (iii) $c^{\log_C x} = x$

(i)
$$\log_{c} 1 = 0$$

(ii)
$$\log_{\mathbf{c}} \mathbf{c}^{x} = x$$

(iii)
$$c^{\log_{c} x} = x$$

Did You Know?

The input value for a logarithm is called an argument. For example, in the expression log₆ 1, the argument is 1.

(1)
$$\log_5 1 = 0$$
 (11) $\log_5 3^3 = 3$

$$(i\hat{n}) = \frac{109}{19} = \frac{19}{19}$$

Product Law of Logarithms (Page 394)

The logarithm of a product of numbers can be expressed as the sum of the logarithms of the numbers.

$$\frac{\log_c MN = \log_c M + \log_c N}{Proof}$$
 Fx: $\log_3 8 + \log_3 3 = \log_3 (8 \times 3) = \log_3 8 + \log_3 3 = \log_3 (8 \times 3) = \log_3 8 + \log_3 3 = \log_3 (8 \times 3) = \log_3 8 + \log_3 3 = \log_3 (8 \times 3) = \log_3 8 + \log_3 3 = \log_3 (8 \times 3) = \log_3 8 + \log_3 3 = \log_3 (8 \times 3) = \log_3 8 + \log_3 3 = \log_3 (8 \times 3) = \log_3 8 + \log_3$

Let $\log_c M = x$ and $\log_c N = y$, where M, N, and c are positive real numbers with $c \neq 1$.

Write the equations in exponential form as $M = c^x$ and $N = c^y$:

$$\begin{split} MN &= (c^x)(c^y) \\ MN &= c^{x+y} \end{split} \qquad \text{Apply the product law of powers.} \\ \log_c MN &= x+y \\ \log_c MN &= \log_c M + \log_c N \end{split} \qquad \text{Substitute for x and y.}$$

Quotient Law of Logarithms

The logarithm of a quotient of numbers can be expressed as the difference of the logarithms of the dividend and the divisor.

$$\frac{\log_c \frac{M}{N} = \log_c M - \log_c N}{Proof}$$

$$E_{\mathbf{x}}: \log 400 - \log 4 = \log(\frac{400}{4}) = \log 100$$

$$= 2$$

Let $\log_c M = x$ and $\log_c N = y$, where M, N, and c are positive real numbers with $c \neq 1$.

Write the equations in exponential form as $M = c^x$ and $N = c^y$:

$$\frac{M}{N} = \frac{c^x}{c^y}$$

$$\frac{M}{N} = c^{x-y}$$
 Apply the quotient law of powers.
$$\log_c \frac{M}{N} = x - y$$
 Write in logarithmic form.
$$\log_c \frac{M}{N} = \log_c M - \log_c N$$
 Substitute for x and y .

Power Law of Logarithms

The logarithm of a power of a number can be expressed as the exponent times the logarithm of the number.

$$\log_c M^p = P \log_c M$$

How could you prove the quotient law using the product law and the power law?

Proof

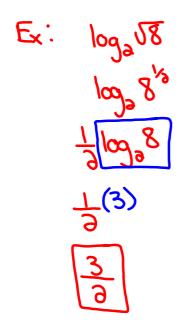
Let $\log_c M = x$, where M and c are positive real numbers with $c \neq 1$.

Write the equation in exponential form as $M = c^{x}$.

Let P be a real number.

$$M=c^x$$
 $M^p=(c^x)^p$ $M^p=c^{xp}$ Simplify the exponents. $\log_c M^p=xP$ Write in logarithmic form. $\log_c M^p=(\log_c M)P$ Substitute for x . $\log_c M^p=P\log_c M$

The laws of logarithms can be applied to logarithmic functions, expressions, and equations.



Questions from Homework

(a) c)
$$\log_{10}(3x+5) = 3$$
 Logar ithmic Form
$$10^3 = 3x+5$$
 Exponential Form
$$100 = 3x+5$$

$$06 = 3x$$

$$\frac{95 = x}{3}$$

h)
$$10^{5^{\times}} = 3$$
 Exponential Form $(\log_{10} 3) = 5^{\times}$ Logarithmic Form ans.

$$\int \log_5(\log^3) = \times$$

9)
$$\log_{3}(\log_{3}x) = 4$$

 $\partial^{4} = \log_{3}x$
 $16 = \log_{3}x$
 $3^{6} = x$
 $43.046731 = x$

9)
$$\log_{3}(\log_{3}x) = 4$$
e) $\lambda^{1-x} = 3$

$$\lambda^{4} = \log_{3}x$$

$$\log_{3} 3 = 1-x$$

$$16 = \log_{3}x$$

$$x = 1 - \log_{3}3$$

$$x = 1 - \log_{3}3$$

Example 1

Use the Laws of Logarithms to Expand Expressions

Write each expression in terms of individual logarithms of x, y, and z.

a)
$$\log_5 \frac{xy}{z} = \log_5 x + \log_5 y - \log_5 z$$

b)
$$\log_7 \sqrt[3]{x} = \log_7 x^{1/3} = \frac{1}{3} \log_7 x$$

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$$\log_7 \sqrt[3]{x} = \log_7 x^{1/3} = \frac{1}{3} \log_7 x$$

c) $\log_6 \frac{1}{x^2} = \log_6 1 - \log_6 x^3 = 0 - 2\log_6 x = -2\log_6 x$
d) $\log \frac{x^3}{x}$

d)
$$\log \frac{x^3}{y\sqrt{z}}$$

$$= \log x^3 - [\log y + \log z^3]$$

$$= 3\log x - \log y - \frac{1}{5}\log z$$

Example 2

Use the Laws of Logarithms to Evaluate Expressions

Use the laws of logarithms to simplify and evaluate each expression.

- a) $\log_6 8 + \log_6 9 \log_6 2$
- **b)** $\log_{7} 7\sqrt{7}$
- c) $2 \log_2 12 (\log_2 6 + \frac{1}{3} \log_2 27)$

a)
$$\log_{6} 8 + \log_{9} 9 - \log_{9} 1$$
 $\log_{7} 7 + \log_{7} 7$
 $\log_{7} 36$
 $1 + \frac{1}{2}(1)$
 $\frac{3}{2} + \frac{1}{2}$

Example 3

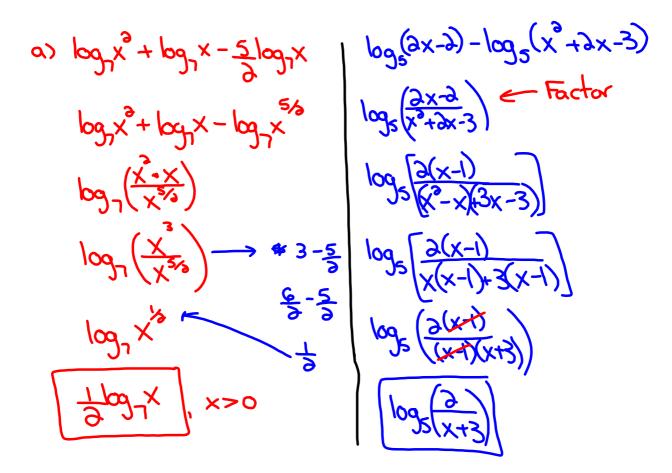
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Use the Laws of Logarithms to Simplify Expressions

Write each expression as a single logarithm in simplest form. State the restrictions on the variable.

a)
$$\log_7 x^2 + \log_7 x - \frac{5 \log_7 x}{2}$$

b)
$$\log_5 (2x - 2) - \log_5 (x^2 + 2x - 3)$$



For the original expression to be defined, both logarithmic terms must be defined.

The conditions x > 1 and x < -3 or x > 1 are both satisfied when x > 1.

Hence, the variable x needs to be restricted to x > 1 for the original expression to be defined and then written as a single logarithm.

Therefore,
$$\log_5 (2x-2) - \log_5 (x^2 + 2x - 3) = \log_5 \frac{2}{x+3}, x > 1.$$

Key Ideas

• Let P be any real number, and M, N, and c be positive real numbers with $c \neq 1$. Then, the following laws of logarithms are valid.

Name	Law	Description
Product	$\log_c MN = \log_c M + \log_c N$	The logarithm of a product of numbers is the sum of the logarithms of the numbers.
Quotient	$\log_c \frac{M}{N} = \log_c M - \log_c N$	The logarithm of a quotient of numbers is the difference of the logarithms of the dividend and divisor.
Power	$\log_c M^p = P \log_c M$	The logarithm of a power of a number is the exponent times the logarithm of the number.

Many quantities in science are measured using a logarithmic scale. Two
commonly used logarithmic scales are the decibel scale and the pH scale.

Homework

Finish Exercise 3 and #1-3, 5, 8 and 11 on page 400

Do I really understand??...

- a) Express the following as a single logarithm... $2 \log_2 3^2 + \log_2 6 3 \log_2 3$
- b) Evaluate the following... $\log_2(32)^{\frac{1}{3}}$
- c) Express the following as a single logarithm... $\frac{1}{2} [(\log_5 a + 2\log_5 b) 3\log_5 c]$
- d) Express as a single logarithm in simplest form...

$$\frac{3}{4} \left[12 (\log_b x^2 - 2\log_b x) + 8\log_b \sqrt{x} - 4\log_b \frac{1}{x^7} \right]$$