① k) $y = \ln\sqrt{\dfrac{x}{2x+3}} = \ln\left(\dfrac{x}{2x+3}\right)^{1/2} = \ln \dfrac{x^{1/2}}{(2x+3)^{1/2}}$

$$y' = \frac{1}{\frac{x^{1/2}}{(2x+3)^{1/2}}} \cdot \left[\frac{1}{2}\left(\frac{x}{2x+3}\right)^{-1/2}\left(\frac{1(2x+3)-2x}{(2x+3)^2}\right)\right]$$

$$y' = \frac{(2x+3)^{1/2}}{x^{1/2}} \cdot \left[\frac{1}{2} \cdot \frac{(2x+3)^{1/2}}{x^{1/2}} \cdot \frac{3}{(2x+3)^2}\right]$$

$$y' = \frac{3\cancel{(2x+3)}}{2x(2x+3)^{\cancel{2}}} = \frac{3}{2x(2x+3)}$$

f) $y = \ln(\sin x)$

$$y' = \frac{1}{\sin x} \cdot \cos x = \frac{\cos x}{\sin x} = \cot x$$

Laws of Logarithms

$y = b^x \longleftrightarrow \log_b y = x$

exponential $\longleftrightarrow$ logarithmic

$\log_b M + \log_b N = \log_b (MN)$ Product Law

$\log_b M - \log_b N = \log_b \left(\frac{M}{N}\right)$ Quotient Law

$\log_b (N^p) = p \log_b (N)$ Power Law

3 Basic Properties

① $\log_b b^m = m$

Ex: $\ln e^3$

$\log_e e^3 = 3$

② $\log_b 1 = 0$

Ex: $\ln 1$

$\log_e 1 = 0$

③ $b^{\log_b n} = n$

Ex: $5^{\log_5 6} = 6$

Warm Up

Review of laws of logarithms...

Given that $\log_x M = -3$, $\log_x N = 5$ **and** $\log_x P = 4$, **evaluate the following logarithmic expression:**

$$\log_x\left[\frac{(M^3N)^2\sqrt{P}}{MP}\right] \rightarrow \log_x\left[\frac{M^6N^2P^{1/2}}{MP}\right]$$

$$\log_x M^6 + \log_x N^2 + \log_x P^{1/2} - \log_x M - \log_x P$$

$$6\log_x M + 2\log_x N + \frac{1}{2}\log_x P - \log_x M - \log_x P$$

$$6(-3) + 2(5) + \frac{1}{2}(4) - (-3) - (4)$$

$$-18 + 10 + 2 + 3 - 4$$

$$-7$$

Solve the following equation: $\dfrac{3^{x-1}}{5 * 2^{3x}} = 6^{1-2x}$

$$\ln\left[\frac{3^{x-1}}{5\cdot 2^{3x}}\right] = \ln 6^{1-2x}$$

$$\ln 3^{x-1} - \ln 5 - \ln 2^{3x} = \ln 6^{1-2x}$$

$$(x-1)\ln 3 - \ln 5 - 3x\ln 2 = (1-2x)\ln 6$$

Bring "x's" to the left

$$x\ln 3 - \ln 3 - \ln 5 - 3x\ln 2 = \ln 6 - 2x\ln 6$$

Factor an "x" out

$$x\ln 3 - 3x\ln 2 + 2x\ln 6 = \ln 6 + \ln 3 + \ln 5$$

$$x(\ln 3 - 3\ln 2 + 2\ln 6) = \ln 6 + \ln 3 + \ln 5$$

$$x = \frac{\ln 6 + \ln 3 + \ln 5}{\ln 3 - 3\ln 2 + 2\ln 6}$$

$$x = \frac{\ln 6 + \ln 3 + \ln 5}{\ln 3 - \ln 2^3 + \ln 6^2} \rightarrow \ln\left(\frac{3\cdot 6^2}{2^3}\right)$$

$$\ln(13.5)$$

$$x = \frac{\ln 90}{\ln 13.5}$$

$$\boxed{x \doteq 1.7289}$$

Questions from Homework

Rule: $d(\ln u) = \dfrac{1}{u}\,du$

$u = \ln x \quad du = \frac{1}{x}\cdot 1 = \frac{1}{x}$

1 e) $y = \sin(\ln x)$

$y' = \cos(\ln x)\cdot\frac{1}{x}$

$y' = \dfrac{\cos(\ln x)}{x}$

$u = \sin x \quad du = \cos x\,(1)$

f) $y = \ln(\sin x)$

$y' = \frac{1}{\sin x}\cdot\cos x = \dfrac{\cos x}{\sin x} = \cot x$

h) $y = (x + \ln x)^3$

$y' = 3(x+\ln x)^2\left(1 + \frac{1}{x}\right)$

$y' = 3(x+\ln x)^2\left(\dfrac{x+1}{x}\right)$

$y' = \dfrac{3(x+\ln x)^2(x+1)}{x}$

k) $y = \ln\sqrt{\dfrac{x}{2x+3}} = \ln\dfrac{x^{1/2}}{(2x+3)^{1/2}}$

$y' = \dfrac{1}{\frac{x^{1/2}}{(2x+3)^{1/2}}}\cdot\left[\dfrac{(2x+3)^{1/2}\left(\frac{1}{2}\right)x^{-1/2} - x^{1/2}\left(\cancel{\frac{1}{2}}\right)(2x+3)^{-1/2}(\cancel{2})}{2x+3}\right]$

$y' = \dfrac{(2x+3)^{1/2}}{x^{1/2}}\left[\dfrac{x^{-1/2}(2x+3)^{-1/2}\left[\frac{1}{2}(2x+3) - x\right]}{(2x+3)}\right]$

$y' = \dfrac{(2x+3)^{1/2}}{x^{1/2}}\left[\dfrac{x^{-1/2}\left(\cancel{x} + 3/2 - \cancel{x}\right)}{(2x+3)^{3/2}}\right]$

$y' = \dfrac{\frac{3}{2}}{x(2x+3)}$

$y' = \frac{3}{2}\cdot\dfrac{1}{x(2x+3)} = \boxed{\dfrac{3}{2x(2x+3)}}$

We have now covered base "e"...both as an exponential and logarithmic function...

What about other bases??

Will need to know the change of base formula for logarithms:

$$\log_N M = \frac{\log_b M}{\log_b N}$$

base "e" or base "10"

Whatever new base you choose

$\log_5 13 = \frac{\log 13}{\log 5} \approx 1.59$ | $\log_5 13 = \frac{\ln 13}{\ln 5} \approx 1.59$

Rule: $d(\log_b u) = \frac{1}{u \ln b} du$

Differentiate:

$b=6 \quad u=x^3 \quad du=3x^2$

$$y = \log_6 x^3$$

$y' = \frac{1}{x^3 \ln 6} \cdot 3x^2$

$y' = \frac{3x^2}{x^3 \ln 6}$

$y' = \frac{3}{x \ln 6}$

$b=10 \quad u=5x^4 \quad du=20x^3$

$$y = \log(5x^4)$$

$y' = \frac{20x^3}{5x^4 \ln 10}$

$y' = \frac{4}{x \ln 10}$

This leaves one form of exponential function remaining...

- What about a function such as $y = 3^{9x}$

Rule:

$$d(b^u) = b^u(\ln b)du \text{ , } where \;\; b \in R$$

$b=3 \quad u=9x \quad du=9$

$y = 3^{9x}$

$y' = 3^{9x}(\ln 3)(9)$

$b=\pi \quad u=x^5 \quad du=5x^4$

Try this one... $y = \pi^{x^5}$

$y' = \pi^{x^5}(\ln \pi)(5x^4)$

Practice Problems:

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#1 #2 a #3 #4

#5 #6 #7 #8

① b) $y = \ln\left(\frac{x}{\sqrt{x^2+1}}\right)$ $u = \frac{x}{(x^2+1)^{1/2}}$ du ⇒ quotient/chain

$$y' = \frac{(x^2+1)^{1/2}}{x}\left[\frac{(x^2+1)^{1/2}(1) - x\left(\frac{1}{\cancel{2}}\right)(x^2+1)^{-1/2}(\cancel{2}x)}{x^2+1}\right]$$

$$y' = \frac{(x^2+1)^{1/2}}{x}\left[\frac{(x^2+1)^{1/2} - x^2(x^2+1)^{-1/2}}{x^2+1}\right]$$

$$y' = \frac{(x^2+1)^{1/2}}{x}\left[\frac{(x^2+1)^{-1/2}\overset{1}{\left[\cancel{x^2}+1-\cancel{x^2}\right]}}{x^2+1}\right]$$

$$y' = \frac{1}{x(x^2+1)}$$

② $y = \ln(\ln x)$ $u = \ln x$ $du = \frac{1}{x}$

$$y' = \frac{1}{\ln x} \cdot \frac{1}{x} = \frac{1}{x\ln x}$$

③ d) $g(x) = \dfrac{1+\log_3 x}{x}$

$$g'(x) = \frac{\cancel{x}\left(\frac{1}{\cancel{x}\ln 3}\cdot 1\right) - (1+\log_3 x)(1)}{x^2}$$

$$g'(x) = \frac{\frac{1}{\ln 3}^{\ln 3} - 1^{\ln 3} - \log_3 x^{\ln 3}}{x^2 \ln 3}$$

Change of base formula.

$$g'(x) = \frac{1 - \ln 3 - \ln 3(\log_3 x)}{x^2 \ln 3}$$

$$\log_N M = \frac{\log_b M}{\log_b N}$$

$$g'(x) = \frac{1 - \ln 3 - \cancel{\ln 3}\left(\frac{\ln x}{\cancel{\ln 3}}\right)}{x^2 \ln 3}$$

$$\boxed{g'(x) = \frac{1 - \ln 3 - \ln x}{x^2 \ln 3}}$$