Questions from Homework

Remember!

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If $f(x) = x^2 + 7x$, find $f'(3)$

Hint: find the derivative first then substitute 3 into that

$$f(x+h) = (x+h)^2 + 7(x+h)$$
$$= x^2 + 2xh + h^2 + 7x + 7h$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{7x} + 7h - (\cancel{x^2} + \cancel{7x})}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{2xh + h^2 + 7h}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{h}(2x + h + 7)}{\cancel{h}} = 2x + 7$$

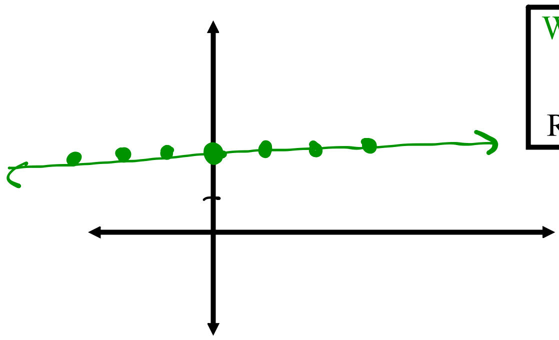
$$f'(x) = 2x + 7$$

$$f'(3) = 2(3) + 7 = 13$$ ← "m" slope of tangent

Differentiation Rules

I. Constant Functions

- Sketch the function $y = 2$



What is the slope of the tangent to this graph?

Recall: slope of the tangent is the derivative

$e = 2.71828$
eulers constant

The derivative of a constant will always be equal to "0".

$f(x) = 6$ $\quad f(x) = 32$ $\quad y = \pi$ $\quad f(x) = 3e$

$f'(x) = 0$ $\quad f'(x) = 0$ $\quad y' = 0$ $\quad f'(x) = 0$

Formal Proof:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{c - c}{h}$$

$$= \lim_{h \to 0} 0 = 0$$

II. Power Functions

We want to come up with a rule to differentiate functions of the form $f(x) = x^n \ , x \in R$

Using the definition of a derivative to differentiate $f(x) = x^4$ would lead to ...

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^4 - x^4}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^4} + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 - \cancel{x^4}}{h}$$

$$= \lim_{h \to 0} \frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4}{h}$$

$$= \lim_{h \to 0} (4x^3 + 6x^2h + 4xh^2 + h^3) = 4x^3$$

Other examples we have looked at so far

$f(x) = x^2$	$f(x) = x^3$	$f(x) = x^4$	$f(x) = x^5$
$f'(x) = 2x$	$f'(x) = 3x^2$	$f'(x) = 4x^3$	$f'(x) = 5x^4$

Do you see a pattern emerging?

The Power Rule (General Version) If n is any real number, then

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$f(x) = 5x^2$

$f'(x) = 10x$

$f(x) = 14x$

$f'(x) = 14$

$f(x) = 7x$

$f(x) = 7x^0 = 7(1) = 7$

$f(x) = 3x^4 - 5x^3 + 2x^2 - 7x + 38$

$f'(x) = 12x^3 - 15x^2 + 4x - 7$

$f(x) = 3x^2 + \frac{1}{x^{10}} = 3x^2 + 1x^{-10}$

$f'(x) = 6x - 10x^{-11} = 6x - \frac{10}{x^{11}}$

The Power Rule (General Version) If n is any real number, then

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Let's practice using the power rule...

Differentiate each of the following functions:

1. $f(x) = x^{25}$

$f'(x) = 25x^{24}$

2. $f(x) = x^{-5}$

$f'(x) = -5x^{-6} = \frac{-5}{x^6}$

3. $f(x) = \frac{1}{x^{10}} = 1x^{-10}$

$f'(x) =$

4. $f(x) = \sqrt{x} = x^{1/2}$

$f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2x^{1/2}} = \frac{1}{2\sqrt{x}}$

Constant Multiples

- The following formula says that <u>the derivative of a constant multiplied by a function is the constant multiplied by the derivative of the function</u>:

The Constant Multiple Rule If c is a constant and f is a differentiable function, then

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$$

EXAMPLE 4

(a) $\frac{d}{dx}(3x^4) = 3\frac{d}{dx}(x^4) = 3(4x^3) = 12x^3$

(b) $\frac{d}{dx}(-x) = \frac{d}{dx}[(-1)x] = (-1)\frac{d}{dx}(x) = -1(1) = -1$

Examples:

1. $f(x) = 4x^3$

$f'(x) = 12x^2$

2. $f(x) = \frac{8}{x^2} = 8x^{-2}$

$f'(x) = -16x^{-3} = \frac{-16}{x^3}$

3. $f(x) = 5x^{\frac{6}{5}}$

$f'(x) = \frac{30}{5}x^{1/5} = 6x^{1/5}$

4. $f(x) = (3x^2)^2 = 9x^4$

$f'(x) = 36x^3$

Recall the derivative of a function is equal to the slope of a line that is tangent to the function.

derivative or $f'(x)$

Find the slope of the tangent line to the function at the given "x" coordinate!

$$f(x) = 3x^2 \quad \text{at } x = 4$$

$f'(x) = 6x$

$f'(4) = 6(4) = 24$

"m"

e) $y = \sqrt{x^3}$

$y = x^{3/2}$

$y' = \frac{3}{2}x^{1/2} = \frac{3\sqrt{x}}{2}$

Homework

equation of tangent line

1. slope of tangent line
2. point (x_1, y_1)
3. $y - y_1 = m(x - x_1)$

4 a) $y = x^5$ $(2, 32)$

① $y' = 5x^4$

② $y'(2) = 5(2)^4$
$= 5(16)$
$= 80$

③ $y - 32 = 80(x - 2)$
$y - 32 = 80x - 160$
$y = 80x - 128$
or $\boxed{80x - y - 128 = 0}$