Questions from Homework

$$F'(x) = f(x)g'(x) + f'(x)g(x)$$

$$F'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

Chain Rule:

The Chain Rule If f and g are both differentiable and $F = f \circ g$ is the composite function defined by $F(x) = f(g(x))$, then F is differentiable and F' is given by the product

$$F'(x) = f'(g(x))g'(x)$$

Work from the outside to the inside

Examples:

$f(x) = (5x^3 + 1)^{10}$

$f'(x) = 10(5x^3+1)^9 (15x^2)$

$f'(x) = 150x^2(5x^3+1)^9$

$F(x) = \sqrt{2x^2+3} = (2x^2+3)^{1/2}$

$F'(x) = \frac{1}{2}(2x^2+3)^{-1/2}(4x)$

$F'(x) = 2x(2x^2+3)^{-1/2}$

$F'(x) = \frac{2x}{(2x^2+3)^{1/2}} = \frac{2x}{\sqrt{2x^2+3}}$

$h(x) = \sqrt[3]{5-3x^4} = (5-3x^4)^{1/3}$

$h'(x) = \frac{1}{3}(5-3x^4)^{-2/3}(-12x^3)$

$h'(x) = \frac{-4x^3}{(5-3x^4)^{2/3}} = \frac{-4x^3}{\sqrt[3]{(5-3x^4)^2}}$

Combining the Chain Rule With the Product and Quotient Rule:

The Chain Rule If f and g are both differentiable and $F = f \circ g$ is the composite function defined by $F(x) = f(g(x))$, then F is differentiable and F' is given by the product

$$F'(x) = f'(g(x))g'(x)$$

Differentiate the following function and simplify your answer:

$$y = (x^2+1)^3(2-3x)^4$$

$y' = f'g + fg'$

$y' = 3(x^2+1)^2(2x)(2-3x)^4 + (x^2+1)^3(4)(2-3x)^3(-3)$

$y' = 6x(x^2+1)^2(2-3x)^4 - 12(x^2+1)^3(2-3x)^3$

$y' = 6(x^2+1)^2(2-3x)^3[x(2-3x) - 2(x^2+1)]$ ($2x-3x^2$, $-2x^2-2$)

$y' = 6(x^2+1)^2(2-3x)^3(-5x^2+2x-2)$

$$g(x) = \frac{(3x+2)^2}{2x}$$

$g'(x) = \frac{(2x)(2)(3x+2)^1(3) - 2(3x+2)^2}{(2x)^2}$

$g'(x) = \frac{12x(3x+2) - 2(3x+2)^2}{4x^2}$

$g'(x) = \frac{2(3x+2)[6x - (3x+2)]}{4x^2}$ ($6x-3x-2$)

$g'(x) = \frac{2(3x+2)(3x-2)}{2 \cdot 4x^2} = \frac{(3x+2)(3x-2)}{2x^2}$

$\therefore$

Differentiate the following functions and simplify your answers:

$$s = \left(\frac{2t-1}{t+2}\right)^6$$

$y' = \frac{f'g - fg'}{g^2}$

$$s' = 6\left(\frac{2t-1}{t+2}\right)^5 \left[\frac{2(t+2) - 1(2t-1)}{(t+2)^2}\right]$$

$2t+4-2t+1$

$$s' = 6 \cdot \frac{(2t-1)^5}{(t+2)^5} \cdot \frac{5}{(t+2)^2} = \frac{30(2t-1)^5}{(t+2)^7}$$

Homework

$$g(x)=9x^{-3}\left(5x^3-1\right)^6$$

$$g(x)=\frac{\left(x^2-5x+1\right)^8}{\left(1-x^{-7}\right)^{20}}$$