Principal Angles

The smallest positive coterminal angle between 0 and 360°or 2π.

1) Divide By 360°(how many rotations?).

2) Get rid of # of full rotations.

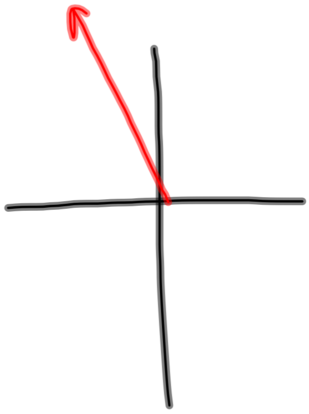
3) Mulitply decimal by 360°to find principal angle.

Ex: 13784°

① $13784° \div 360° = 38.\overline{28}$

② $38.\overline{28} - 38 = 0.\overline{28}$

③ $0.\overline{28} \times 360° = \boxed{104°}$



Ex: $\frac{1058\pi}{3}$

① $\frac{1058\pi}{3} \div 2\pi = \frac{1058\pi}{3} \times \frac{1}{2\pi} = \frac{1058\cancel{\pi}}{6\cancel{\pi}} = \frac{529}{3} = 176.\overline{3}$

$= 176\frac{1}{3}$

② $176.\overline{3} - 176 = \frac{1}{3}$

③ $\frac{1}{3} \times 2\pi = \frac{2\pi}{3}$

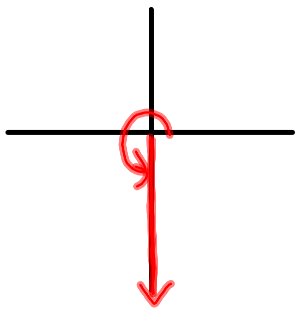
For each angle in standard position, determine one positive and one negative angle measure that is coterminal with it.

a) $270°$ **b)** $-\frac{5\pi}{4}$ **c)** $740°$

a) $270°$

① $270° + 360° = 630°$

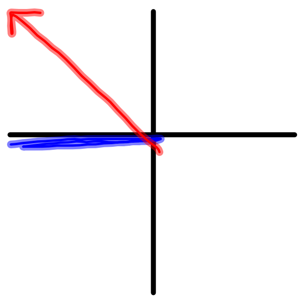
② $270° - 360° = -90°$



b) $-\frac{5\pi}{4}$

① $-\frac{5\pi}{4} + \frac{2\pi}{1} = -\frac{5\pi}{4} + \frac{8\pi}{4} = \boxed{\frac{3\pi}{4}}$

② $-\frac{5\pi}{4} - \frac{2\pi}{1} = -\frac{5\pi}{4} - \frac{8\pi}{4} = \boxed{-\frac{13\pi}{4}}$

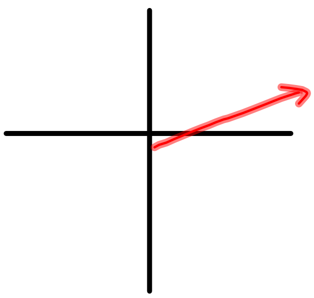


$\frac{2\pi}{4}, \frac{3\pi}{4}, \frac{4\pi}{4} \leftarrow 1\pi$

c) $740°$

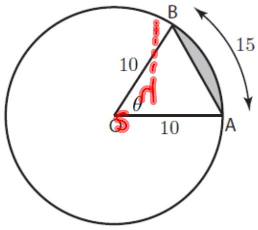
① $740° - 360° = \boxed{380°}$

② $740° - 720° = \boxed{20°}$



Example

Refer to Figure 8. Suppose we have a circle of radius 10cm and an arc of length 15cm. Suppose we want to find (a) the angle θ, (b) the area of the sector OAB, (c) the area of the minor segment (shaded).



Given:
$r = 10$
$a = 15$
Area of Circle $= \pi r^2$

Figure 8. The shaded area is called the minor segment.

a) $a = \theta r$

$15 = \theta(10)$

$1.5 = \theta$

radians

b) $\dfrac{\text{Area of Sector}}{\text{Area of Circle}} = \dfrac{\text{Central Angle}}{\text{Complete Rotation}}$

$$\frac{x}{\pi(10)^2} = \frac{1.5}{2\pi}$$

$$\cancel{100\pi} \cdot \frac{x}{\cancel{100\pi}} = \frac{1.5}{2\pi} \cdot 100\pi$$

$$x = \frac{150\cancel{\pi}}{2\cancel{\pi}} = 75\text{cm}^2$$

c) Step 1:

Area of Triangle $= \frac{1}{2} r^2 \sin\theta$

Area $= \frac{1}{2}(10)^2 \sin(1.5)$ radians
↳ convert calculator to radians

Area $= 50\sin(1.5)$

Area $= 50(0.9975)$

Area $= 49.88\text{ cm}^2$

Step 2:

Area of segment = Area of Sector − Area of triangle

$= 75 - 49.88$

$= 25.12\text{ cm}^2$

Key Ideas

- Angles can be measured using different units, including degrees and radians.
- An angle measured in one unit can be converted to the other unit using the relationships 1 full rotation = $360° = 2\pi$.
- An angle in standard position has its vertex at the origin and its initial arm along the positive x-axis.
- Angles that are coterminal have the same initial arm and the same terminal arm.
- An angle θ has an infinite number of angles that are coterminal expressed by $\theta \pm (360°)n$, $n \in \mathrm{N}$, in degrees, or $\theta \pm 2\pi n$, $n \in \mathrm{N}$, in radians.
- The formula $a = \theta r$, where a is the arc length; θ is the central angle, in radians; and r is the length of the radius, can be used to determine any of the variables given the other two, as long as a and r are in the same units.

Homework

Page 176: #7, 9, 11, 13, 17, 18

Extra Practice

Sketch the following angles.

a) $\frac{7\pi}{6}$ b) $\frac{8\pi}{3}$ c) 5.7 d) $-\frac{11\pi}{4}$

Quad 3 Quad 2 Quad 4 Quad 3

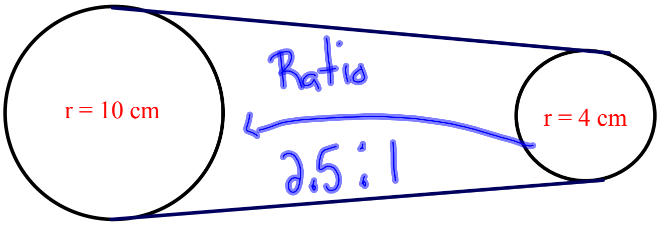
Questions from Homework

Applying our knowledge of rotations and radians...

Ex. (a) If the large wheel rotates $2\pi/3$ radians, how many radians does the smaller wheel rotate?

(b) If the large wheel completes three revolutions, how much does the small wheel rotate in radians?

(c) If the small wheel rotates $-15\pi/4$ radians, how many radians does the larger wheel rotate?



Big Wheel

$a = \theta r$

$a = \left(\frac{2\pi}{3}\right)(10)$

$a = \frac{20\pi}{3}$ cm

Small Wheel

$\theta = \frac{a}{r}$

$\theta = \dfrac{\frac{20\pi}{3}\text{ cm}}{4\text{ cm}}$

$\theta = \frac{20\pi}{3} \cdot \frac{1}{4} = \boxed{\frac{5\pi}{3}\text{ rads}}$

The amount of chain is the same for both wheels.

b) 3 rotations = 6π radians

Big Wheel

$a = \theta r$

$a = (6\pi)(10)$

$a = 60\pi$ cm

Small Wheel

$\theta = \frac{a}{r}$

$\theta = \frac{60\pi}{4}$

$\theta = 15\pi$ rads.

c) Small Wheel

$a = \theta r$

$a = \left(\frac{-15\pi}{\cancel{4}}\right)(\cancel{4})$

$a = -15\pi$ cm

Big Wheel

$\theta = \frac{a}{r}$

$\theta = \frac{-15\pi}{10}$

$\theta = \boxed{\frac{-3\pi}{2}\text{ rads}}$

Angular Velocity

Angular velocity - amount of rotation around a central point per unit of time

$$v_a = \frac{\theta}{t} \qquad \theta = \frac{a}{r}$$

θ = angle (radians)

v_a = angular velocity

t = time

a = arc length

r = radius

Ex. The roller on a computer printer makes 2200 rpm (revolution per minute). Find the roller's angular velocity.

$$2200 \frac{\cancel{\text{revs}}}{\text{min}} \cdot 2\pi \frac{\text{rads}}{\cancel{\text{revs}}} = 4400\pi \text{ rads/min}$$

In rads/sec.

$$\frac{4400\pi \text{ rads}}{\text{min}}$$

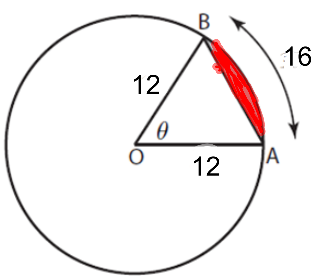
$$\frac{4400\pi \text{ rads}}{60 \text{ sec}}$$

$$\boxed{230.4 \text{ rads/sec}}$$

Homework

Page 176: #14, 15, 16

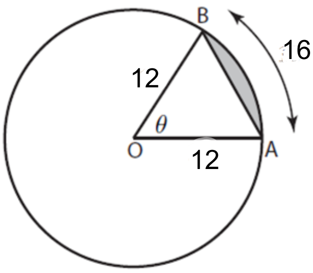
Find the area of the shaded region



Given:
$a = 16cm$
$r = 12cm$

$$\frac{\text{Sector Area}}{\text{Circle Area}} = \frac{\text{Central Angle}}{\text{one revolution}}$$

$$A_{\Delta} = \frac{1}{2}r^2 \sin\theta$$



① Find θ

$\theta = \frac{a}{r}$

$\theta = \frac{16}{12}$

$\theta = \frac{4}{3}$ rads

② $\frac{\text{Sector Area}}{\text{Area of Circle}} = \frac{\text{Central Angle}}{\text{Complete Rev}}$

$\frac{x}{\pi(12)^2} = \frac{4/3}{2\pi}$

$\boxed{x = 96 cm^2}$

③ $A_\Delta = \frac{1}{2} r^2 \sin\theta$

$A_\Delta = \frac{1}{2}(12)^2 \sin\left(\frac{4}{3}\right)$

$A_\Delta = \frac{1}{2}(144)(0.972)$

$\boxed{A_\Delta = 70 cm^2}$

④ $A_{seg} = A_{sec} - A_\Delta$

$A_{seg} = 96 cm^2 - 70 cm^2$

$\boxed{A_{seg} = 26 cm^2}$