

$$f(x) = x^3 - 9x^2 + 24x - 10$$

① y-intercept (x=0) ② Asymptotes: None
 (Polynomial function)

$$f(0) = (0)^3 - 9(0)^2 + 24(0) - 10 = -10$$

(0, -10)

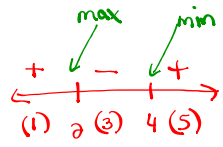
③ $f(x) = x^3 - 9x^2 + 24x - 10$

$$f'(x) = 3x^2 - 18x + 24$$

$$f'(x) = 3(x^2 - 6x + 8)$$

$$f'(x) = 3(x-4)(x-2)$$

Increasing on $(-\infty, 2) \cup (4, \infty)$
 Decreasing on $(2, 4)$



CV: $x=2, 4$

Max/min

$$f(x) = x^3 - 9x^2 + 24x - 10$$

$$f(2) = (2)^3 - 9(2)^2 + 24(2) - 10 = 10 \quad (2, 10) \text{ max}$$

$$f(4) = (4)^3 - 9(4)^2 + 24(4) - 10 = 6 \quad (4, 6) \text{ min}$$

Concavity:

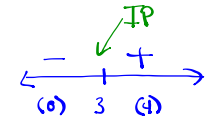
$$f(x) = x^3 - 9x^2 + 24x - 10$$

$$f'(x) = 3x^2 - 18x + 24$$

$$f''(x) = 6x - 18 \quad \text{CD on } (-\infty, 3)$$

$$f''(x) = 6(x-3) \quad \text{CU on } (3, \infty)$$

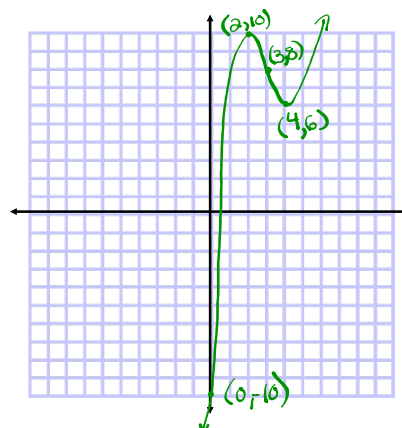
CV: $x=3$



IP:

$$f(x) = x^3 - 9x^2 + 24x - 10$$

$$f(3) = (3)^3 - 9(3)^2 + 24(3) - 10 = 8 \quad (3, 8) \text{ IP}$$



$$f(x) = 3x^5 - 5x^3$$

$$f(x) = x^3(3x^2 - 5)$$

$$x^3 = 0 \quad | \quad 3x^2 - 5 = 0$$

$$x = 0 \quad | \quad 3x^2 = 5$$

$$x^2 = \frac{5}{3}$$

$$(0, 0)$$

$$x = \pm \sqrt{\frac{5}{3}}$$

$$\left(\sqrt{\frac{5}{3}}, 0\right) + \left(-\sqrt{\frac{5}{3}}, 0\right)$$

1. Consider the function: $f(x) = \frac{2+x-x^2}{(x-1)^2}$
 CV: $x=1, 5$
 $f'(x) = \frac{x-5}{(x-1)^3}$ and $f''(x) = \frac{2(7-x)}{(x-1)^4}$ CV: $x=1, 7$

x-int: ($y=0$) y-int: ($x=0$)
 $2+x-x^2=0$ $f(0) = \frac{2+(0)-(0)^2}{(0-1)^2}$
 $-x^2+x+2=0$ $f(0) = \frac{2}{1} = 2$
 $-(x^2-x-2)=0$ $(0, 2)$
 $-(x+1)(x-2)=0$
 $x+1=0 \mid x-2=0$
 $x=-1 \mid x=2$
 $(-1, 0) \quad (2, 0)$

VA: (set denom = 0)

$f(x) = \frac{2+x-x^2}{(x-1)^2} = -\frac{(x-2)(x+1)}{(x-1)^2}$

$(x-1)^2=0$
 $x-1=0$
 $x=1$

Test behaviour near VA

$\lim_{x \rightarrow 1^-} f(x) = \frac{-(-)(-)}{(+)} = +\infty$
 $x=0.99$

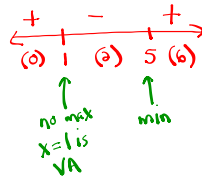
$\lim_{x \rightarrow 1^+} f(x) = \frac{-(-)(+)}{+} = -\infty$
 $x=1.01$

HA: (compare the degree of the num. and denom.)

$f(x) = \frac{2+x-x^2}{(x-1)^2} = \frac{2+x-x^2}{x^2-2x+1}$
 $\lim_{x \rightarrow \infty} \frac{2+x-x^2}{x^2-2x+1} = -1 \quad y = -1$

$f'(x) = \frac{x-5}{(x-1)^3}$

CV: $x=1, 5$



$f(x) = \frac{2+x-x^2}{(x-1)^2}$

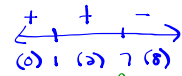
$f(5) = \frac{2+(5)-(5)^2}{(5-1)^2}$

$f(5) = \frac{-18}{16} = -\frac{9}{8}$

$(5, -\frac{9}{8})$ or $(5, -1.125)$

$f''(x) = \frac{2(7-x)}{(x-1)^4}$

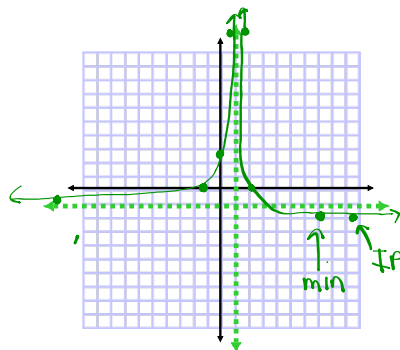
CV: $x=1, 7$



$f(7) = \frac{2+(7)-(7)^2}{(7-1)^2}$

$f(7) = \frac{-40}{36} = -\frac{10}{9}$

$(7, -\frac{10}{9})$ or $(7, -1.11)$



Sketch the following function $f(x) = \frac{(x+2)^2}{(x^2+4)}$ if

$$f'(x) = \frac{16 - 4x^2}{(x^2 + 4)^2} \quad \text{and} \quad f''(x) = \frac{8x(x^2 - 12)}{(x^2 + 4)^3}$$

- Intercepts
- Asymptotes
- Intervals of Increase or Decrease
- Local Maximum and Minimum values
- Intervals of Concavity
- Points of Inflection