Curve Sketching

Intercepts:

To find the *x* - intercept of *y = f(x),* set *y = 0* and solve for *x.*

To find the y - intercept of *y = f(x),* set *x = 0;* the *y* - intercept is *f(0).*

Example: $y = \dfrac{x^2 - x - 6}{x + 1}$

(i) x-int ($y=0$)

$\dfrac{0}{1} = \dfrac{x^2 - x - 6}{x+1}$

$0 = x^2 - x - 6$

$2 \times -3 = -6$

$2 + -3 = -1$

$0 = (x+2)(x-3)$

$x+2=0$ | $x-3=0$

$x=-2$ | $x=3$

$(-2,0)$ | $(3,0)$

(ii) y-int ($x=0$)

$y = \dfrac{(0)^2 - (0) - 6}{(0)+1}$

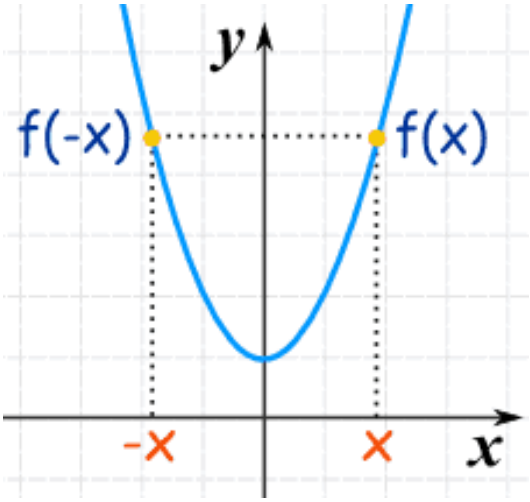
$y = \dfrac{-6}{1} = -6$ $(0,-6)$

Symmetry:

An even function satisfies

$$f(-x) = f(x)$$

for all *x* in its domain. Thus, a function is even if it is unchanged when *x* is replaced by *-x.* The graph of an even function is symmetric about the *y-axis.*



$f(x) = x^2 + 1$

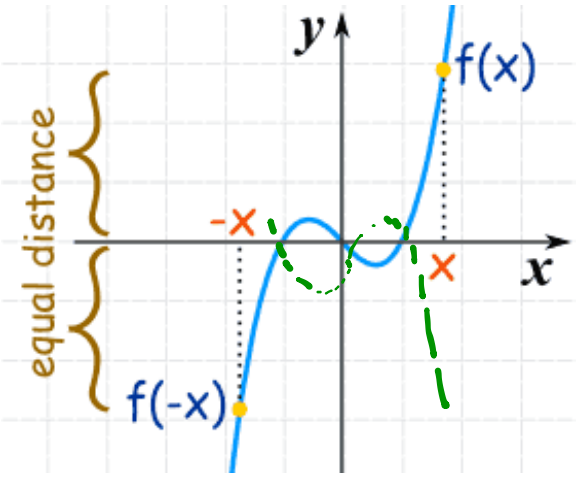
$f(2) = (2)^2 + 1 = 5 \quad (2, 5)$

$f(-2) = (-2)^2 + 1 = 5 \quad (-2, 5)$

An odd function satisfies

$f(-x) = -f(x)$

for all *x* in its domain. The graph of an odd function is symmetric about the *origin.*



$f(x) = x^3$

$f(4) = (4)^3 = 64$ (4,64)

$f(-4) = (-4)^3 = -64$ (4,-64)

Symmetry is used to reduce the amount of work in graphing. If we have graphed an *even function* for $x \geq 0$, we just reflect in the *y-axis* to get the entire graph. For an *odd function* we just rotate through 180 degrees about the origin.

Example:

Determine whether each function is even, odd, or neither

a) $f(x) = x^6$

$f(-x) = (-x)^6 = x^6$

$f(x) = f(-x)$ Even

$f(3) = (3)^6 = 729$

$f(3) = (3)^6 = 729$ } Even

b) $g(x) = x^3 + \frac{1}{x}$

$g(-x) = (-x)^3 + \frac{1}{(-x)}$

$g(-x) = -x^3 - \frac{1}{x}$

$g(-x) = -\left(x^3 + \frac{1}{x}\right)$

$g(-x) = -g(x)$ Odd

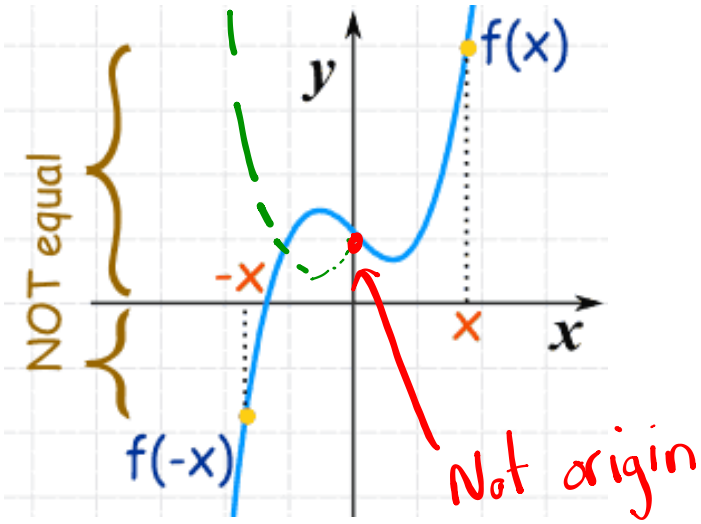
$g(2) = (2)^3 + \frac{1}{2}$

$= 8 + \frac{1}{2} = \frac{17}{2}$

$g(-2) = (-2)^3 + \frac{1}{-2}$

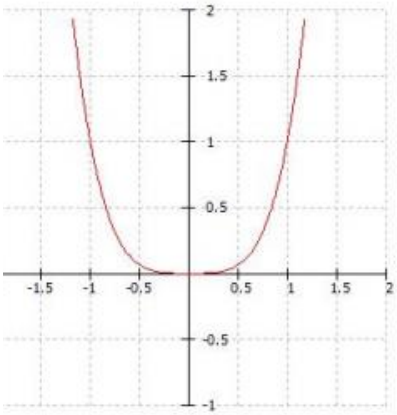
$= -8 - \frac{1}{2} = \frac{-17}{2}$ } odd

Is this function Even or Odd? Neither

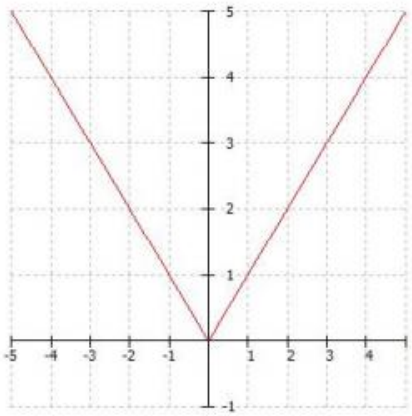


Homework

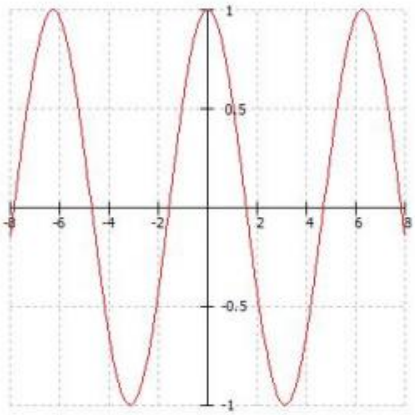
Even Functions



$f(x) = x^4$



$g(x)=|x|$



$h(x) = \cos x$