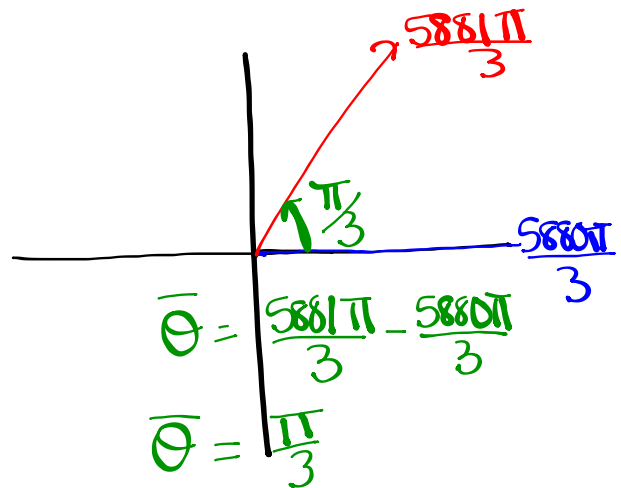


Sketch the following and determine a negative angle co-terminal with:

$$(i) \frac{5881\pi}{3}$$

$$\frac{5880\pi}{3}, \frac{5881\pi}{3}, \frac{5882\pi}{3}$$

$$1960\pi$$



Negative coterminal angle:

$$\frac{5881\pi}{3} - \frac{1962\pi}{1}$$

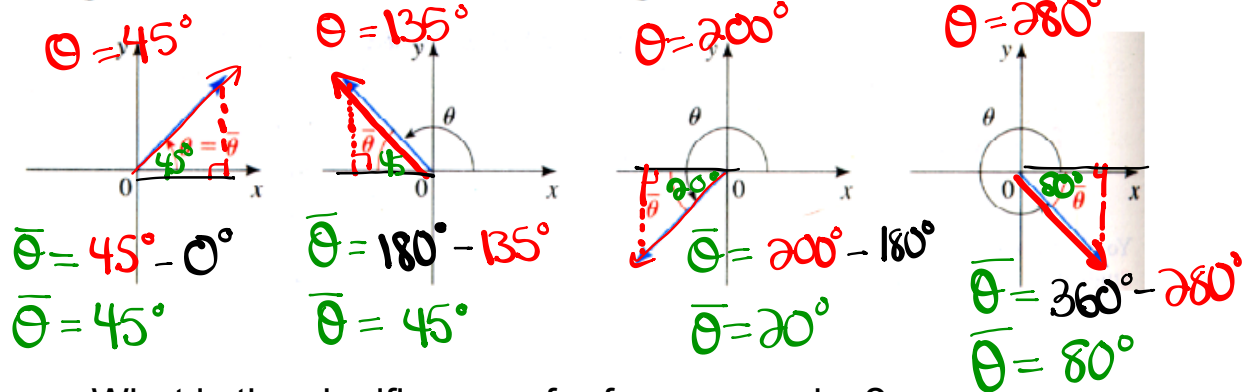
$$\frac{5881\pi}{3} - \frac{5886\pi}{3}$$

$$\frac{-5\pi}{3}$$

## Reference Triangles:

**Definition 17** The reference angle  $\bar{\theta}$  of an angle  $\theta$  in standard position is the acute angle (between 0 and  $90^\circ$ ) the terminal side makes with the x-axis.

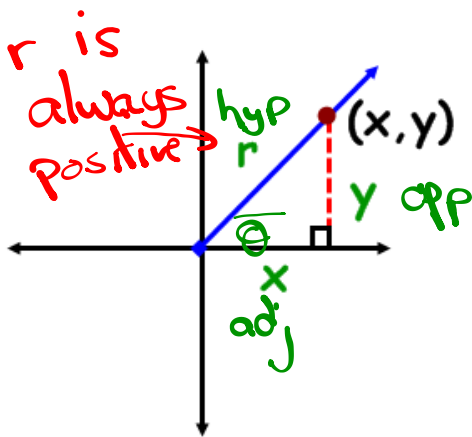
The picture below illustrates this concept.



What is the significance of reference angles?

## Angles on the Cartesian Plane

- **Reference Angle** - an acute angle formed between the terminal arm and the **x-axis**.
- **Reference Triangle** - a triangle formed by drawing a perpendicular line from a point on the terminal to the **x-axis**.



Notice what will happen if the rotation moves into other quadrants?

### TRIG RATIOS on the CARTESIAN PLANE

$$\sin \theta = \frac{y}{r} = \frac{o}{h} \qquad \csc \theta = \frac{r}{y} = \frac{h}{o}$$

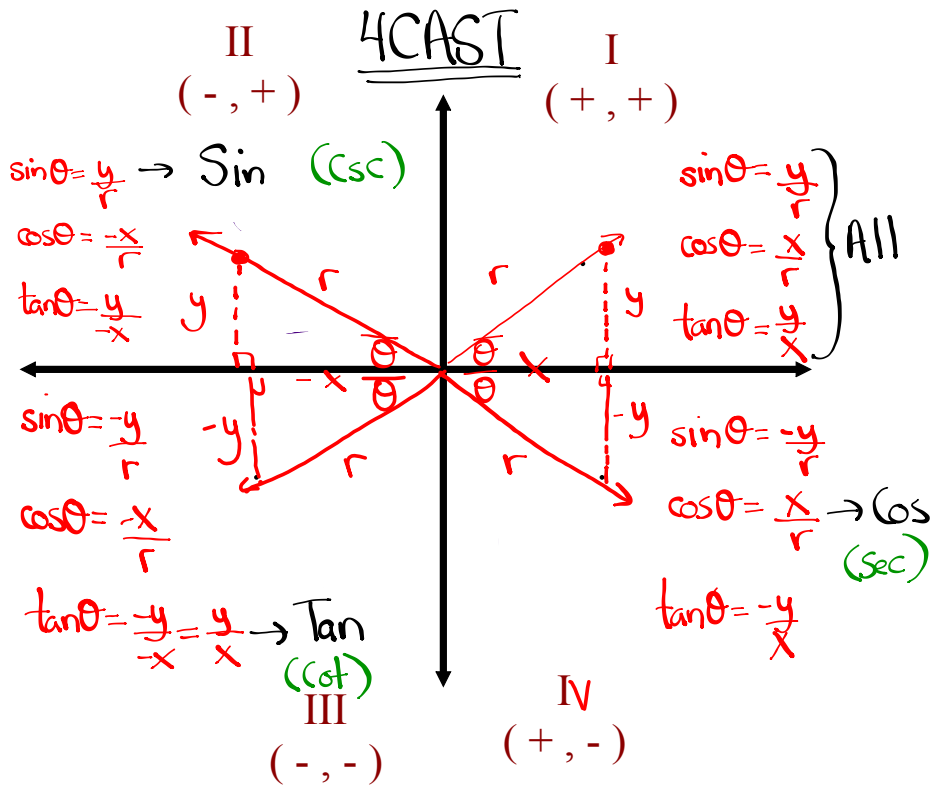
$$\cos \theta = \frac{x}{r} = \frac{a}{h} \qquad \sec \theta = \frac{r}{x} = \frac{h}{a}$$

$$\tan \theta = \frac{y}{x} = \frac{o}{a} \qquad \cot \theta = \frac{x}{y} = \frac{a}{o}$$

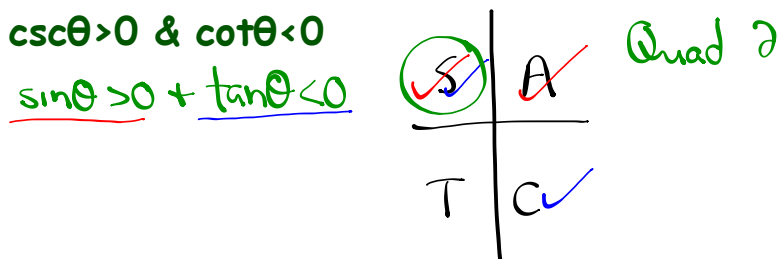
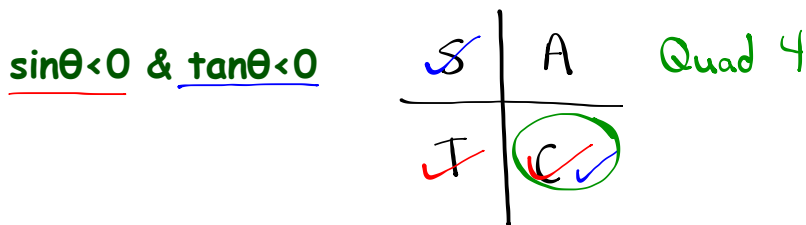
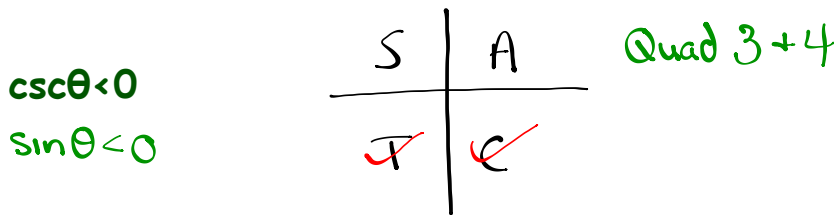
} }  
"Primary"    "Reciprocal"

## TRIG RATIOS IN ALL 4 QUADRANTS

What primary trig ratios are **POSITIVE** in...



Where is  $\theta$  if...



**Homework**

$\sin \theta > 0$	$\sqrt{S}$	A
$\cos \theta < 0$	T	C

Quad 2

$\sec \theta < 0$  →  $\cos \theta < 0$

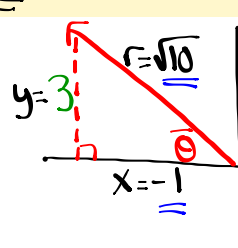
If  $\sec \theta = -\sqrt{10}$  and  $\sin \theta > 0$ , determine the value of  $\csc \theta = \frac{h}{o} = \frac{r}{y}$

$\sec \theta = -\frac{\sqrt{10}}{1} = \frac{\text{hyp}}{\text{adj}} = \frac{r}{x}$

$r = \sqrt{10}$  (always +)

$x = -1$

②  $\csc \theta = \frac{\text{hyp}}{\text{opp}} = \frac{r}{y} = \frac{\sqrt{10}}{3}$



① Find y:

$$x^2 + y^2 = r^2$$

$$(-1)^2 + y^2 = (\sqrt{10})^2$$

$$1 + y^2 = 10$$

$$y^2 = 9$$

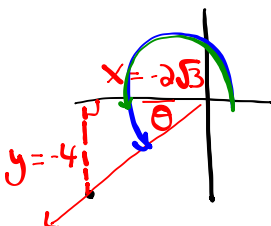
$$y = \pm 3$$

$y = 3$  (Q2)

Determine the measure (in radians) of an angle whose terminal arm passes through the ordered pair  ~~$(-2\sqrt{3}, 4)$~~

$x = -2\sqrt{3}$

$y = -4$



① Find  $\bar{\theta}$

$$\tan \bar{\theta} = \frac{y}{x} = \frac{-4}{-2\sqrt{3}} = \frac{2}{\sqrt{3}}$$

$$\tan \bar{\theta} = 1.1547$$

convert calculator to rads →  $\bar{\theta} = \tan^{-1}(1.1547)$

$\bar{\theta} = \underline{\underline{0.86 \text{ rads}}}$

② Find  $\theta$

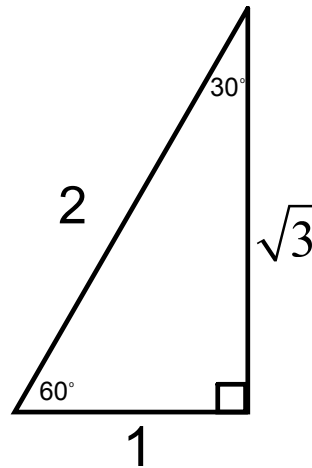
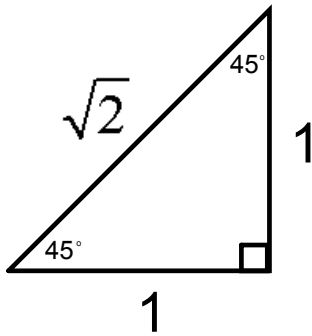
$$\theta = \pi + \bar{\theta}$$

$$\theta = 3.14 + 0.86$$

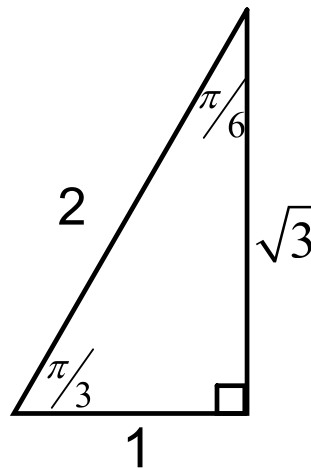
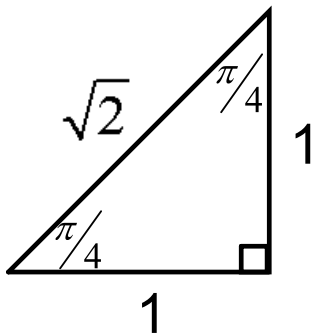
$$\theta = 4 \text{ rads}$$

# Special Angles

**In Degrees:**

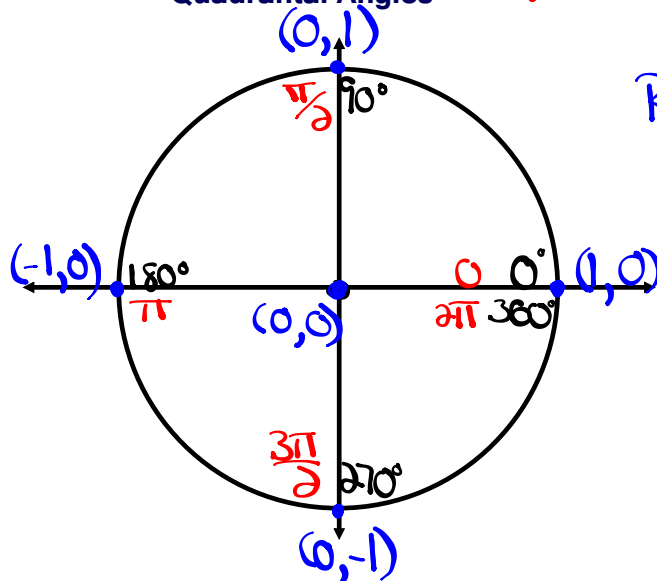


**In Radians:**



Quadrantal Angles

(Multiples of  $90^\circ$  or  $\pi/2$  rads)

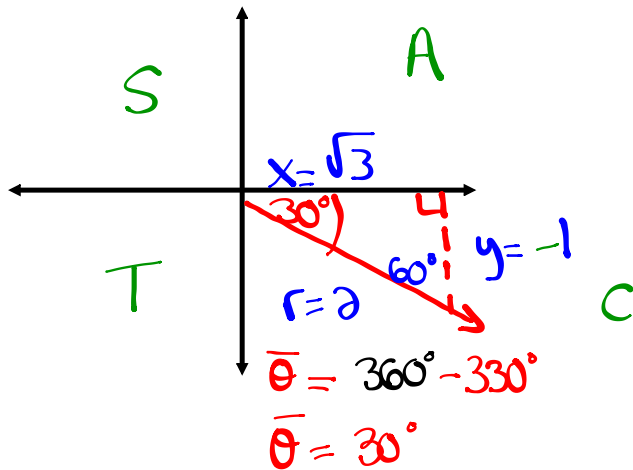


Radius = 1

$$690^\circ - 360^\circ = 330^\circ$$

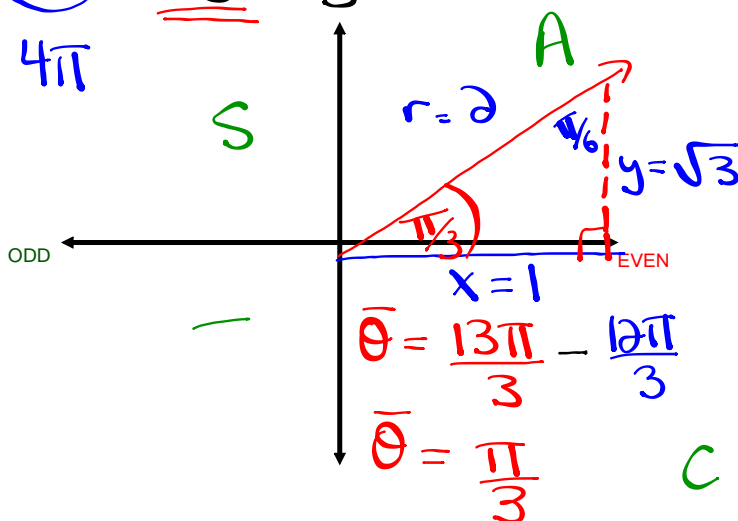
### Solving Trig Expressions by Sketching Angles

Ex. Evaluate the  $\sin 690^\circ = \sin 330^\circ = \frac{-1}{2}$



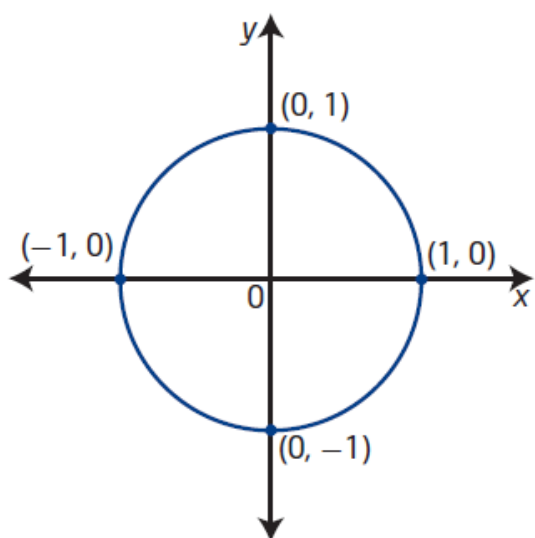
Ex.  $\cos \frac{13\pi}{3} = \frac{1}{2}$

$\frac{12\pi}{3}, \frac{13\pi}{3}, \frac{14\pi}{3}$



1

# Unit Circle



## unit circle

- a circle with radius 1 unit ( $r=1$ )
- a circle of radius 1 unit with centre at the origin on the Cartesian plane is known as *the* unit circle

$$\sin \theta = \frac{o}{h} = \frac{y}{r} = \frac{y}{1} = y$$

$$\cos \theta = \frac{a}{h} = \frac{x}{r} = \frac{x}{1} = x$$

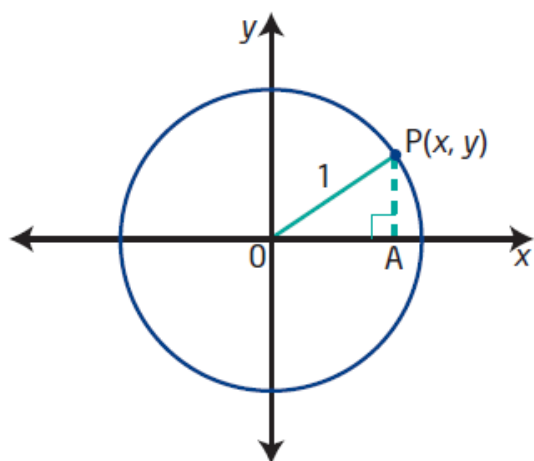
$$\tan \theta = \frac{o}{a} = \frac{y}{x}$$

$$\csc \theta = \frac{1}{y}$$

$$\sec \theta = \frac{1}{x}$$

$$\cot \theta = \frac{x}{y}$$





$$r = 1$$

$$x^2 + y^2 = r^2$$

The equation of the unit circle is  $x^2 + y^2 = 1$ .

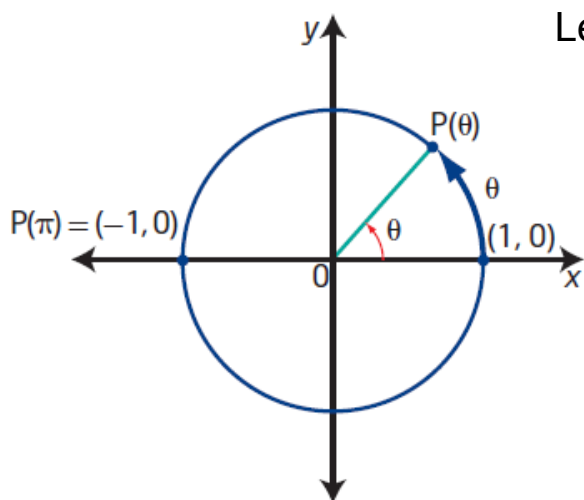
Determine the equation of a circle with centre at the origin and radius 6.

$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = (6)^2$$

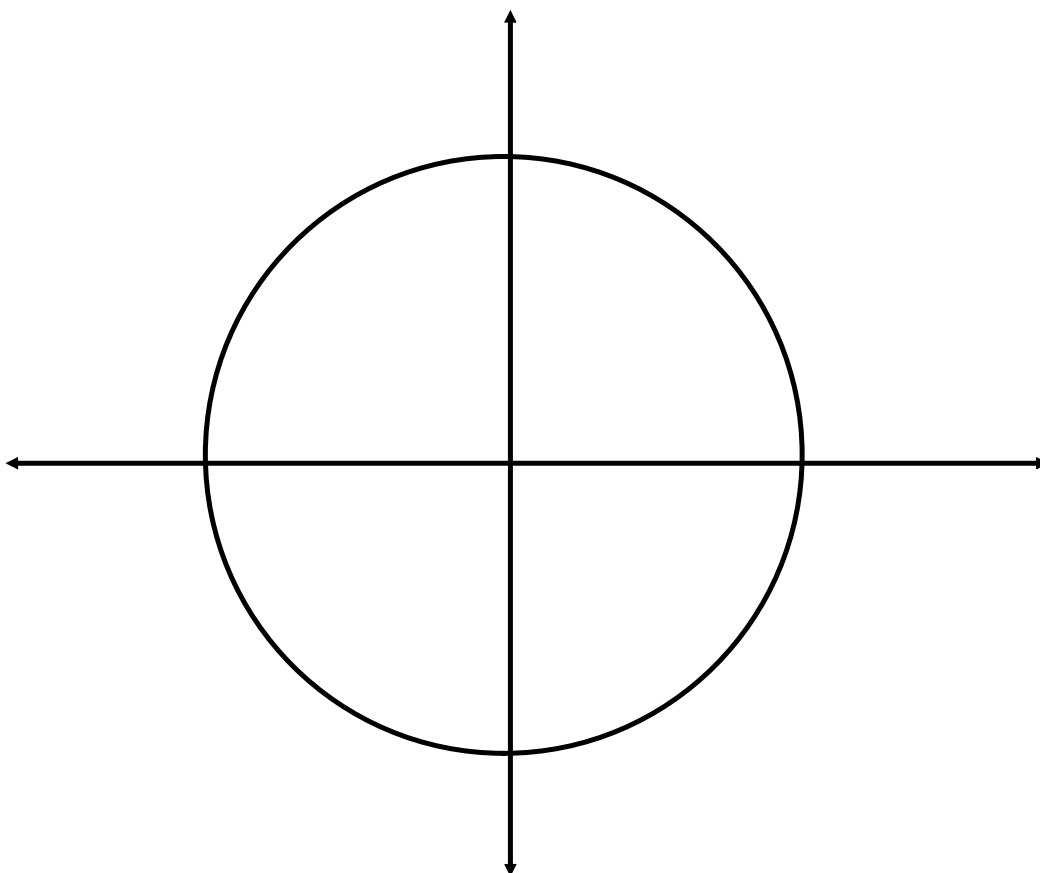
$$x^2 + y^2 = 36$$

## Special Angles on the Unit Circle:

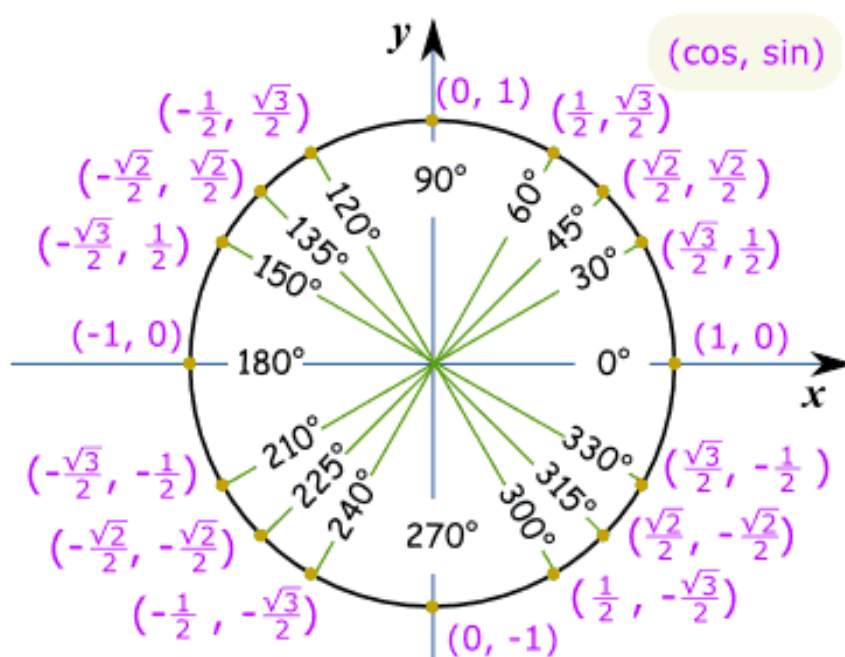


Let's use  $\frac{\pi}{4}$  as our reference angle

Construct reference triangles  
for all multiples of  $\pi/4$   
between 0 and  $2\pi$

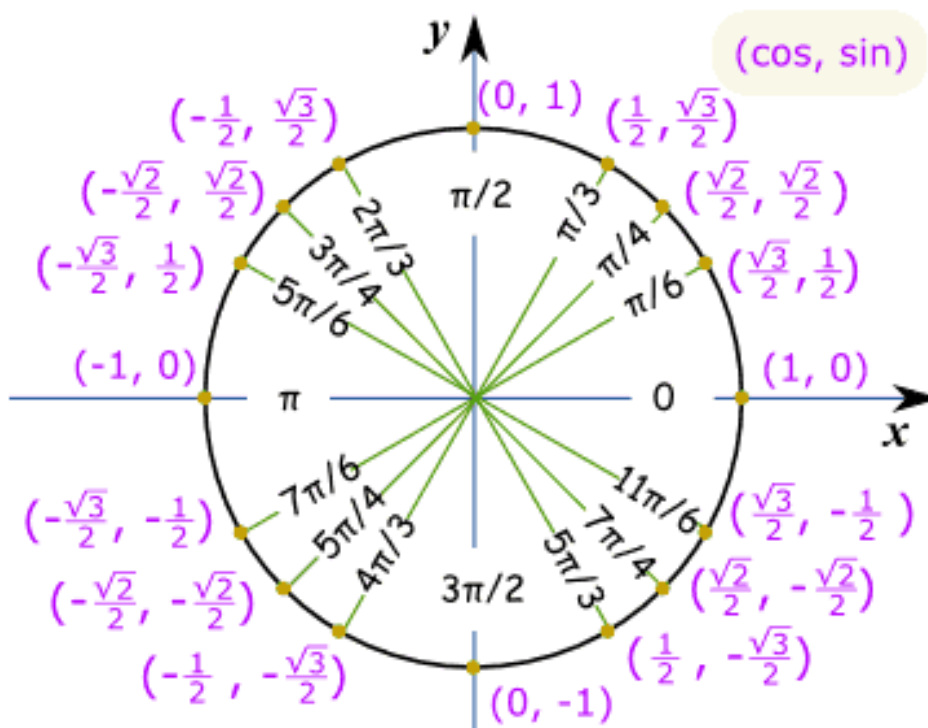


## Unit Circle of Special Angles in Degrees



This is lovely...so what is it used for????

## Unit Circle of Special Angles in Radians

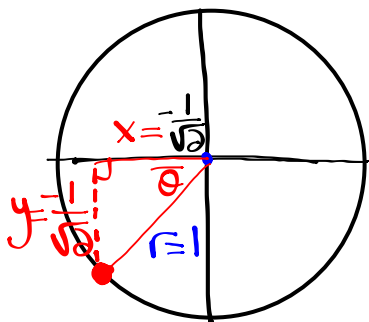


## Problems Involving the Unit Circle:

### Determine Coordinates for Points of the Unit Circle

Determine the coordinates for all points on the unit circle that satisfy the conditions given. Draw a diagram in each case.

- the y-coordinate is  $-\frac{1}{\sqrt{2}}$  and the point is in quadrant III



$$x^2 + y^2 = r^2$$

$$x^2 + \left(\frac{-1}{\sqrt{2}}\right)^2 = (1)^2$$

$$x^2 + \frac{1}{2} = 1$$

$$x^2 = 1 - \frac{1}{2}$$

$$x^2 = \frac{2}{2} - \frac{1}{2}$$

$$x^2 = \frac{1}{2}$$

$$x = \pm \frac{\sqrt{1}}{\sqrt{2}}$$

$$x = \pm \frac{1}{\sqrt{2}}$$

$$x = -\frac{1}{\sqrt{2}} \text{ or } -\frac{\sqrt{2}}{2} \quad (\text{Quad 3})$$

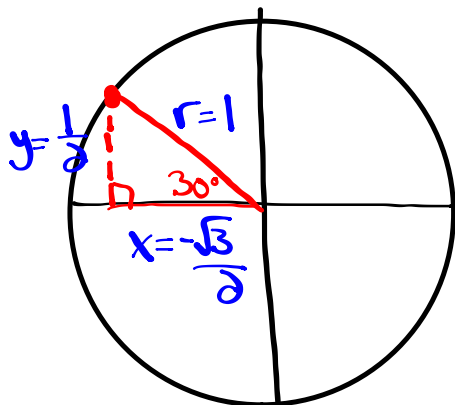
Coordinates are:

$$\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) \text{ or } \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$$

## Problems Involving the Unit Circle:

If  $P(150^\circ)$  is the point at which the terminal arm of an angle  $\theta$  in standard position intersects the unit circle, determine the exact coordinates of...

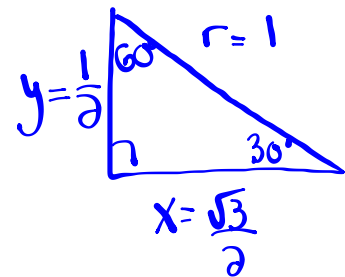
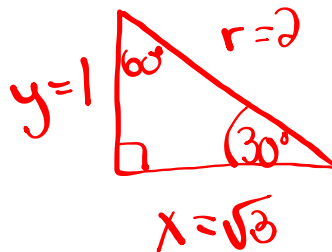
$(x, y)$



$$\bar{\theta} = 180^\circ - 150^\circ$$

$$\bar{\theta} = 30^\circ$$

We will have to scale the special triangle so that  $r=1$



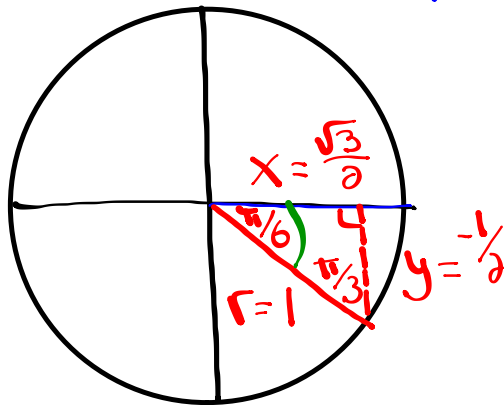
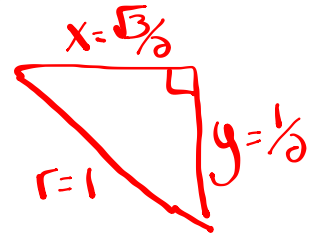
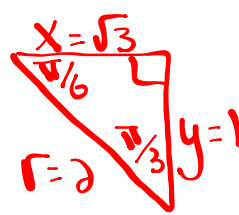
Coordinates are:

$$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$

## Questions from Homework

① c)  $\frac{10\pi}{6}, \frac{11\pi}{6}, \frac{12\pi}{6}$   
 $2\pi$

Scale to fit unit circle



Coordinates are:  
 $(\frac{\sqrt{3}}{2}, -\frac{1}{2})$

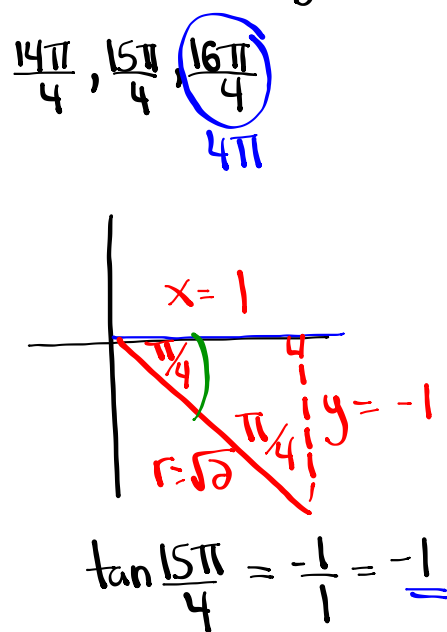
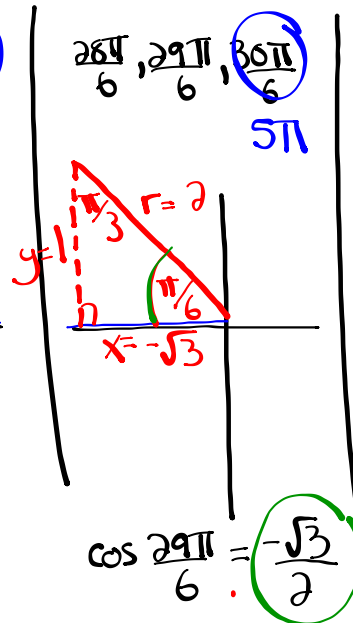
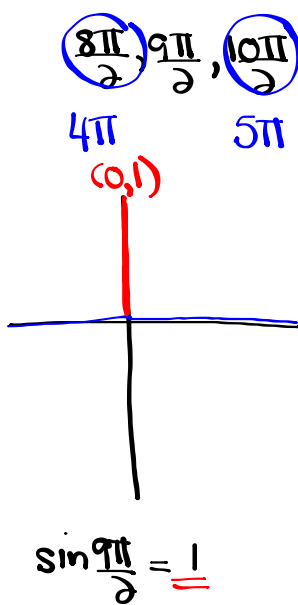
$$\bar{\theta} = \frac{12\pi}{6} - \frac{11\pi}{6}$$

$$\bar{\theta} = \frac{\pi}{6}$$

Evaluate without the use of a calculator:

$$\sin \frac{9\pi}{2} - \cos^2 \left( \frac{29\pi}{6} \right) \tan \left( \frac{15\pi}{4} \right)$$

- ① Sketch angles
- ② Find  $\theta$
- ③ Fill in ref. triangle
- ④ Find trig ratio



$$\sin \frac{9\pi}{2} - \cos^2 \left( \frac{29\pi}{6} \right) \tan \left( \frac{15\pi}{4} \right)$$

$$(1) - \left( \frac{-\sqrt{3}}{2} \right)^2 (-1)$$

$$1 - \left( \frac{3}{4} \right) \left( \frac{-1}{1} \right) \rightarrow \text{or } 1 - \left( \frac{-3}{4} \right)$$

$$1 + \frac{3}{4}$$

$$\frac{4}{4} + \frac{3}{4}$$

$$\frac{7}{4}$$



Evaluate without the use of a calculator:

$$\cos\left(\frac{16\pi}{3}\right) \tan^2\left(\frac{23\pi}{6}\right) + \csc\left(\frac{11\pi}{2}\right) + \sin^2\left(\frac{27\pi}{4}\right)$$

(Q3)
(Q4)
Border of 3 and 4
(Q2)

$$\left(-\frac{1}{2}\right) \left(-\frac{1}{\sqrt{3}}\right)^2 + \left(-\frac{1}{-1}\right) + \left(+\frac{1}{\sqrt{2}}\right)^2$$

$$-\frac{1}{2} \left(\frac{1}{3}\right) + (-1) + \frac{1}{2}$$

$$-\frac{1}{6} - 1 + \frac{1}{2}$$

$$-\frac{1}{6} - \frac{6}{6} + \frac{3}{6}$$

$$-\frac{4}{6}$$

-
3

# Homework:

Worksheet - Sketching Angles in Radians.doc

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## Solutions...

1.  $-\frac{5}{3}$

5.  $\frac{4+3\sqrt{3}}{6}$

2.  $\frac{-\sqrt{6}}{3}$

6.  $\frac{-10}{3}$

3.  $-2-\sqrt{3}$

7. 0

4.  $\frac{-5}{3}$

8.  $\frac{3+3\sqrt{3}}{-2}$

## Attachments

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Worksheet - Sketching Angles in Radians.doc