Questions from Homework

8 $\dfrac{\tan\left(-\frac{15\pi}{4}\right)+\sec\left(\frac{301\pi}{3}\right)}{\csc\left(\frac{25\pi}{2}\right)+\cot\left(-\frac{31\pi}{6}\right)}$

$-\frac{15\pi}{4}+\frac{16\pi}{4}=\frac{\pi}{4}$

$-\frac{31\pi}{6}+\frac{36\pi}{6}=\frac{5\pi}{6}$

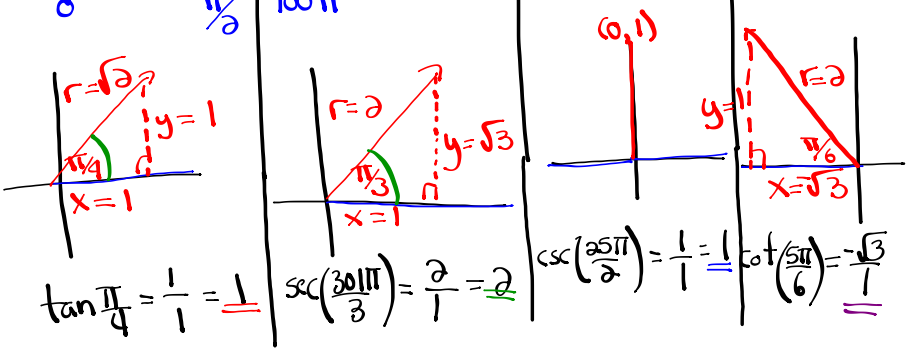
$\dfrac{\tan\left(\frac{\pi}{4}\right)+\sec\left(\frac{301\pi}{3}\right)}{\csc\left(\frac{25\pi}{2}\right)+\cot\left(\frac{5\pi}{6}\right)}$

$\frac{0\pi}{4}, \frac{1\pi}{4}, \frac{2\pi}{4}$ → 0, $\pi/2$

$\frac{300\pi}{3}, \frac{301\pi}{3}, \frac{302\pi}{3}$ → 100π

$\frac{24\pi}{2}, \frac{25\pi}{2}, \frac{26\pi}{2}$ → 12π, 13π

$\frac{4\pi}{6}, \frac{5\pi}{6}, \frac{6\pi}{6}$ → π



$\tan\frac{\pi}{4}=\frac{1}{1}=1$

$\sec\left(\frac{301\pi}{3}\right)=\frac{2}{1}=2$

$\csc\left(\frac{25\pi}{2}\right)=\frac{1}{1}=1$

$\cot\left(\frac{5\pi}{6}\right)=\frac{-\sqrt{3}}{1}$

$\dfrac{\tan\left(\frac{\pi}{4}\right)+\sec\left(\frac{301\pi}{3}\right)}{\csc\left(\frac{25\pi}{2}\right)+\cot\left(\frac{5\pi}{6}\right)}$

$\dfrac{1+2}{1+(-\sqrt{3})}$

$\dfrac{3}{(1-\sqrt{3})}\cdot\dfrac{(1+\sqrt{3})}{(1+\sqrt{3})}$

$\dfrac{3+3\sqrt{3}}{1+\sqrt{3}-\sqrt{3}-3}$

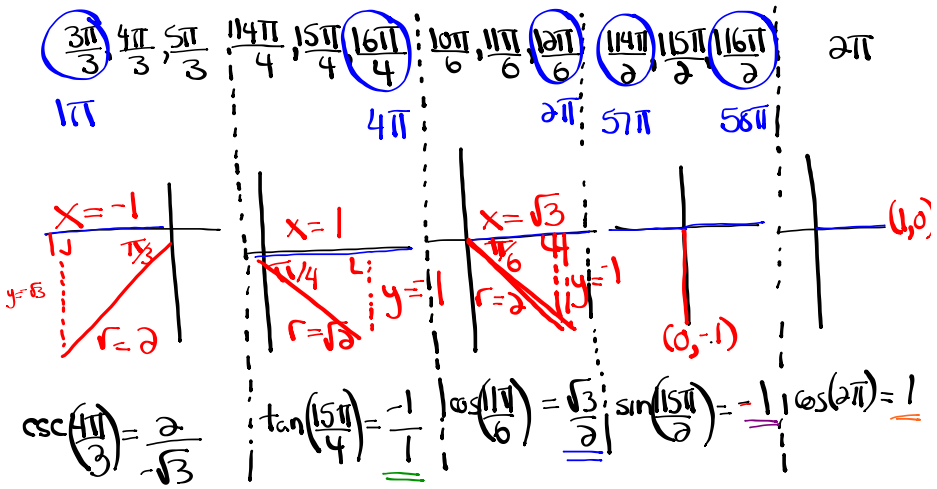
$\dfrac{3+3\sqrt{3}}{-2}$ or $-\dfrac{3-3\sqrt{3}}{2}$

Questions from Homework

$-\frac{13\pi}{6}+\frac{24\pi}{6}=\frac{11\pi}{6}$ $\quad -14\pi+16\pi=2\pi$

⑤ $\csc^2\left(\frac{4\pi}{3}\right)\tan\left(\frac{15\pi}{4}\right)+\cos\left(-\frac{13\pi}{6}\right)-\sin\left(\frac{115\pi}{2}\right)+\cos(-14\pi)$

$\csc^2\left(\frac{4\pi}{3}\right)\tan\left(\frac{15\pi}{4}\right)+\cos\left(\frac{11\pi}{6}\right)-\sin\left(\frac{115\pi}{2}\right)+\cos(2\pi)$



$\csc\left(\frac{4\pi}{3}\right)=\frac{2}{-\sqrt{3}}$ $\quad \tan\left(\frac{15\pi}{4}\right)=\frac{-1}{1}$ $\quad \cos\left(\frac{11\pi}{6}\right)=\frac{\sqrt{3}}{2}$ $\quad \sin\left(\frac{15\pi}{2}\right)=-1$ $\quad \cos(2\pi)=1$

$\csc^2\left(\frac{4\pi}{3}\right)\tan\left(\frac{15\pi}{4}\right)+\cos\left(\frac{11\pi}{6}\right)-\sin\left(\frac{15\pi}{2}\right)+\cos(2\pi)$

$\left(-\frac{2}{\sqrt{3}}\right)^2(-1)+\left(\frac{\sqrt{3}}{2}\right)-(-1)+(1)$

$\left(\frac{4}{3}\right)(-1)+\frac{\sqrt{3}}{2}+1+1$

$-\frac{4}{3}+\frac{\sqrt{3}}{2}+\frac{2}{1}$

$-\frac{8}{6}+\frac{3\sqrt{3}}{6}+\frac{12}{6}$

$\boxed{\frac{4+3\sqrt{3}}{6}}$ or $\frac{3\sqrt{3}+4}{6}$

Introduction to Trigonometric Equations

trigonometric equation

- an equation involving trigonometric ratios

Focus on...

- algebraically solving first-degree and second-degree trigonometric equations in radians and in degrees
- verifying that a specific value is a solution to a trigonometric equation
- identifying exact and approximate solutions of a trigonometric equation in a restricted domain
- determining the general solution of a trigonometric equation

Did You Know?

In equations, mathematicians often use the notation $\cos^2 \theta$. This means the same as $(\cos \theta)^2$.

Let's start with basic LINEAR trigonometric equations...

...Pre-Calculus 110

Solve: $\sin\theta = 0.9659\ ,\ -360° < \theta < 720°$ (Degrees)

- Reference angle?
- Which quadrants?
- Any co-terminal angles acceptable?

- If the domain is in degrees, give solutions in degrees.
- If the domain is in radians, give solutions in radians.

$\sin\theta = 0.9659$ use positive for $\bar{\theta}$ where is $\sin\theta > 0$ (positive)

$\bar{\theta} = \sin^{-1}(0.9659)$

$\bar{\theta} = 75°$

Q1	Q2
$\theta = \bar{\theta}$	$\theta = 180° - \bar{\theta}$
$\theta = 75°$	$\theta = 180° - 75° = 105°$
$\theta = 75° - 360° = -285°$	$\theta = 105° - 360° = -255°$
$\theta = 75° + 360° = 435°$	$\theta = 105° + 360° = 465°$

Solve: $\sec\theta = \frac{-1.3054}{1}$, $-2\pi \le \theta \le 2\pi$ (Radians)

-6.28 6.28

$\cos\theta = \frac{1}{-1.3054}$

$\cos\theta = -0.7660$

where is $\cos < 0$ (negative)

use positive for $\bar{\theta}$

$\bar{\theta} = \cos^{-1}(0.7660)$

$\bar{\theta} = 0.7$ rads.

Q2	Q3
$\theta = \pi - \bar{\theta}$	$\theta = \pi + \bar{\theta}$
$\theta = 3.14 - 0.7 = 2.44$	$\theta = 3.14 + 0.7 = 3.84$
$\theta = 2.44 - 6.28 = -3.84$	$\theta = 3.84 - 6.28 = -2.44$

Exact Value

Ex. $\sqrt{2}\cos\theta + 1 = 0, \quad -360° \le \theta \le 720°$ (Degrees)

$\frac{\sqrt{2}\cos\theta}{\sqrt{2}} = \frac{-1}{\sqrt{2}}$

$\cos\theta = \frac{-1}{\sqrt{2}}$

where is $\cos < 0$ (negative)

$\cos\theta = -0.7071$

use positive for $\bar{\theta}$

$\bar{\theta} = \cos^{-1}(0.7071)$

$\bar{\theta} = 45°$

Q2	Q3
$\theta = 180° - \bar{\theta}$	$\theta = 180° + \bar{\theta}$
$\theta = 180° - 45° = 135°$	$\theta = 180° + 45° = 225°$
$\theta = 135° - 360° = -225°$	$\theta = 225° - 360° = -135°$
$\theta = 135° + 360° = 495°$	$\theta = 225° + 360° = 585°$

$-\frac{4\pi}{2} \leq x \leq \frac{8\pi}{2}$

Ex. $\sin x + 1 = 0, \quad -2\pi \leq x \leq 4\pi$ (Radians)

$\sin x = -1$

(Unit Circle)

$x = \frac{3\pi}{2}$

$\frac{3\pi}{2} - \frac{2\pi}{1}$ | $\frac{3\pi}{2} + \frac{2\pi}{1}$

$\frac{3\pi}{2} - \frac{4\pi}{2}$ | $\frac{3\pi}{2} + \frac{4\pi}{2}$

$\frac{-\pi}{2}$ | $\frac{7\pi}{2}$

Your Turn

Solve each trigonometric equation in the specified domain.

a) $3\cos\theta - 1 = \cos\theta + 1$, $-2\pi \le \theta \le 2\pi$

b) $4\sec x + 8 = 0$, $0° \le x < 360°$

a) $3\cos\theta - 1 = \cos\theta + 1$, $-2\pi \le \theta \le 2\pi$ (Radians)

$3\cos\theta - \cos\theta = 1 + 1$

$2\cos\theta = 2$

$\cos\theta = 1$

(Unit Circle)

$\theta = 0, 2\pi, -2\pi$

$\theta = 0 - 2\pi = -2\pi$

b) $4\sec x + 8 = 0$ $0° \le x < 360°$ (Degrees)

$\frac{4\sec x}{4} = \frac{-8}{4}$

$\sec x = \frac{-2}{1}$

* $\cos x = -\frac{1}{2}$ where is $\cos x < 0$ (Negative)

$\bar{x} = 60°$

Q2	Q3
$\theta = 180° - \bar{\theta}$	$\theta = 180° + \bar{\theta}$
$\theta = 180° - 60°$	$\theta = 180° + 60°$
$\theta = 120°$	$\theta = 240°$

Homework

Page 211 #1-5

Front

① $\sin\theta = -\frac{\sqrt{3}}{2}$ opp / hyp

where is $\sin\theta < 0$ (negative)

$\sin\theta = -0.8660$

$\bar{\theta} = \sin^{-1}(0.8660)$

$\bar{\theta} = 60°$

Q3	Q4
$\theta = 180° + \bar{\theta}$	$\theta = 360° - \bar{\theta}$
$\theta = 180° + 60°$	$\theta = 360° - 60°$
$\theta = 240°$	$\theta = 300°$
$240° \pm 360°n, n \in \mathbb{N}$	$300° \pm 360°n, n \in \mathbb{N}$

From back of sheet:

⑨ $\sin\theta = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

$\sin\theta = \frac{1}{\sqrt{2}}$

where is $\sin\theta > 0$ (Positive)

$\bar{\theta} = \frac{\pi}{4}$

Q1	Q2
$\theta = \bar{\theta}$	$\theta = \pi - \bar{\theta}$
$\theta = \frac{\pi}{4}$	$\theta = \pi - \frac{\pi}{4}$
	$\theta = \frac{4\pi}{4} - \frac{\pi}{4}$
	$\theta = \frac{3\pi}{4}$

Questions from Homework

Back

③ $\tan\theta = -\frac{\sqrt{3}}{3} = -\frac{1}{\sqrt{3}}$ Where is $\tan\theta < 0$ (Negative)

(Triangle) $\bar{\theta} = 30°$

Q2	Q4
$\theta = 180° - \bar{\theta}$	$\theta = 360° - \bar{\theta}$
$\theta = 180° - 30° = 150°$	$\theta = 360° - 30° = 330°$
$150° \pm 360°n, n \in N$	$330° \pm 360°n, n \in N$

Back

⑫ $\tan\theta =$ undefined $\tan\theta = \frac{o}{a} = \frac{y}{x}$

(Unit Circle)

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

Front

⑫ $5\sin\theta - 4 = 0$ (Approximate Value)

$\frac{5\sin\theta}{5} = \frac{4}{5}$

$\sin\theta = 0.8$

$\bar{\theta} = \sin^{-1}(0.8)$

$\bar{\theta} = 53.1°$

Where is $\sin\theta > 0$ (Positive)

Q1	Q2
$\theta = \bar{\theta}$	$\theta = 180° - \bar{\theta}$
$\theta = 53.1°$	$\theta = 180° - 53.1°$
	$\theta = 126.9°$
$53.1° \pm 360°n, n \in N$	$126.9° \pm 360°n, n \in N$

Check-Up:

Solve:

① $\cot\theta = 0.7834, \quad -\frac{\pi}{2} < \theta < -\pi$

② $3\cos x + 5 = 6, \quad -360^\circ \leq x \leq 720^\circ$

③ $2\sin x(1-\sin x) = 0, \quad -4\pi < x < 4\pi$

① $\cot\theta = 0.7834$, $-\frac{\pi}{2} < \theta < -\pi$

② $3\cos x + 5 = 6$, $-360° \leq x \leq 720°$

③ $2\csc x(1-\csc x)=0, \quad -4\pi<x<4\pi$

Let's move onto QUADRATIC trigonometric equations...

...Pre-Calculus 110

- What strategies can we use to solve quadratic equations?
- Quadratic trigonometric equations will ultimately become TWO linear trigonometric equations.

Solve: $2x^2 + x = 1$

$2x^2 + x - 1 = 0$ $\quad 2 \times -1 = -2$, $2 + -1 = 1$

$(x+2)(x-1) = 0$

$(x+1)(2x-1) = 0$

$x+1=0 \quad | \quad 2x-1=0$

$x = -1 \quad | \quad \frac{2x}{2} = \frac{1}{2}$

$x = \frac{1}{2}$

Solve: $2\sin^2 x + \sin x = 1, \quad 0 \le x \le 4\pi$ (Radians)

$2\sin^2 x + \sin x - 1 = 0$ $\quad 2 \times -1 = -2$, $2 + -1 = 1$

$(\sin x + 2)(\sin x - 1) = 0$

$(\sin x + 1)(2\sin x - 1) = 0$

$\sin x + 1 = 0$ | $2\sin x - 1 = 0$

$\sin x = -1$ (Unit Circle) | $\frac{2\sin x}{2} = \frac{1}{2}$

$\sin x = \frac{1}{2}$ (Triangle)

$x = \frac{3\pi}{2}$ | $\bar{x} = \frac{\pi}{6}$

$x = \frac{3\pi}{2} + 2\pi$ | Where is $\sin\theta > 0$

$= \frac{3\pi}{2} + \frac{4\pi}{2}$ | Q1 | Q2

$= \frac{7\pi}{2}$ | $x = \bar{x}$ | $x = \pi - \bar{x}$

$x = \frac{\pi}{6}$ | $x = \pi - \frac{\pi}{6}$

$x = \frac{6\pi}{6} - \frac{\pi}{6}$

$x = \frac{5\pi}{6}$

$x = \frac{\pi}{6} + 2\pi$ | $x = \frac{5\pi}{6} + 2\pi$

$x = \frac{\pi}{6} + \frac{12\pi}{6}$ | $x = \frac{5\pi}{6} + \frac{12\pi}{6}$

$x = \frac{13\pi}{6}$ | $x = \frac{17\pi}{6}$

Factoring trinomials:

① Decomposition

$2x^2+7x+6$ $\quad 3 \times 4 = 12$, $\quad 3+4=7$

$(x+3)(x+4)$

$(2x+3)(x+2)$

② Simple trinomial

x^2+7x+6 $\quad 6 \times 1 = 6$, $\quad 6+1=7$

$(x+1)(x+6)$

Ex. $\cos^2\theta - \frac{1}{2}\cos\theta = 0, -2\pi \le \theta \le 4\pi$ (Common Factor)

Ex. $6\sin^2 x - \sin x = 2, -2\pi \le \theta \le 4\pi$ (Decomposition)

Your Turn

Solve for θ.

$\cos^2\theta - \cos\theta - 2 = 0,\ 0° \leq \theta < 360°$ (Simple Trinomial)

Give solutions as exact values where possible. Otherwise, give approximate measures to the nearest thousandth of a degree.

General Solution of a Trigonometric Equation

$Solve: \quad 3\cos^2\theta - \cos\theta = 2; \ \theta \in \mathbb{R}$ (Decomposition)

Determine the general solution for $\sin^2 x - 1 = 0$ over the real numbers if x is measured in radians. (Difference of Squares)

Did You Know?

$2n$, where $n \in \mathrm{I}$, represents all even integers.

$2n + 1$, where $n \in \mathrm{I}$, is an expression for all odd integers.

$\sin^2 x - 1 = 0$

$(\sin x + 1)(\sin x - 1) = 0$

$\sin x + 1 = 0$	$\sin x - 1 = 0$
$\sin x = -1$	$\sin x = 1$
(Unit Circle)	(Unit Circle)
$x = \frac{3\pi}{2}$	$x = \frac{\pi}{2}$
$x = \frac{3\pi}{2} + 2\pi n, n \in \mathbb{N}$	$x = \frac{\pi}{2} + 2\pi n, n \in \mathbb{N}$

$x = \frac{\pi}{2} + 2\pi n$, where $n \in \mathrm{I}$

$x = \frac{3\pi}{2} + 2\pi n$, where $n \in \mathrm{I}$

or

$x = \frac{\pi}{2} + \pi n$, where $n \in \mathrm{I}$

or

$(2n + 1)\left(\frac{\pi}{2}\right)$, $n \in \mathrm{I}$

Determine the general solution for $\cos^2 x - 1 = 0$, where the domain is real numbers measured in degrees.

Practice Problems:

Pages 212 - 214
#7-9, 11-13, 16, 18, 22

Check-Up problem...

Solve: $\sin x \sec x + 2\sin x = 0 \ , \ x \in R$ (x is measured in radians)

Unit Review...

What topics have we covered??

Review...

C4 **a)** Determine all solutions for the equation $2\sin^2\theta = 1 - \sin\theta$ in the domain $0° \leq \theta < 360°$.

b) Are your solutions exact or approximate? Why?

c) Show how you can check one of your solutions to verify its correctness.

A grandfather clock shows a time of 7 o'clock. What is the exact radian measure of the angle between the hour hand and the minute hand?

Determine the angular velocity of the minute hand on a clock.

Solve: $6\sin^2\theta - 3\sin\theta = 0$, $0 \le \theta \le 360°$

[A] $0°, 30°, 180°, 330°, 360°$

[B] $0°, 30°, 180°, 150°, 360°$

[C] $30°, 90°, 120°, 270°$

[D] $0°, 180°, 210°, 330°, 360°$

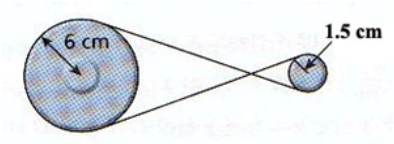
If $\csc\theta < 0$ and $\tan\theta > 0$, then which of the following could be a possible measure of angle θ?

[A] $\frac{11\pi}{6}$ [B] $\frac{4\pi}{3}$ [C] $\frac{3\pi}{4}$ [D] $\frac{\pi}{2}$

What is the principal angle of $-\frac{25\pi}{4}$?

[A] $\frac{3\pi}{4}$ [B] $\frac{\pi}{4}$ [C] $-\frac{\pi}{4}$ [D] $\frac{7\pi}{4}$

If the belt in the pulley system below travels 30 cm, what is the angle of rotation of the smaller pulley?



[A] $\frac{\pi}{9}$ radians [B] 20°

[C] 20 radians [D] 5°

Nibbles the hamster is running at 0.02 m/s on an exercise wheel of radius 8 cm. What is the angular velocity of this wheel?

[A] 0.15 rad/minute [B] 240 rad/minute [C] 0.25 rad/minute [D] 15 radians/minute

Solve: $2(1-\sin\theta)^2+\sin\theta=2(3-4\sin^2\theta)$, $-360°\le\theta\le720°$

Little Johnny has a rock tied to the end of a piece of rope 1.5 m long and he is swinging it around his head in a circular pattern. Mrs. Centripetal, his physics teacher, is watching Johnny out the window of her physics lab and notes that the rock is making 12 revolutions every 48 seconds.

(a) Determine the angular velocity with which little Johnny is twirling the rope above his head. [2]

(b) The rock comes flying from the rope 3 minutes after Mrs. Centripetal started to time little Johnny. How far did the rock travel during the 3 minutes? [2]

Attachments

Worksheet - Sketching Angles in Radians.doc