Questions from Homework

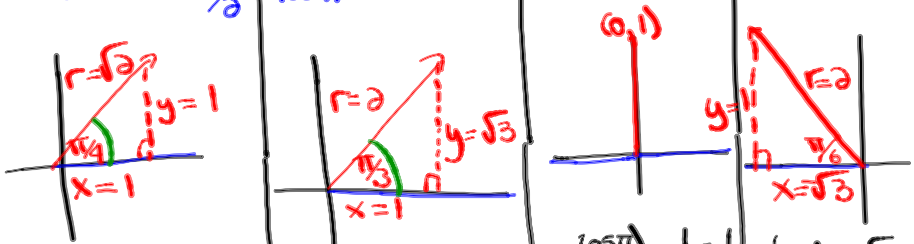
8) $$\frac{\tan\left(-\frac{15\pi}{4}\right) + \sec\left(\frac{301\pi}{3}\right)}{\csc\left(\frac{25\pi}{2}\right) + \cot\left(-\frac{31\pi}{6}\right)}$$

$$-\frac{15\pi}{4} + \frac{16\pi}{4} = \frac{\pi}{4}$$

$$-\frac{31\pi}{6} + \frac{36\pi}{6} = \frac{5\pi}{6}$$

$$\frac{\tan\left(\frac{\pi}{4}\right) + \sec\left(\frac{301\pi}{3}\right)}{\csc\left(\frac{25\pi}{2}\right) + \cot\left(\frac{5\pi}{6}\right)}$$

$\frac{0\pi}{4}, \frac{1\pi}{4}, \frac{2\pi}{4}$	$\frac{300\pi}{3}, \frac{301\pi}{3}, \frac{302\pi}{3}$	$\frac{24\pi}{2}, \frac{25\pi}{2}, \frac{26\pi}{2}$	$\frac{4\pi}{6}, \frac{5\pi}{6}, \frac{6\pi}{6}$
0 $\quad$ $\pi/2$	100π	12π $\quad$ 13π	1π



$\tan\frac{\pi}{4} = \frac{1}{1} = 1$	$\sec\left(\frac{301\pi}{3}\right) = \frac{2}{1} = 2$	$\csc\left(\frac{25\pi}{2}\right) = \frac{1}{1} = 1$	$\cot\left(\frac{5\pi}{6}\right) = \frac{-\sqrt{3}}{1}$

$$\frac{\boxed{\tan\left(\frac{\pi}{4}\right)} + \boxed{\sec\left(\frac{301\pi}{3}\right)}}{\boxed{\csc\left(\frac{25\pi}{2}\right)} + \boxed{\cot\left(\frac{5\pi}{6}\right)}}$$

$$\frac{1+2}{1+(-\sqrt{3})}$$

$$\frac{3}{(1-\sqrt{3})} \cdot \frac{(1+\sqrt{3})}{(1+\sqrt{3})}$$

$$\frac{3+3\sqrt{3}}{1+\cancel{\sqrt{3}}-\cancel{\sqrt{3}}-3}$$

$$\boxed{\frac{3+3\sqrt{3}}{-2}} \quad \text{or} \quad \boxed{-\frac{3-3\sqrt{3}}{2}}$$

Questions from Homework

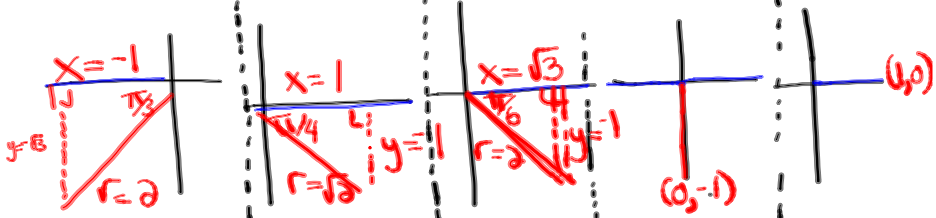
$-\frac{13\pi}{6}+\frac{24\pi}{6}=\frac{11\pi}{6}$ $\quad$ $-14\pi+16\pi=2\pi$

(5) $\csc^2\left(\frac{4\pi}{3}\right)\tan\left(\frac{15\pi}{4}\right)+\cos\left(-\frac{13\pi}{6}\right)-\sin\left(\frac{115\pi}{2}\right)+\cos(-14\pi)$

$\csc^2\left(\frac{4\pi}{3}\right)\tan\left(\frac{15\pi}{4}\right)+\cos\left(\frac{11\pi}{6}\right)-\sin\left(\frac{115\pi}{2}\right)+\cos(2\pi)$

$\frac{3\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ | $\frac{14\pi}{4}, \frac{15\pi}{4}, \frac{16\pi}{4}$ | $\frac{10\pi}{6}, \frac{11\pi}{6}, \frac{12\pi}{6}$ | $\frac{114\pi}{2}, \frac{115\pi}{2}, \frac{116\pi}{2}$ | 2π

1π $\quad$ 4π $\quad$ 2π $\quad$ 57π $\quad$ 58π



$\csc\left(\frac{4\pi}{3}\right)=\frac{2}{-\sqrt{3}}$ | $\tan\left(\frac{15\pi}{4}\right)=\frac{-1}{1}$ | $\cos\left(\frac{11\pi}{6}\right)=\frac{\sqrt{3}}{2}$ | $\sin\left(\frac{115\pi}{2}\right)=-1$ | $\cos(2\pi)=1$

$\boxed{\csc^2\left(\frac{4\pi}{3}\right)}\boxed{\tan\left(\frac{15\pi}{4}\right)}+\boxed{\cos\left(\frac{11\pi}{6}\right)}-\boxed{\sin\left(\frac{115\pi}{2}\right)}+\boxed{\cos(2\pi)}$

$\left(-\frac{2}{\sqrt{3}}\right)^2(-1)+\left(\frac{\sqrt{3}}{2}\right)-(-1)+(1)$

$\left(\frac{4}{3}\right)(-1)+\frac{\sqrt{3}}{2}+1+1$

$-\frac{4}{3}+\frac{\sqrt{3}}{2}+\frac{2}{1}$

$-\frac{8}{6}+\frac{3\sqrt{3}}{6}+\frac{12}{6}$

$\boxed{\frac{4+3\sqrt{3}}{6}}$ or $\frac{3\sqrt{3}+4}{6}$

Introduction to Trigonometric Equations

trigonometric equation

- an equation involving trigonometric ratios

Focus on…

- algebraically solving first-degree and second-degree trigonometric equations in radians and in degrees
- verifying that a specific value is a solution to a trigonometric equation
- identifying exact and approximate solutions of a trigonometric equation in a restricted domain
- determining the general solution of a trigonometric equation

Did You Know?

In equations, mathematicians often use the notation $\cos^2 \theta$. This means the same as $(\cos \theta)^2$.

Let's start with basic LINEAR trigonometric equations...

...Pre-Calculus 110

- Reference angle?
- Which quadrants?
- Any co-terminal angles acceptable?

Solve: $\sin\theta = 0.9659$, $-360° < \theta < 720°$

(Degrees)

- If the domain is in degrees, give solutions in degrees.
- If the domain is in radians, give solutions in radians.

$\sin\theta = 0.9659$ use positive for $\bar{\theta}$

$\bar{\theta} = \sin^{-1}(0.9659)$

$\bar{\theta} = 75°$

where is $\sin\theta > 0$ (positive)

Q1	Q2
$\theta = \bar{\theta}$	$\theta = 180° - \bar{\theta}$
$\theta = 75°$	$\theta = 180° - 75° = 105°$
$\theta = 75° - 360° = -285°$	$\theta = 105° - 360° = -255°$
$\theta = 75° + 360° = 435°$	$\theta = 105° + 360° = 465°$

Solve: $\sec\theta = -1.3054$, $-2\pi \le \theta \le 2\pi$

$-6.28 \qquad 6.28$

(Radians)

$\cos\theta = \frac{1}{-1.3054}$

where is cos < 0 (negative)

$\cos\theta = -0.7660$

use positive for $\bar{\theta}$

$\bar{\theta} = \cos^{-1}(0.7660)$

$\bar{\theta} = 0.7$ rads.

Q2	Q3
$\theta = \pi - \bar{\theta}$	$\theta = \pi + \bar{\theta}$
$\theta = 3.14 - 0.7 = 2.44$	$\theta = 3.14 + 0.7 = 3.84$
$\theta = 2.44 - 6.28 = -3.84$	$\theta = 3.84 - 6.28 = -2.4$

Exact Value

Ex. $\sqrt{2}\cos\theta + 1 = 0, -360^\circ \le \theta \le 720^\circ$ (Degrees)

$$\frac{\sqrt{2}\cos\theta}{\sqrt{2}} = \frac{-1}{\sqrt{2}}$$

$$\cos\theta = -\frac{1}{\sqrt{2}}$$

where is cos<0 (negative)

$$\cos\theta = -0.7071$$

use positive for $\bar{\theta}$

$$\bar{\theta} = \cos^{-1}(0.7071)$$

$$\bar{\theta} = 45^\circ$$

Q2	Q3
$\theta = 180^\circ - \bar{\theta}$	$\theta = 180^\circ + \bar{\theta}$
$\theta = 180^\circ - 45^\circ = 135^\circ$	$\theta = 180^\circ + 45^\circ = 225^\circ$
$\theta = 135^\circ - 360^\circ = -225^\circ$	$\theta = 225^\circ - 360^\circ = -135^\circ$
$\theta = 135^\circ + 360^\circ = 495^\circ$	$\theta = 225^\circ + 360^\circ = 585^\circ$

$$-\frac{4\pi}{2} \le x \le \frac{8\pi}{2}$$

Ex. $\sin x + 1 = 0,\ -2\pi \le x \le 4\pi$ (Radians)

$\sin x = -1$

(Unit Circle)

$x = \frac{3\pi}{2}$

$\frac{3\pi}{2} - \frac{2\pi}{1}$	$\frac{3\pi}{2} + \frac{2\pi}{1}$
$\frac{3\pi}{2} - \frac{4\pi}{2}$	$\frac{3\pi}{2} + \frac{4\pi}{2}$
$\frac{-\pi}{2}$	$\frac{7\pi}{2}$

Your Turn

Solve each trigonometric equation in the specified domain.

a) $3 \cos \theta - 1 = \cos \theta + 1$, $-2\pi \leq \theta \leq 2\pi$

b) $4 \sec x + 8 = 0$, $0° \leq x < 360°$

a) $3\cos\theta - 1 = \cos\theta + 1$, $-2\pi \leq \theta \leq 2\pi$ (Radians)

$3\cos\theta - \cos\theta = 1 + 1$

$2\cos\theta = 2$

$\cos\theta = 1$

$\theta = 0 - 2\pi = -2\pi$

(Unit Circle)

$\theta = 0, 2\pi, -2\pi$

b) $4\sec x + 8 = 0$ $0° \leq x < 360°$ (Degrees)

$\frac{4\sec x}{4} = \frac{-8}{4}$

$\sec x = \frac{-2}{1}$

* $\cos x = -\frac{1}{2}$ where is $\cos x < 0$ (Negative)

$\bar{x} = 60°$

Q2	Q3
$\theta = 180° - \bar{\theta}$	$\theta = 180° + \bar{\theta}$
$\theta = 180° - 60°$	$\theta = 180° + 60°$
$\theta = 120°$	$\theta = 240°$

Homework

Page 211 #1-5

Front

① $\sin\theta = \frac{-\sqrt{3}}{2}$ $\frac{opp}{hyp}$

where is $\sin\theta < 0$ (negative)

$\sin\bar{\theta} = 0.8660$

$\bar{\theta} = \sin^{-1}(0.8660)$

$\bar{\theta} = 60°$

Q3	Q4
$\theta = 180° + \bar{\theta}$	$\theta = 360° - \bar{\theta}$
$\theta = 180° + 60°$	$\theta = 360° - 60°$
$\theta = 240°$	$\theta = 300°$
$\boxed{240° \pm 360°n, n \in N}$	$300° \pm 360°n, n \in N$

From back of sheet:

① $\sin\theta = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

where is $\sin\theta > 0$ (Positive)

$\sin\bar{\theta} = \frac{1}{\sqrt{2}}$

$\bar{\theta} = \frac{\pi}{4}$

Q1	Q2
$\theta = \bar{\theta}$	$\theta = \pi - \bar{\theta}$
$\theta = \frac{\pi}{4}$	$\theta = \pi - \frac{\pi}{4}$
	$\theta = \frac{4\pi}{4} - \frac{\pi}{4}$
	$\theta = \frac{3\pi}{4}$

Questions from Homework

Back

③ $\tan\theta = -\frac{\sqrt{3}}{3} = \frac{-1}{\sqrt{3}}$

Where is $\tan\theta < 0$ (Negative)

(Triangle) $\bar{\theta} = 30^\circ$

Q2	Q4
$\theta = 180^\circ - \bar{\theta}$	$\theta = 360^\circ - \bar{\theta}$
$\theta = 180^\circ - 30^\circ = 150^\circ$	$\theta = 360^\circ - 30^\circ = 330^\circ$
$\boxed{150^\circ \pm 360^\circ n, n \in N}$	$\boxed{330^\circ \pm 360^\circ n, n \in N}$

Back

⑫ $\tan\theta = \text{undefined}$

$\tan\theta = \frac{o}{a} = \frac{y}{x}$

(Unit Circle)

$$\boxed{\theta = \frac{\pi}{2}, \frac{3\pi}{2}}$$

Front

⑫ $5\sin\theta - 4 = 0$ (Approximate Value)

$\frac{5\sin\theta}{5} = \frac{4}{5}$

$\sin\theta = 0.8$

Where is $\sin\theta > 0$ (Positive)

$\bar{\theta} = \sin^{-1}(0.8)$

$\bar{\theta} = 53.1^\circ$

Q1	Q2
$\theta = \bar{\theta}$	$\theta = 180^\circ - \bar{\theta}$
$\theta = 53.1^\circ$	$\theta = 180^\circ - 53.1^\circ$
	$\theta = 126.9^\circ$
$\boxed{53.1^\circ \pm 360^\circ n, n \in N}$	$\boxed{126.9^\circ \pm 360^\circ n, n \in N}$

Let's move onto QUADRATIC trigonometric equations...

...Pre-Calculus 110

- What strategies can we use to solve quadratic equations?
- Quadratic trigonometric equations will ultimately become TWO linear trigonometric equations.

Solve: $2x^2 + x = 1$

$2x^2 + x - 1 = 0$ $2 \times -1 = -2$, $2 + -1 = 1$

$(x_{+2})(x_{-1}) = 0$

$(x+1)(2x-1) = 0$

$x + 1 = 0$, $x = -1$

$2x - 1 = 0$, $\frac{2x}{2} = \frac{1}{2}$, $x = \frac{1}{2}$

Solve: $2\sin^2 x + \sin x = 1$, $0 \le x \le 4\pi$

(Radians)

$2\sin^2 x + \sin x - 1 = 0$ $2 \times -1 = -2$, $2 + -1 = 1$

$\left(\sin x \frac{+2}{2}\right)\left(\sin x \frac{-1}{2}\right) = 0$

$(\sin x + 1)(2\sin x - 1) = 0$

$\sin x + 1 = 0$

$\sin x = -1$

(Unit Circle)

$x = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + 2\pi$

$= \frac{3\pi}{2} + \frac{4\pi}{2}$

$= \frac{7\pi}{2}$

$2\sin x - 1 = 0$

$\frac{2\sin x}{2} = \frac{1}{2}$

$\sin x = \frac{1}{2}$ (Triangle)

$\bar{x} = \frac{\pi}{6}$

Where is $\sin\theta > 0$

Q1	Q2
$x = \bar{x}$	$x = \pi - \bar{x}$
$x = \frac{\pi}{6}$	$x = \pi - \frac{\pi}{6}$
	$x = \frac{6\pi}{6} - \frac{\pi}{6}$
$x = \frac{\pi}{6} + 2\pi$	$x = \frac{5\pi}{6}$
$x = \frac{\pi}{6} + \frac{12\pi}{6}$	$x = \frac{5\pi}{6} + \frac{12\pi}{6}$
$x = \frac{13\pi}{6}$	$x = \frac{17\pi}{6}$

Factoring trinomials:

① Decomposition

$2x^2 + 7x + 6$ $\qquad 3 \times 4 = 12$

$(x+\frac{3}{2})(x+\frac{4}{2})$ $\qquad 3 + 4 = 7$

$(2x+3)(x+2)$

② Simple trinomial

$x^2 + 7x + 6$ $\qquad 6 \times 1 = 6$

$(x+1)(x+6)$ $\qquad 6 + 1 = 7$

(Radians)

Ex. $\cos^2\theta - \frac{1}{2}\cos\theta = 0, \; -2\pi \le \theta \le 4\pi$ (Common Factor)

$\cos\theta\left(\cos\theta - \frac{1}{2}\right) = 0$

$\cos\theta = 0$

(Unit circle)

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$\theta = -\frac{3\pi}{2}, -\frac{\pi}{2}$

$\theta = \frac{5\pi}{2}, \frac{7\pi}{2}$

$\cos\theta - \frac{1}{2} = 0$

$\cos\theta = \frac{1}{2}$

where is $\cos\theta > 0$ (Positive)

(Triangle)

$\bar{\theta} = \frac{\pi}{3}$

Q1	Q4
$\theta = \bar{\theta}$	$\theta = 2\pi - \bar{\theta}$
$\theta = \frac{\pi}{3}$	$\theta = \frac{6\pi}{3} - \frac{\pi}{3} = \frac{5\pi}{3}$
$\theta = -\frac{5\pi}{3}$	$\theta = -\frac{\pi}{3}$
$\theta = \frac{7\pi}{3}$	$\theta = \frac{11\pi}{3}$

Ex. $6\sin^2 x - \sin x = 2, -2\pi \le \theta \le 4\pi$ (Decomposition)

Your Turn

Solve for θ.

$\cos^2 \theta - \cos \theta - 2 = 0,\ 0° \le \theta < 360°$ (Simple Trinomial)

Give solutions as exact values where possible. Otherwise, give approximate measures to the nearest thousandth of a degree.

$\cos^2\theta - \cos\theta - 2 = 0$ $\quad -2 \times 1 = -2$; $-2 + 1 = -1$

$(\cos\theta - 2)(\cos\theta + 1) = 0$

$\cos\theta - 2 = 0$ | $\cos\theta + 1 = 0$

$\cos\theta = 2$ | $\cos\theta = -1$

Not Possible | (Unit Circle)

$-1 \le \cos\theta \le 1$ | $\boxed{\theta = 180°}$

General Solution of a Trigonometric Equation

(Radians)

Solve: $3\cos^2\theta - \cos\theta = 2;\ \theta \in \mathbb{R}$ (Decomposition)

$3\cos^2\theta - \cos\theta - 2 = 0$ $\quad -3 \times 2 = -6$

$-3 + 2 = -1$

$\left(\cos\theta - \frac{3}{3}\right)\left(\cos\theta + \frac{2}{3}\right) = 0$

$(\cos\theta - 1)(3\cos\theta + 2) = 0$

$\cos\theta - 1 = 0$	$3\cos\theta + 2 = 0$
$\cos\theta = 1$	$\frac{3\cos\theta}{3} = \frac{-2}{3}$
(Unit Circle)	$\cos\theta = -0.6667$ (Approximate Value)
$\theta = 0, 2\pi$	$\bar{\theta} = \cos^{-1}(0.6667)$
$\boxed{\theta = 0 \pm 2\pi n, n \in \mathbb{N}}$	$\bar{\theta} = 0.84$

where is $\cos\theta < 0$ (Negative)

Q2	Q3
$\theta = \pi - \bar{\theta}$	$\theta = \pi + \bar{\theta}$
$\theta = 3.14 - 0.84$	$\theta = 3.14 + 0.84$
$\theta = 2.3$	$\theta = 3.98$

$\boxed{\theta = 2.3 \pm 2\pi n, n \in \mathbb{N}}$ $\quad$ $\boxed{\theta = 3.98 \pm 2\pi n, n \in \mathbb{N}}$

Determine the general solution for $\sin^2 x - 1 = 0$ over the real numbers if x is measured in radians. (Difference of Squares)

Did You Know?

$2n$, where $n \in I$, represents all even integers.

$2n + 1$, where $n \in I$, is an expression for all odd integers.

$\sin^2 x - 1 = 0$

$(\sin x + 1)(\sin x - 1) = 0$

$\sin x + 1 = 0$ | $\sin x - 1 = 0$

$\sin x = -1$ | $\sin x = 1$

(Unit Circle) | (Unit Circle)

$x = \frac{3\pi}{2}$ | $x = \frac{\pi}{2}$

$\boxed{x = \frac{3\pi}{2} + 2\pi n, n \in N}$ | $\boxed{x = \frac{\pi}{2} + 2\pi n, n \in N}$

$$x = \frac{\pi}{2} + 2\pi n, \text{ where } n \in I$$

$$x = \frac{3\pi}{2} + 2\pi n, \text{ where } n \in I$$

or

$$x = \frac{\pi}{2} + \pi n, \text{ where } n \in I$$

or

$$(2n + 1)\left(\frac{\pi}{2}\right), n \in I$$

Unit Review...

What topics have we covered??

- Radian Measure
- Co-terminal angles
- Principal Angles
- Angular Velocity (Open Response)
- The Unit Circle
- Trig Expressions (Open Response)
- Trig Equations (Open Response)

Solve: $6\sin^2\theta - 3\sin\theta = 0$, $0 \le \theta \le 360°$

[A] $0°, 30°, 180°, 330°, 360°$

[B] $0°, 30°, 180°, 150°, 360°$

[C] $30°, 90°, 120°, 270°$

[D] $0°, 180°, 210°, 330°, 360°$

$6\sin^2\theta - 3\sin\theta = 0$

$3\sin\theta(2\sin\theta - 1) = 0$

$\frac{3\sin\theta}{3} = \frac{0}{3}$

$\sin\theta = 0$

(Unit Circle)

$\theta = 0°, 180°, 360°$

$2\sin\theta - 1 = 0$

$\frac{2\sin\theta}{2} = \frac{1}{2}$

$\sin\theta = \frac{1}{2}$

(Triangle)

$\bar{\theta} = 30°$

where is $\sin\theta > 0$

Q1	Q2
$\theta = \bar{\theta}$	$\theta = 180° - \bar{\theta}$
$\theta = 30°$	$\theta = 150°$

If $\csc\theta < 0$ and $\tan\theta > 0$, then which of the following could be a possible measure of angle θ?

Q4 Q3

[A] $\frac{11\pi}{6}$ $\frac{10\pi}{6}, \frac{11\pi}{6}, \frac{12\pi}{6}$ 2π

[B] $\frac{4\pi}{3}$ $\rightarrow \frac{3\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ π

[C] $\frac{3\pi}{4}$ Q2

[D] $\frac{\pi}{2}$ Between 1 + 2

Given:

$\csc\theta < 0$ (negative)

$\tan\theta > 0$ (positive)

S	A
T	C

θ must be in quadrant 3

What is the principal angle of $-\frac{25\pi}{4}$?

[A] $\frac{3\pi}{4}$

[B] $\frac{\pi}{4}$

[C] $-\frac{\pi}{4}$

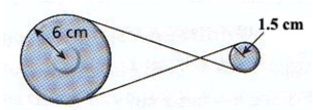
[D] $\frac{7\pi}{4}$

$-\frac{25\pi}{4} + 8\pi$

$-\frac{25\pi}{4} + \frac{32\pi}{4}$

$\frac{7\pi}{4}$

If the belt in the pulley system below travels 30 cm, what is the angle of rotation of the smaller pulley?



[A] $\frac{\pi}{9}$ radians [B] 20° [C] 20 radians [D] 5°

Given:
$a = 30\text{cm}$
$r = 1.5\text{cm}$

$\theta = \frac{a}{r} = \frac{30\text{cm}}{1.5\text{cm}} = \underline{\underline{20 \text{ rads}}}$

Nibbles the hamster is running at 0.02 m/s on an exercise wheel of radius 8 cm. What is the ~~angular velocity of this wheel~~?
[A] 0.15 rad/minute [B] 240 rad/minute [C] 0.25 rad/minute [D] 15 radians/minute

Given:
$r = 8\text{cm} = 0.08\text{m}$
$a = 0.02\text{m}$ (after 1sec)

① $\theta = \frac{a}{r} = \frac{0.02 \text{ m}}{0.08 \text{ m}} = 0.25 \text{ rads}$

② $V_a = \frac{\theta}{t} = \frac{0.25\text{rads}}{1 \text{ sec}} = \frac{15 \text{ rads}}{60\text{sec}}$

Solve: $2(1-\sin\theta)^2 + \sin\theta = 2(3 - 4\sin^2\theta)$, $-360° \le \theta \le 720°$

$2(1 - 2\sin\theta + \sin^2\theta) + \sin\theta = 6 - 8\sin^2\theta$
$2 - 4\sin\theta + 2\sin^2\theta + \sin\theta = 6 - 8\sin^2\theta$
$10\sin^2\theta - 3\sin\theta - 4 = 0$
$(10\sin^2\theta + 5\sin\theta)(-8\sin\theta - 4) = 0$
$5\sin\theta(2\sin\theta + 1) - 4(2\sin\theta + 1) = 0$
$(5\sin\theta - 4)(2\sin\theta + 1) = 0$

$5\sin\theta - 4 = 0$
(Calc) $\sin\theta = \frac{4}{5} = 0.8$
$\bar{\theta} = \sin^{-1}(0.8)$
$\bar{\theta} = 53.1°$

where is $\sin\theta > 0$

Q1	Q2
$\theta = 53.1°$	$\theta = 180° - 53.1°$
$-306.9°$	$\theta = 126.9°$
$413.1°$	$-233.1°$
	$486.9°$

$2\sin\theta + 1 = 0$
$\sin\theta = -\frac{1}{2}$ (Triangle)
$\bar{\theta} = 30°$

where is $\sin\theta < 0$

Q3	Q4
$\theta = 180° + 30°$	$\theta = 360° - 30°$
$\theta = 210°$	$\theta = 330°$
$-150°$	$-30°$
$570°$	$690°$

Ch. 4 Review

① e) $\cos^2\theta + \frac{1}{2}\cos\theta = 0$, $0° \le \theta \le 360°$

$\cos\theta\left(\cos\theta + \frac{1}{2}\right) = 0$

$\cos\theta = 0$

(Unit Circle)

$\theta = 90°, 270°$

$\cos\theta + \frac{1}{2} = 0$

$\cos\theta = -\frac{1}{2}$

where is $\cos\theta < 0$

(Triangle)

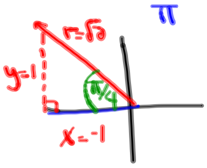
$\bar{\theta} = 60°$

Q2	Q3
$\theta = 180° - 60°$	$\theta = 180° + 60°$
$\theta = 120°$	$\theta = 240°$

② a) $\dfrac{3}{1 - 2\boxed{\sin\frac{3\pi}{4}}}$

$\frac{2\pi}{4}, \frac{3\pi}{4}, \frac{4\pi}{4}$

π



$\dfrac{3}{1 - \cancel{2}\left(\frac{\sqrt{2}}{\cancel{2}}\right)}$

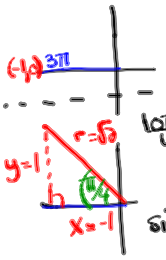
$\sin\frac{3\pi}{4} = \frac{1}{\sqrt{2}} = \boxed{\frac{\sqrt{2}}{2}}$

$\dfrac{3}{(1-\sqrt{2})(1+\sqrt{2})} \cdot (1+\sqrt{2})$ Rationalize

$\dfrac{3+3\sqrt{2}}{1+\cancel{\sqrt{2}}-\cancel{\sqrt{2}}-2}$

$\dfrac{3+3\sqrt{2}}{-1}$ or $-3-3\sqrt{2}$

② d) $\dfrac{2\boxed{\cos 3\pi} + \boxed{\sin 11\pi/4}}{\boxed{\cos^2 \pi/6}}$



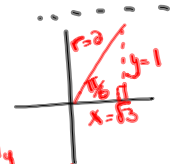
$\cos 3\pi = -1$

$\frac{10\pi}{4}, \frac{11\pi}{4}, \frac{12\pi}{4}$

3π

$\sin\frac{11\pi}{4} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\dfrac{2(-1) + \frac{\sqrt{2}}{2}}{\left(\frac{\sqrt{3}}{2}\right)^2}$



$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\dfrac{-2+\frac{\sqrt{2}}{2}}{\frac{3}{4}}$

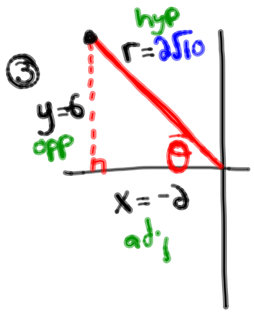
$\left(-\frac{4}{2} + \frac{\sqrt{2}}{2}\right) \cdot \frac{4}{3}$ multiply by reciprocal

$\left(\frac{-4+\sqrt{2}}{2}\right) \cdot \frac{4}{3}$

$\frac{-16+4\sqrt{2}}{6} \rightarrow \frac{-8+2\sqrt{2}}{3}$ reduce

Ch. 4 Review

③



① Find r:

$x^2+y^2=r^2$

$(-2)^2+(6)^2=r^2$

$4+36=r^2$

$40=r^2$

$\sqrt{40}=r$

$\sqrt{2\cdot2\cdot2\cdot5}=r$

$2\sqrt{10}=r$

② $\sin\theta=\frac{6}{2\sqrt{10}}=\frac{3}{\sqrt{10}}=\frac{3\sqrt{10}}{10}$

$\cos\theta=\frac{-2}{2\sqrt{10}}=\frac{-1}{\sqrt{10}}=\frac{-\sqrt{10}}{10}$

$\tan\theta=\frac{6}{-2}=-3$

$\csc\theta=\frac{\sqrt{10}}{3}$

$\sec\theta=-\sqrt{10}$

$\cot\theta=-\frac{1}{3}$

Reciprocal Ratios

④ $\sec\theta=-\frac{\sqrt{11}}{3}$ hyp / adj and $\tan\theta>0$

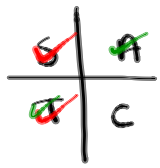
$\cos\theta<0$

Given:

$r=\text{hyp}=\sqrt{11}$

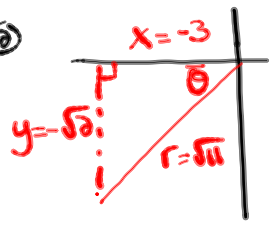
$x=\text{adj}=-3$

① • $\cos\theta<0$
• $\tan\theta>0$



θ is in Q3

②



$x^2+y^2=r^2$

$(-3)^2+y^2=(\sqrt{11})^2$

$9+y^2=11$

$y^2=2$

$y=\pm\sqrt{2}$

$y=-\sqrt{2}$ Q3

③ $\sin\theta=\frac{-\sqrt{2}}{\sqrt{11}}=\frac{-\sqrt{22}}{11}$

$\cos\theta=\frac{-3}{\sqrt{11}}=\frac{-3\sqrt{11}}{11}$

$\tan\theta=\frac{-\sqrt{2}}{-3}=\frac{\sqrt{2}}{3}$

$\csc\theta=-\frac{\sqrt{11}}{\sqrt{2}}=-\frac{\sqrt{22}}{2}$

$\cot\theta=\frac{3}{\sqrt{2}}=\frac{3\sqrt{2}}{2}$

Attachments

Worksheet - Sketching Angles in Radians.doc