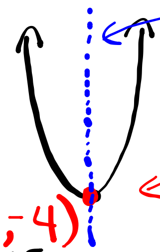
6.2 Properties of Graphs of Quadratic Functions

GOAL

Identify the characteristics of graphs of quadratic functions, and use the graphs to solve problems.

$y = ax^2 + bx + c$ c = the y-intercept (0, c)

- if $a > 0$ (positive) the parabola opens up:



axis of symmetry ($x = 2$)

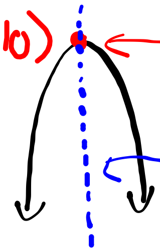
vertex is a minimum

(2, -4)

- if $a < 0$ (negative) the parabola opens down:

(-5, 10)

vertex is a maximum



axis of symmetry ($x = -5$)

LEARN ABOUT the Math

Nicolina plays on her school's volleyball team. At a recent match, her Nonno, Marko, took some time-lapse photographs while she warmed up. He set his camera to take pictures every 0.25 s. He started his camera at the moment the ball left her arms during a bump and stopped the camera at the moment that the ball hit the floor. Marko wanted to capture a photo of the ball at its greatest height. However, after looking at the photographs, he could not be sure that he had done so. He decided to place the information from his photographs in a table of values.

From his photographs, Marko observed that Nicolina struck the ball at a height of 2 ft above the ground. He also observed that it took about 1.25 s for the ball to reach the same height on the way down.

x y

Time (s)	Height (ft)
0.00	2
0.25	6
0.50	8
0.75	8
1.00	6
1.25	2

max is between 0.5 and 0.75 s

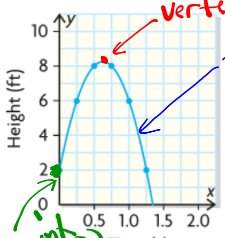
When did the volleyball reach its greatest height?

EXAMPLE 1 Using symmetry to estimate the coordinates of the vertex

Marko's Solution

vertex is a max

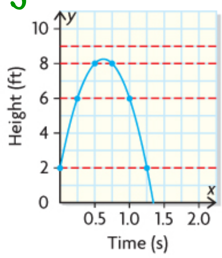
parabola



y-int=2

I plotted the points from my table, and then I sketched a graph that passed through all the points.

The graph looked like a parabola, so I concluded that the relation is probably quadratic.



I knew that I could draw horizontal lines that would intersect the parabola at two points, except at the **vertex**, where a horizontal line would intersect the parabola at only one point.

Using a ruler, I drew horizontal lines and estimated that the coordinates of the vertex are around (0.6, 8.2).

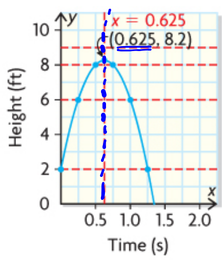
This means that the ball reached maximum height at just over 8 ft, about 0.6 s after it was launched.

vertex The point at which the quadratic function reaches its maximum or minimum value.

Equation of the axis of symmetry: *

$$x = \frac{0 + 1.25}{2}$$

$x = 0.625$



I used points that have the same *y*-value, (0, 2) and (1.25, 2), to determine the equation of the **axis of symmetry**. I knew that the axis of symmetry must be the same distance from each of these points.

axis of symmetry A line that separates a 2-D figure into two identical parts. For example, a parabola has a vertical axis of symmetry passing through its vertex.

axis of symmetry: x = 0.625

From the equation, the *x*-coordinate of the vertex is 0.625. From the graph, the *y*-coordinate of the vertex is close to 8.2.

Therefore, 0.625 s after the volleyball was struck, it reached its maximum height of approximately 8 ft 2 in.

I revised my estimate of the coordinates of the vertex.

APPLY the Math

EXAMPLE 2 Reasoning about the maximum value of a quadratic function

Some children are playing at the local splash pad. The water jets spray water from ground level. The path of water from one of these jets forms an arch that can be defined by the function

parabola

$$f(x) = -0.12x^2 + 3x$$

opens down
$a = -0.12$
has a max

where x represents the horizontal distance from the opening in the ground in feet and $f(x)$ is the height of the sprayed water, also measured in feet. What is the maximum height of the arch of water, and how far from the opening in the ground can the water reach?

Manuel's Solution

$f(x) = -0.12x^2 + 3x$

I knew that the degree of the function is 2, so the function is quadratic. The arch must be a parabola.

I also knew that the coefficient of x^2, a, is negative, so the parabola opens down. This means that the function has a **maximum value,** associated with the y-coordinate of the vertex.

$f(0) = 0$

maximum value
The greatest value of the dependent variable in a relation.

$x = \frac{12+13}{2} = 12.5$

I started to create a table of values by determining the y-intercept. I knew that the constant, zero, is the y-intercept. This confirms that the stream of water shoots from ground level.

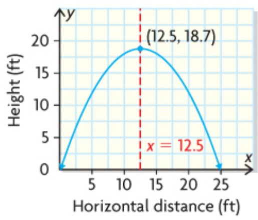
$f(1) = -0.12(1)^2 + 3(1)$
$f(1) = -0.12 + 3$
$f(1) = 2.88$

I continued to increase x by intervals of 1 until I noticed a repeat in my values. A height of 18.72 ft occurs at horizontal distances of 12 ft and 13 ft.

x	0	1	2	12	13
f(x)	0	2.88	5.52	18.72	18.72

Based on symmetry and the table of values, the maximum value of $f(x)$ will occur halfway between (12, 18.72) and (13, 18.72).

The arch of water will reach a maximum height between 12 ft and 13 ft from the opening in the ground.



I used my table of values to sketch the graph. I extended the graph to the x-axis. I knew that my sketch represented only part of the function, since I am only looking at the water when it is above the ground.

$$x = \frac{12 + 13}{2}$$
$x = 12.5$

I used two points with the same y-value, (12, 18.72) and (13, 18.72), to determine the equation of the axis of symmetry.

Equation of the axis of symmetry:
$x = 12.5$

Height at the vertex:
$f(x) = -0.12x^2 + 3x$
$f(12.5) = -0.12(12.5)^2 + 3(12.5)$
$f(12.5) = -0.12(156.25) + 37.5$
$f(12.5) = -18.75 + 37.5$
$f(12.5) = 18.75$

I knew that the x-coordinate of the vertex is 12.5, so I substituted 12.5 into the equation to determine the height of the water at this horizontal distance.

Vertex:
(12.5, 18.75)

Axis of symmetry:
$x = 12.5$

The water reaches a maximum height of 18.75 ft when it is 12.5 ft from the opening in the ground.

The water can reach a maximum horizontal distance of 25 ft from the opening in the ground.

Due to symmetry, the opening in the ground must be the same horizontal distance from the axis of symmetry as the point on the ground where the water lands. I simply multiplied the horizontal distance to the axis of symmetry by 2.

The domain of this function is $0 \le x \le 25$, where $x \in R$.

EXAMPLE 3 Graphing a quadratic function using a table of values

Sketch the graph of the function:

$$y = x^2 + x - 2$$

Determine the y-intercept, any x-intercepts, the equation of the axis of symmetry, the coordinates of the vertex, and the domain and range of the function.

Anthony's Solution

$y = x^2 + x - 2$

The function is a quadratic function in the form
$y = ax^2 + bx + c$
$a = 1$ opens up
$b = 1$
$c = -2$ y int = -2

$y = x^2 + x - 2$

The degree of the given equation is 2, so the graph will be a parabola.

Since the coefficient of x^2 is positive, the parabola opens up.

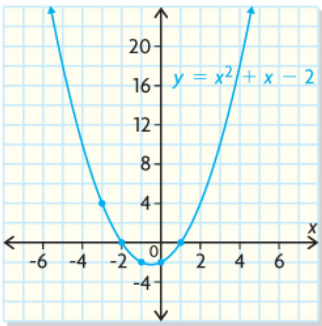
Since the y-intercept is less than zero and the parabola opens up, there must be two x-intercepts and a **minimum value**.

minimum value The least value of the dependent variable in a relation.

x	−3	−2	−1	0	1
y	4	0	−2	−2	0

I made a table of values. I included the y-intercept, (0, −2), and determined some other points by substituting values of x into the equation.

I stopped determining points after I had identified both x-intercepts, because I knew that I had enough information to sketch an accurate graph.



I graphed each coordinate pair and then drew a parabola that passed through all the points.

Equation of the axis of symmetry:

$x = \frac{-2 + 1}{2}$

$x = \frac{-1}{2}$

$x = -0.5$

I used the x-intercepts to determine the equation of the axis of symmetry.

y-coordinate of the vertex:
$y = (-0.5)^2 + (-0.5) - 2$
$y = 0.25 - 0.5 - 2$
$y = -2.25$
The vertex is $(-0.5, -2.25)$.

I knew that the vertex is a point on the axis of symmetry. The x-coordinate of the vertex must be −0.5. To determine the y-coordinate of the vertex, I substituted −0.5 for x in the given equation.

The y-intercept is −2.
The x-intercepts are −2 and 1.
The equation of the axis of symmetry is $x = -0.5$.
The vertex is $(-0.5, -2.25)$.

The vertex, $(-0.5, -2.25)$, defines the minimum value of y.

Domain and range:
$\{(x, y) \mid x \in R, y \geq -2.25, y \in R\}$

No restrictions were given for x, so the domain is all real numbers.

EXAMPLE 3 Graphing a quadratic function using a table of values

Sketch the graph of the function:

$$y = x^2 + x - 2$$

Determine the y-intercept, any x-intercepts, the equation of the axis of symmetry, the coordinates of the vertex, and the domain and range of the function.

Your Turn

Explain how you could decide if the graph of the function $y = -x^2 + x + 2$ has x-intercepts.

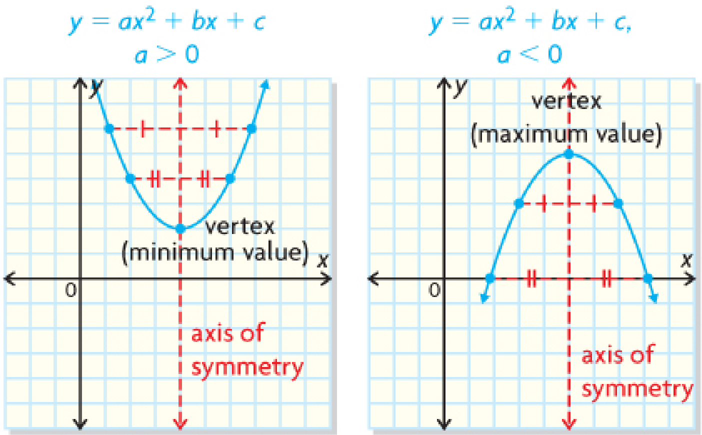
Answer

First, look at the direction of the opening. The given function must open downward, because the coefficient of the x^2 term is negative. Next, look at the y-intercept, which is 2 in this case. Since the parabola opens downward and the y-intercept is positive, the vertex must lie above the x-axis, and the parabola must have two x-intercepts.

In Summary

Key Idea

- A parabola that is defined by the equation $y = ax^2 + bx + c$ has the following characteristics:
  - If the parabola opens down ($a < 0$), the vertex of the parabola is the point with the greatest y-coordinate. The y-coordinate of the vertex is the maximum value of the function.
  - If the parabola opens up ($a > 0$), the vertex of the parabola is the point with the least y-coordinate. The y-coordinate of the vertex is the minimum value of the function.
  - The parabola is symmetrical about a vertical line, the axis of symmetry, through its vertex.



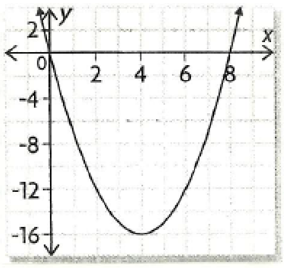
Need to Know

- For all quadratic functions, the domain is the set of real numbers, and the range is a subset of real numbers.
- When a problem can be modelled by a quadratic function, the domain and range of the function may need to be restricted to values that have meaning in the context of the problem.

Assignment: **pages 287 - 288**

Questions 1, 3, 4, 6, 7, 9

SOLUTIONS ⇒ 6.2 Properties of Graphs of Quadratic Functions

1. 

a) Determine the equation of the axis of symmetry for the parabola.

↳ $x = 4$

b) Determine the coordinates of the vertex of the parabola.

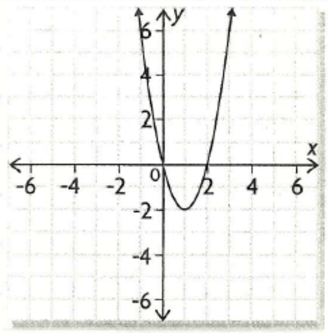
↳ $(4, -16)$

c) State the domain and range of the function.

↳ Domain: $\{x \mid x \in R\}$

↳ Range: $\{y \mid y \geq -16, y \in R\}$

3. For each function, identify the x- and y-intercepts, determine the equation of the axis of symmetry and the coordinates of the vertex, and state the domain and range.

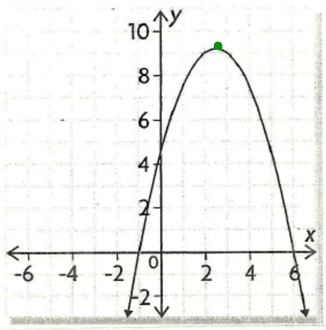
a) 

x-intercepts: (0,0) and (2,0)
y-intercept: (0,0)

Equation of axis of symmetry => $x = 1$

Vertex => $(1,-2)$

Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y \geq -2, y \in R\}$

b) 

x-intercepts: (-1,0) and (6,0)
y-intercept: (0, 4.5)

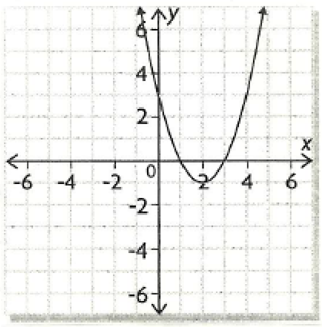
Equation of axis of symmetry => $x = 2.5$

Vertex => (2.5, 9.2)

Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y \leq 9.2, y \in R\}$

4. For each function, identify the equation of the axis of symmetry, determine the coordinates of the vertex, and state the domain and range.

a)

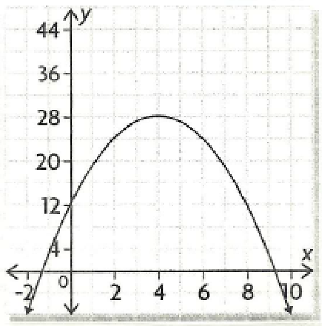


Equation of axis of symmetry ⇒ $x=2$

Vertex ⇒ $(2,-1)$

Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y \geq -1, y \in R\}$

b)

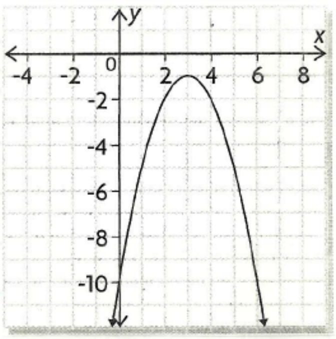


Equation of axis of symmetry ⇒ $x=4$

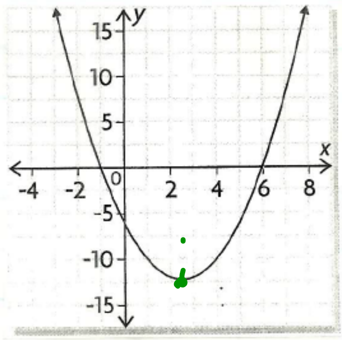
Vertex ⇒ $(4,28)$

Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y \leq 28, y \in R\}$

c)



d)



Equation of axis of symmetry => $x=3$

Vertex => $(3,-1)$

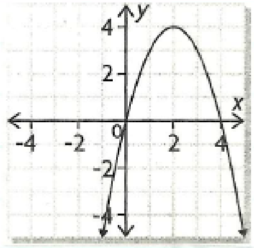
Domain : $\{x \mid x \in R\}$
Range : $\{y \mid y \leq -1, y \in R\}$

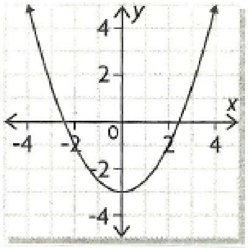
Equation of axis of symmetry => $x=2.5$

Vertex => $(2.5,-12.25)$

Domain : $\{x \mid x \in R\}$
Range : $\{y \geq -12.25, y \in R\}$

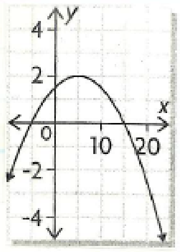
6. State whether each parabola has a minimum or maximum value, and then determine this value.

a) 

b) 

Maximum Value => 4

Minimum Value => -3

c) 

Maximum Value => 2

7.a) Complete the table of values shown for each of the following functions.

x	-4	-2	0	2	4
y					

i) $y = -\frac{1}{2}x^2 + 5$

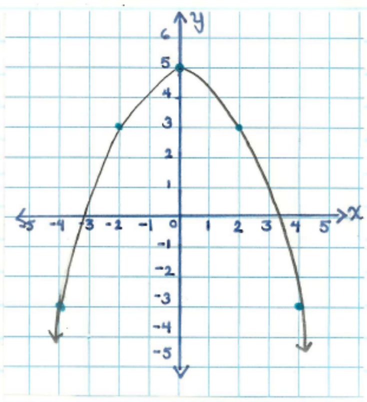
x	-4	-2	0	2	4
y	-3	3	5	3	-3

ii) $y = \frac{3}{2}x^2 - 2$

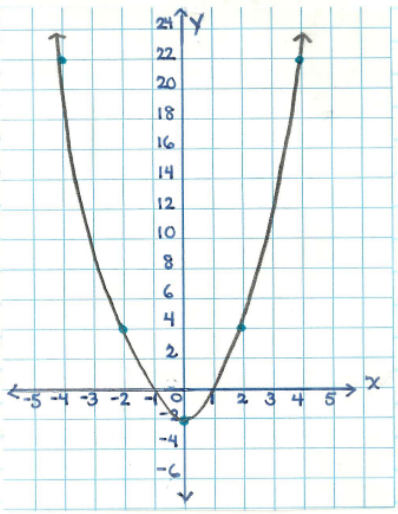
x	-4	-2	0	2	4
y	22	4	-2	4	22

b) Graph the points in your table of values.

i)



ii)



d) State the domain and range of the function.

i) Domain: $\{x \mid x \in R\}$

Range: $\{y \mid y \leq 5, y \in R\}$

ii) Domain: $\{x \mid x \in R\}$

Range: $\{y \mid y \geq -2, y \in R\}$

9. For each of the following, both points, (x,y), are located on the same parabola. Determine the equation of the axis of symmetry for each parabola.

a) (0,2) and (6,2).

↳ $x = \frac{0+6}{2}$

$x = \frac{6}{2}$

$x = 3$

b) (1,-3) and (9,-3)

↳ $x = \frac{1+9}{2}$

$x = \frac{10}{2}$

$x = 5$

c) (-6,0) and (2,0)

↳ $x = \frac{-6+2}{2}$

$x = -\frac{4}{2}$

$x = -2$

d) (-5,-1) and (3,-1)

↳ $x = \frac{-5+3}{2}$

$x = -\frac{2}{2}$

$x = -1$

Attachments

7s2e2 final.mp4

7s2e3 final.mp4

7s2e4 final.mp4

fm7s2-p8.tns