6.5 Solving Quadratic Equations by Factoring

Ex: $f(x) = 2(x+5)(2x-1)$

$f(x) = y$
functions height

$a = 2 \rightarrow$ opens up.

x-int: $(y=0)$ $0 = 2(x+5)(2x-1)$

$x+5=0$	$2x-1=0$
$x=-5$	$\frac{2x}{2} = \frac{1}{2}$
$(-5,0)$	$x = 0.5$
	$(0.5,0)$

$y = -1(x+1)(x-4)$

① a) $f(x) = -1(x+1)(x-4)$

① $a = -1$ → opens down

② x-int: $x+1=0$, $x=-1$, $(-1,0)$ | $x-4=0$, $x=4$, $(4,0)$

③ y-int: $c=(a)(r)(s)$

$c=(-1)(1)(-4) = 4$ $(0,4)$

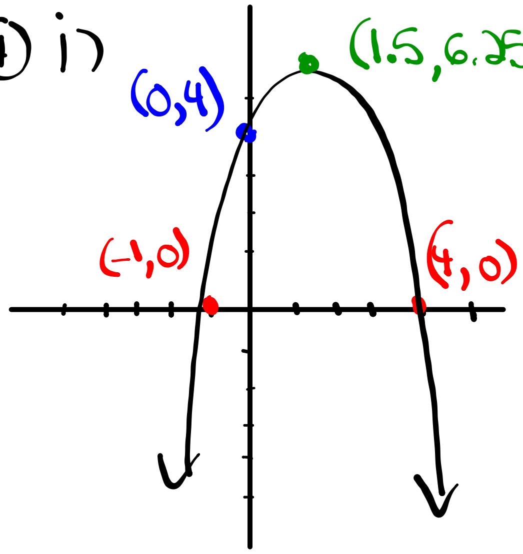
④ axis of symmetry: use $(-1,0)$ and $(4,0)$

$x=1.5$ (x coordinate of vertex) $\frac{-1+4}{2} = \frac{3}{2} = 1.5$

⑤ vertex: $y = -1(1.5+1)(1.5-4)$

$y = -1(2.5)(-2.5) = 6.25$

④ i)



Solving Quadratic Equations ($ax^2 + bx + c = 0$)

Simple Trinomial

Add Multiply

Example 1: $x^2 + \underline{6}x + \underline{8} = 0$

$2 \times 4 = 8$

$2 + 4 = 6$

8
1x8
2x4

$(x+2)(x+4) = 0$

$x+2=0 \mid x+4=0$

$\boxed{x=-2} \mid \boxed{x=-4}$

Example: $x^2 + 4x - 32 = 0$

$-4 \times 8 = -32$

$-4 + 8 = 4$

32
1x32
2x16
4x8

$(x-4)(x+8) = 0$

$x-4=0 \mid x+8=0$

$\boxed{x=4} \mid \boxed{x=-8}$

* Decomposition

Example 2: $2x^2 + 1x - 15 = 0$

$-5 \times 6 = -30$

$-5 + 6 = 1$

30
1x30
2x15
3x10
5x6

$(x-\frac{5}{2})(x+\frac{6}{2})=0$

$(2x-5)(x+3)=0$

$2x-5=0$ | $x+3=0$

$\frac{2x}{2}=\frac{5}{2}$ | $\boxed{x=-3}$

$\boxed{x=2.5}$

Let's try a few more...

$3x^2 + 5x + 2 = 0$

$2 \times 3 = 6$

$2 + 3 = 5$

6
1x6
2x3

$(x+\frac{2}{3})(x+\frac{3}{3})=0$

$(3x+2)(x+1)=0$

$3x+2=0$ | $x+1=0$

$\frac{3x}{3}=\frac{-2}{3}$ | $\boxed{x=-1}$

$\boxed{x=\frac{-2}{3}}$

$6x^2 + 14x + 8 = 0$

EXTRA STEP

___ x ___ = ___

___ + ___ = ___

Common Factor

Example 3: $7x^2 + 4x = 0$

$x(7x+4)=0$

$\boxed{x=0}$ | $7x+4=0$

$\frac{7x}{7} = \frac{-4}{7}$

$\boxed{x=-\frac{4}{7}} = -0.57$

*****Sometimes you may remove a common factor first and then end up with a simple trinomial, a hard trinomial, or a difference of squares.**

$3x^2+6x=0$

$3x(x+2)=0$

$\frac{3x}{3}=\frac{0}{3}$ | $x+2=0$

$\boxed{x=0}$ | $\boxed{x=-2}$

$\frac{3x^2}{3x}=1x$

$\frac{6x}{3x}=2$

$5x^2-25x+10$

$5(x^2-5x+2)$

Difference of Squares

Example 4: $4x^2 - 9 = 0$

Perfect Square   Perfect Square

$(2x+3)(2x-3)=0$ (use opposite signs)

$2x+3=0$ | $2x-3=0$

$2x=-3$ | $2x=3$

$\boxed{x=\frac{-3}{2}}$ | $\boxed{x=\frac{3}{2}}$

perfect squares

$x^2-36=0$

$(x+6)(x-6)=0$

$x+6=0$ | $x-6=0$

$\boxed{x=-6}$ | $\boxed{x=6}$

Using reasoning to write an equation from its roots

Tori says she solved a quadratic equation by graphing. She says the roots were −5 and 7. How can you determine an equation that she might have solved?

(x-intercepts)

Philip's Solution

$x = -5$ or $x = 7$

The x-intercepts of the quadratic function are the roots of the equation.

$x + 5 = 0$ $\quad x - 7 = 0$
One factor is $x + 5$.
The other factor is $x - 7$.

I decided to use the roots to help me write the factors of the equation.

$$(x + 5)(x - 7) = 0$$
$$x^2 + 5x - 7x - 35 = 0$$
$$x^2 - 2x - 35 = 0$$

I wrote the factors as a product. Since each root is equal to 0, their product is also equal to 0.

I simplified to write the equation in standard form.

In Summary

Key Idea

- Some quadratic equations can be solved by factoring.

Need to Know

- To factor an equation, start by writing the equation in standard form.
- You can set each factor equal to zero and solve the resulting linear equations. Each solution is a solution to the original equation.
- If the two roots of a quadratic equation are equal, then the quadratic equation is said to have one solution.

Assignment: pages 323 - 324

Questions 1, 2(a-d), 6, 7, 10, 11

SOLUTIONS => 6.5 Solving Quadratic Equations by Factoring

1.

a) $x^2 - 11x + 28 = 0$ (A: -11, M: 28)

$(x-4)(x-7)=0$

$x-4=0$ or $x-7=0$

$x=4$ $x=7$

$\underline{-4} \times \underline{-7} = 28$

$\underline{-4} + \underline{-7} = -11$

b) $x^2 - 7x - 30 = 0$ (A: -7, M: -30)

$(x+3)(x-10)=0$

$x+3=0$ or $x-10=0$

$x=-3$ $x=10$

$\underline{3} \times \underline{-10} = -30$

$\underline{3} + \underline{-10} = -7$

c) $2y^2 + 11y + 5 = 0$

$(y + \frac{1}{2})(y + \frac{10}{2})$

$\frac{1}{} \times \frac{10}{} = 10$

$\frac{1}{} + \frac{10}{} = 11$

$(2y+1)(y+5) = 0$

$2y+1=0$ or $y+5=0$

$\frac{\cancel{2}y}{\cancel{2}} = \frac{-1}{2}$ $\quad y = -5$

$y = \frac{-1}{2}$

d) $4t^2 + 7t - 15 = 0$

$(t - \frac{5}{4})(t + \frac{12}{4})$

$\frac{-5}{} \times \frac{12}{} = -60$

$\frac{-5}{} + \frac{12}{} = 7$

$(4t-5)(t+3) = 0$

$4t-5=0$ or $t+3=0$

$\frac{\cancel{4}t}{\cancel{4}} = \frac{5}{4}$ $\quad t = -3$

$t = \frac{5}{4}$

2.

{a-d} a) $x^2 - 121 = 0$ (Difference of Squares)

$(x-11)(x+11)=0$

$x-11=0$ or $x+11=0$

$x=11$ $\quad$ $x=-11$

b) $9r^2 - 100 = 0$ (Difference of Squares)

$(3r-10)(3r+10)=0$

$3r-10=0$ or $3r+10=0$

$\frac{\cancel{3}r}{\cancel{3}} = \frac{10}{3}$ $\quad$ $\frac{\cancel{3}r}{\cancel{3}} = \frac{-10}{3}$

$r = \frac{10}{3}$ $\quad$ $r = -\frac{10}{3}$

c) $x^2 - 15x = 0$ (Common Factor)

$x(x - 15) = 0$

$x = 0$ or $x - 15 = 0$

$x = 15$

d) $3y^2 + 48y = 0$ (Common Factor)

$3y(y + 16) = 0$

$\frac{\cancel{3}y}{\cancel{3}} = \frac{0}{3}$ or $y + 16 = 0$

$y = -16$

$y = 0$

6. Determine the roots of each equation.

a) $5u^2-10u-315=0$

$5(u^2-2u-63)=0$

$5(u+7)(u-9)=0$

$u+7=0$ or $u-9=0$

$u=-7$ $\quad$ $u=9$

$\underline{7} \times \underline{-9} = -63$

$\underline{7} + \underline{-9} = -2$

b) $0.25x^2+1.5x+2=0$

$0.25(x^2+6x+8)=0$

$0.25(x+4)(x+2)=0$

$x+4=0$ or $x+2=0$

$x=-4$ $\quad$ $x=-2$

$\underline{4} \times \underline{2} = 8$

$\underline{4} + \underline{2} = 6$

c) $1.4y^2 + 5.6y - 16.8 = 0$

$1.4(y^2 + 4y - 12) = 0$ $\quad$ $\underline{6} \times \underline{-2} = -12$

$1.4(y + 6)(y - 2) = 0$ $\quad$ $\underline{6} + \underline{-2} = 4$

$y + 6 = 0$ or $y - 2 = 0$

$y = -6$ $\quad$ $y = 2.$

d) $\frac{1}{2}K^2 + 5K + 12.5 = 0$

$\frac{1}{2}(K^2 + 10K + 25) = 0$ $\quad$ $\underline{5} \times \underline{5} = 25$

$\underline{5} + \underline{5} = 10$

$\frac{1}{2}(K + 5)(K + 5) = 0$

$\frac{1}{2}(K + 5)^2 = 0$

$K + 5 = 0$

$K = -5$

7. The graph of a quadratic function has x-intercepts -5 and -12. Write a quadratic equation that has these roots.

$y = a(x-r)(x-s)$

Assuming $a=1$:

$y = (x - -5)(x - -12)$

$y = (x+5)(x+12)$

$y = x^2 + 12x + 5x + 60$

$y = x^2 + 17x + 60$

Quadratic Equation => $x^2 + 17x + 60 = 0$

10. Identify and correct any errors in the following solution.

$$5a^2 - 100 = 0$$
$$5a^2 = 100$$
$$a^2 = 25 \longleftarrow \text{Error}$$
$$\sqrt{a^2} = \sqrt{25}$$
$$a = 5 \longleftarrow \text{Error}$$

Correction:

$$5a^2 - 100 = 0$$
$$\frac{\cancel{5}a^2}{\cancel{5}} = \frac{100}{5}$$
$$a^2 = 20$$
$$\sqrt{a^2} = \sqrt{20}$$
$$a = \pm\sqrt{20}$$

11. Identify and correct the errors in this solution:

$$4r^2 - 9r = 0$$
$$(2r-3)(2r+3)=0 \rightarrow \text{Error}$$
$$2r-3=0 \quad \text{or} \quad 2r+3=0$$
$$2r=3 \qquad 2r=-3$$
$$r=1.5 \quad \text{or} \quad r=-1.5$$

Correction:

$$4r^2-9r=0$$
$$r(4r-9)=0$$
$$r=0 \quad \text{or} \quad 4r-9=0$$
$$\frac{\cancel{4}r}{\cancel{4}}=\frac{9}{4}$$
$$r=\frac{9}{4}$$

7s5e2 finalt.mp4

7s5e3 finalt.mp4

7s5e4 finalt.mp4

7s5e5 finalt.mp4

FM11-7s5.gsp