6.7 Solving Quadratic Equations Using the Quadratic Formula

GOAL

Use the quadratic formula to determine the roots of a quadratic equation.

THE QUADRATIC FORMULA

The Quadratic Formula can be used to solve **ALL** quadratic equations, even the ones that you cannot factor!

The solution to any quadratic equation: $ax^2 + bx + c = 0$; where $a \neq 0$, is given by:

The Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Since the solutions to quadratic equations are linked to the x-intercepts of quadratic functions, it makes sense that quadratic equations may also have 0, 1, or 2 solutions.

In the next few examples, we will use the quadratic formula to find the solution to various quadratic equations. These examples will illustrate the three possible results that can be obtained when solving quadratics.

Example 1: Two REAL Solutions

Solve $x^2 + 7x + 12 = 0$

opens up a=1

Solution:

a = 1; b = 7; c = 12 y-int

Therefore, $x = \frac{-7 \pm \sqrt{(7)^2 - 4(1)(12)}}{2(1)}$

$x = \frac{-7 \pm \sqrt{49-48}}{2}$

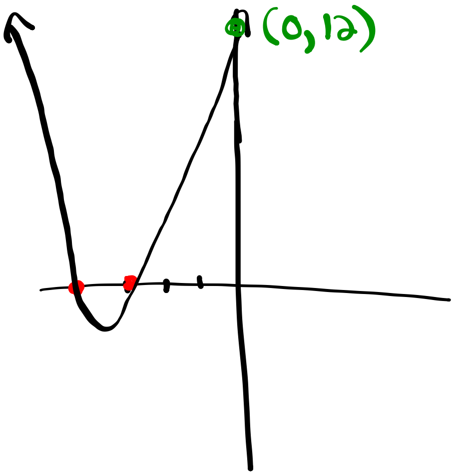
$x = \frac{-7 \pm \sqrt{1}}{2}$

$x = \frac{-7 \pm 1}{2}$

$x = \frac{-6}{2}$ or $x = \frac{-8}{2}$

$x = -3$ or $x = -4$

(2 x-int)



Example 2: One REAL Solution

Solve $2x^2 + 24x + 72 = 0$

opens up a=2

Solution:

a = 2; b = 24; c = 72 y-int

Therefore, $x = \frac{-24 \pm \sqrt{(24)^2 - 4(2)(72)}}{2(2)}$

$x = \frac{-24 \pm \sqrt{576-576}}{4}$

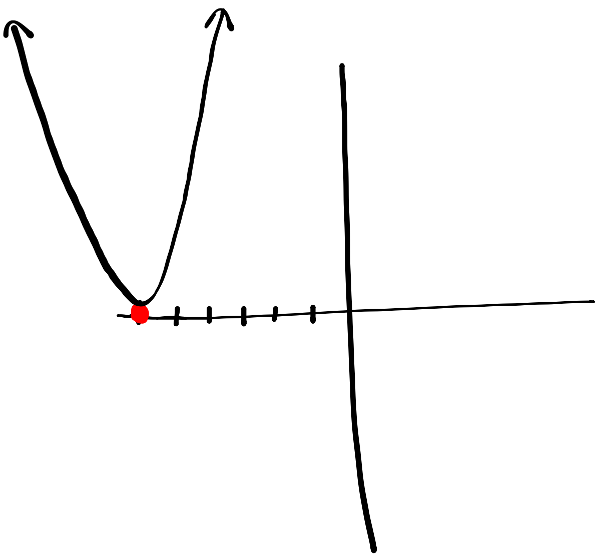
$x = \frac{-24 \pm \sqrt{0}}{4}$

$x = \frac{-24 \pm 0}{4}$

$x = \frac{-24}{4}$

$x = -6$

(1 x-int)



<u>Example 3: No REAL Solutions.</u> (Negative under the square root)

Solve $x^2 - 4x + 8 = 0$

$b^2 - 4ac < 0$

<u>Solution:</u> open up $a=1$

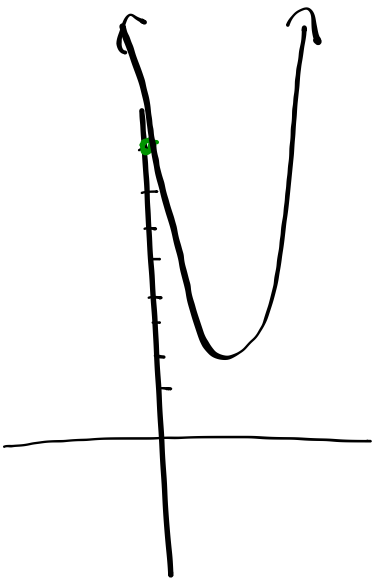
$a = 1$; $b = -4$; $c = 8$ y-int

Therefore, $x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(8)}}{2(1)}$

$x = \frac{4 \pm \sqrt{16-32}}{2}$

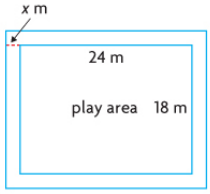
$x = \frac{4 \pm \sqrt{-16}}{2}$

(0 x-int)



LEARN ABOUT the Math

Ian has been hired to lay a path of uniform width around a rectangular play area, using crushed rock. He has enough crushed rock to cover 145 m^2.



? If Ian uses all the crushed rock, how wide will the path be?

Using the quadratic formula to solve a quadratic equation

Determine the width of the path that will result in an area of 145 m^2.

Alima's Solution

Area of border = Total area − Play area
The play area is a constant, (length)(width) or (24 m)(18 m) or 432 m^2.

The total area of the playground, P, can be represented as

$P = (\text{length})(\text{width})$
$P = (2x + 24)(2x + 18)$

I wrote a function that describes how the area of the path, A square metres, changes as the width of the path, x metres, changes.

The area of the path, $A(x)$, can be represented as

$A(x) = (2x + 24)(2x + 18) - 432$
$A(x) = 4x^2 + 84x + 432 - 432$
$A(x) = 4x^2 + 84x$
$145 = 4x^2 + 84x$

I substituted the area of 145 m^2 for $A(x)$.

$4x^2 + 84x - 145 = 0$
$a = 4$, $b = 84$, and $c = -145$

I rewrote the equation in standard form:

$ax^2 + bx + c = 0$

Then I determined the values of the coefficients a, b, and c.

Quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-84 \pm \sqrt{84^2 - 4(4)(-145)}}{2(4)}$$

$$x = \frac{-84 \pm \sqrt{9376}}{8}$$

$$x = \frac{-84 + \sqrt{9376}}{8} \quad \text{or} \quad x = \frac{-84 - \sqrt{9376}}{8}$$

$$x = 1.603\ldots \quad \text{or} \quad x = -22.603\ldots$$

The **quadratic formula** can be used to solve any quadratic equation. I wrote the quadratic formula and then substituted the values of a, b, and c from my equation into the formula.

I simplified the right side.

I separated the quadratic expression into two solutions.

quadratic formula
A formula for determining the roots of a quadratic equation in the form $ax^2 + bx + c = 0$, where $a \neq 0$; the quadratic formula is written using the coefficients of the variables and the constant in the quadratic equation that is being solved:

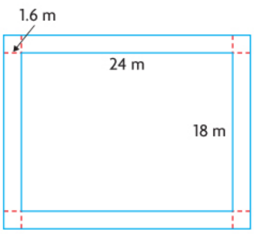
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This formula is derived from $ax^2 + bx + c = 0$ by isolating x.

The solution −22.603 is inadmissible.

I knew that the width of the path couldn't be negative, so −22.603 ... is an **inadmissible solution**.

inadmissible solution
A root of a quadratic equation that does not lead to a solution that satisfies the original problem.



I sketched the path and verified my solution by determining the area of the path. To do this, I added the areas of all the rectangles that make up the path.

$18(1.603\ldots) = 28.866\ldots\ m^2$
$24(1.603\ldots) = 38.489\ldots\ m^2$
$(1.603\ldots)(1.603\ldots) = 2.571\ldots\ m^2$

Area of path = $2(28.866\ldots) + 2(38.489\ldots) + 4(2.571\ldots)$
Area of path = $144.999\ldots\ m^2$

The total area is very close to 145 m^2.
The path should be about 1.6 m wide.

Reflecting

A. Why did Alima need to write her equation in standard form?

B. Which part of the quadratic formula shows that there are two possible solutions?

C. Why did Alima decide not to use the negative solution?

D. In this chapter, you have learned three methods for solving quadratic equations: graphing, factoring, and using the quadratic formula. What are some advantages and disadvantages of each method?

Answers

A. She needed to know the values of a, b, and c so she could substitute them into the formula.

B. The $\pm$ sign tells you that you determine one solution by adding in the numerator and the other solution by subtracting, so there are two solutions.

C. A border cannot be -22 m wide.

D.

Graphing	Factoring	Quadratic Formula
Advantages: ▪ It can be used to solve any equation. ▪ It gives you a quick way to check that solutions make sense.	Advantages: ▪ You can use it if you do not have a graphing calculator. ▪ It works quickly if the equation is easy to factor.	Advantages: ▪ It can be used to solve any equation. ▪ It is quick to use. ▪ It can be used if you have trouble factoring an expression.
Disadvantages: ▪ You have to have a graphing calculator. ▪ You have to make sure the window settings are right. ▪ People sometimes make errors entering equations. ▪ It can be slow.	Disadvantages: ▪ You have to start by rewriting the equation in standard form. ▪ Some equations cannot be factored. ▪ Factoring can be difficult, and you cannot always tell if an equation is factorable.	Disadvantages: ▪ You have to start by rewriting the equation in standard form ▪ You have to remember to consider the positive and negative roots. ▪ You have to be careful not to make sign errors.

APPLY the Math

Connecting the quadratic formula to factoring

Solve the following equation:

$$6x^2 - 3 = 7x$$

Adrianne's Solution

$$6x^2 - 3 = 7x$$
$$6x^2 - 7x - 3 = 0$$
$$a = 6,\ b = -7, \text{ and } c = -3$$

First, I rewrote the equation in standard form to determine the values of *a*, *b*, and *c*.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(6)(-3)}}{2(6)}$$
$$x = \frac{7 \pm \sqrt{121}}{12}$$
$$x = \frac{7 \pm 11}{12}$$

I wrote the quadratic formula and substituted the values of *a*, *b*, and *c*.

I simplified the right side. I realized that 121 is a perfect square.

$$x = \frac{18}{12} \quad \text{or} \quad x = \frac{-4}{12}$$
$$x = \frac{3}{2} \quad \text{or} \quad x = \frac{-1}{3}$$

I determined the two solutions.

Verify:

$$6x^2 - 7x - 3 = 0$$
$$(3x + 1)(2x - 3) = 0$$

$$3x + 1 = 0 \quad \text{or} \quad 2x - 3 = 0$$
$$3x = -1 \qquad\qquad 2x = 3$$
$$x = \frac{-1}{3} \qquad\qquad x = \frac{3}{2}$$

The solutions match those I got using the quadratic formula.

If the radicand in the quadratic formula is a perfect square, then the original equation can be factored. I decided to verify my solution by factoring the original equation.

Using the Quadratic Equation:

$6x^2 - 3 = 7x$

$6x^2 - 7x - 3 = 0$

$a = 6 \quad b = -7 \quad c = -3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(6)(-3)}}{2(6)}$$

$$x = \frac{7 \pm \sqrt{49 + 72}}{12}$$

$$x = \frac{7 \pm \sqrt{121}}{12}$$

$$x = \frac{7 \pm 11}{12}$$

$x = \frac{18}{12} = \frac{3}{2}$ | $x = \frac{-4}{12} = \frac{-1}{3}$

Using Decomposition:

$6x^2 - 3 = 7x$

$6x^2 - 7x - 3 = 0$

$2 \times -9 = -18$
$2 + -9 = -7$

18
1×18
2×9
3×6

Reduce

$(x + \frac{2}{6})(x - \frac{9}{6}) = 0$

$(x + \frac{1}{3})(x - \frac{3}{2}) = 0$

$(3x + 1)(2x - 3) = 0$

$3x + 1 = 0$ | $2x - 3 = 0$

$\frac{3x}{3} = \frac{-1}{3}$ | $\frac{2x}{2} = \frac{3}{2}$

$\boxed{x = -1/3}$ | $\boxed{x = 3/2}$

In Summary

Key Idea

- The roots of a quadratic equation in the form $ax^2 + bx + c = 0$, where $a \neq 0$, can be determined by using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Need to Know

- The quadratic formula can be used to solve any quadratic equation, even if the equation is not factorable.
- If the radicand in the quadratic formula simplifies to a perfect square, then the equation can be solved by factoring.
- If the radicand in the quadratic formula simplifies to a negative number, then there is no real solution for the quadratic equation.

Assignment: pages 345 - 346

Questions 2, 3, 5, 6

SOLUTIONS ⇒ 6.7 Solving Quadratic Equations Using the Quadratic Formula

a) $x^2 + 5x - 6 = 0$

$a = 1,\ b = 5,\ c = -6$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(1)(-6)}}{2(1)}$$

$$x = \frac{-5 \pm \sqrt{25 + 24}}{2}$$

$$x = \frac{-5 \pm \sqrt{49}}{2}$$

$$x = \frac{-5 \pm 7}{2}$$

$x = \frac{-5+7}{2}$ or $x = \frac{-5-7}{2}$

$x = \frac{2}{2}$ $\qquad$ $x = \frac{-12}{2}$

$x = 1$ $\qquad$ $x = -6$

b) $4x + 9x^2 = 0$

$9x^2 + 4x = 0$

$a = 9, b = 4, c = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-4 \pm \sqrt{(4)^2 - 4(9)(0)}}{2(9)}$

$x = \frac{-4 \pm \sqrt{16 - 0}}{18}$

$x = \frac{-4 \pm \sqrt{16}}{18}$

$x = \frac{-4 \pm 4}{18}$

$x = \frac{-4 + 4}{18}$ or $x = \frac{-4 - 4}{18}$

$x = \frac{0}{18}$ $\quad x = \frac{-8}{18}$

$x = 0$ $\quad x = \frac{-4}{9}$

c) $25x^2 - 121 = 0$

$a = 25, b = 0, c = -121$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{0 \pm \sqrt{(0)^2 - 4(25)(-121)}}{2(25)}$

$x = \frac{0 \pm \sqrt{0 + 12100}}{50}$

$x = \frac{\pm \sqrt{12100}}{50}$

$x = \pm \frac{110}{50}$

$x = \pm \frac{11}{5}$

$x = \frac{11}{5}$ or $x = \frac{-11}{5}$

d) $12x^2-17x-40=0$

$a=12, b=-17, c=-40$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{17\pm\sqrt{(-17)^2-4(12)(-40)}}{2(12)}$$

$$x=\frac{17\pm\sqrt{289+1920}}{24}$$

$$x=\frac{17\pm\sqrt{2209}}{24}$$

$$x=\frac{17\pm 47}{24}$$

$x=\frac{17+47}{24}$ or $x=\frac{17-47}{24}$

$x=\frac{64}{24}$ $x=\frac{-30}{24}$

$x=\frac{8}{3}$ $x=\frac{-5}{4}$

3. Solve each equation in question 2 by factoring. Which method did you prefer for each equation? Explain.

a) $x^2+5x-6=0$ $\quad \underline{-1} \times \underline{6} = -6$

$(x-1)(x+6)=0$ $\quad \underline{-1} + \underline{6} = 5$

$x-1=0$ or $x+6=0$

$x=1$ $\quad x=-6$

b) $4x+9x^2=0$

$9x^2+4x=0$

$x(9x+4)=0$

$x=0$ or $9x+4=0$

$\frac{\cancel{9}x}{\cancel{9}}=\frac{-4}{9}$

$x=-\frac{4}{9}$

c) $25x^2-121=0$

$(5x-11)(5x+11)=0$

$5x-11=0$ or $5x+11=0$

$\frac{\cancel{5}x}{\cancel{5}}=\frac{11}{5}$ $\quad \frac{\cancel{5}x}{\cancel{5}}=\frac{-11}{5}$

$x=\frac{11}{5}$ $\quad x=-\frac{11}{5}$

d) $12x^2 - 17x - 40 = 0$

$\frac{15}{12} \times \frac{-32}{12} = -480$

$\frac{15}{12} + \frac{-32}{12} = -17$

$(x + \frac{15}{12})(x - \frac{32}{12}) = 0$

$(x + \frac{5}{4})(x - \frac{8}{3}) = 0$

*Must reduce fractions!

$(4x+5)(3x-8) = 0$

$4x + 5 = 0$ or $3x - 8 = 0$

$\frac{\cancel{4}x}{\cancel{4}} = \frac{-5}{4}$ $\qquad$ $\frac{\cancel{3}x}{\cancel{3}} = \frac{8}{3}$

$x = -\frac{5}{4}$ $\qquad$ $x = \frac{8}{3}$

5. The roots for the quadratic equation

$$1.44a^2 + 2.88a - 21.6 = 0$$

are $a = 3$ and $a = -5$. Verify these roots.

[a=3]

L.S	R.S	
$1.44a^2 + 2.88a - 21.6$	0	Since L.S = R.S, $a = 3$ is a valid solution.
$1.44(3)^2 + 2.88(3) - 21.6$		
$1.44(9) + 8.64 - 21.6$		
$12.96 - 12.96$		
0		

[a=-5]

L.S.	R.S.	
$1.44a^2 + 2.88a - 21.6$	0	Since L.S = R.S, $a = -5$ is a valid solution.
$1.44(-5)^2 + 2.88(-5) - 21.6$		
$1.44(25) - 14.4 - 21.6$		
$36 - 36$		
0		

6. Solve each equation. State the solutions as exact values.

a) $3x^2 - 6x - 1 = 0$

$a=3,\ b=-6,\ c=-1$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{6 \pm \sqrt{(-6)^2-4(3)(-1)}}{2(3)}$$

$$x = \frac{6 \pm \sqrt{36+12}}{6}$$

$$x = \frac{6 \pm \sqrt{48}}{6}$$

We can stop here for now!

b) $x^2 + 8x + 3 = 0$

$a=1,\ b=8,\ c=3$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-8 \pm \sqrt{(8)^2-4(1)(3)}}{2(1)}$$

$$x = \frac{-8 \pm \sqrt{64-12}}{2}$$

$$x = \frac{-8 \pm \sqrt{52}}{2}$$

We can stop here for now!

c) $8x^2+8x-1=0$

$a=8,\ b=8,\ c=-1$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-8\pm\sqrt{(8)^2-4(8)(-1)}}{2(8)}$$

$$x=\frac{-8\pm\sqrt{64+32}}{16}$$

$$x=\frac{-8\pm\sqrt{96}}{16}$$

We can stop here for now!

d) $9x^2-12x-1=0$

$a=9,\ b=-12,\ c=-1$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{12\pm\sqrt{(-12)^2-4(9)(-1)}}{2(9)}$$

$$x=\frac{12\pm\sqrt{144+36}}{18}$$

$$x=\frac{12\pm\sqrt{180}}{18}$$

We can stop here for now!

Attachments

7s7e2 finalt.mp4

7s7e3 finalt.mp4

7s7e4 finalt.mp4