Questions from homework

③ a) $t_1 = 1$

$\boxed{t_n = 2t_{n-1}}$ recursive rule

previous term

$\underline{1}, \underline{2}, \underline{4}, \underline{8}, \underline{16}$

Diverging Sequence
No limit exists

$t_2 = 2t_{2-1} = 2t_1 = 2(1) = 2$

$t_3 = 2t_{3-1} = 2t_2 = 2(2) = 4$

$t_4 = 2t_3 = 2(4) = 8$

$t_5 = 2t_4 = 2(8) = 16$

④ b) $\lim\limits_{n \to \infty} \frac{3n}{n^2+2} = 0$

e) $\lim\limits_{n \to \infty} \left(\frac{3}{4}\right)^n$ $\quad t_n = \left(\frac{3}{4}\right)^n$

$t_1 = \frac{3}{4} = 0.75$

$t_2 = \frac{9}{16} = 0.5625$

$t_3 = \frac{27}{64} = 0.422$

$t_8 = \frac{6561}{65\,536} = 0.1001$

Converging on 0

$\lim\limits_{n \to \infty} \left(\frac{3}{4}\right)^n = 0$

Infinite Geometric Series

How is it possible to find the sum of an infinite series?

Example (Zeno's Paradox)

"A man standing in a room cannot walk to the wall on the other side. In order to do so, he would first have to go half the distance, then half the remaining distance, and then again half of what still remains. The process can always be continued and can never be ended."

Of course we know the the man can actually reach the wall, so this suggests that perhaps the total distance can be expressed as the sum of infinitely many smaller distances as follows:

$$1 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots + \frac{1}{2^n} + \ldots$$

In order to make sense of this equation, we let S_n be the sum of the first n terms of the series. Then..

$$S_1 = \frac{1}{2} = 0.5$$

$$S_2 = \frac{1}{2} + \frac{1}{4} = 0.75$$

$$S_3 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = 0.875$$

$$S_4 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = 0.9375$$

$$S_5 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} = 0.96875$$

$$S_{10} = \frac{1}{2} + \frac{1}{4} + \ldots + \frac{1}{1024} = 0.99902344$$

$$S_{16} = \frac{1}{2} + \frac{1}{4} + \ldots + \frac{1}{2^{16}} = 0.99998474$$

As we add more and more terms, the partial sums become closer and closer to 1. In fact by making n large enough we can make the partial sum S_n as close as we like to the number 1.

Using Limit Notation: $\lim_{n\to\infty} S_n = 1$

If the infinite sequence of partial sums has a Limit as in the previous example, then we say the sum of the series is L and we write:

$$\sum_{n=1}^{\infty} t_n = L$$

If a series has a sum, it is called a *convergent* series. If not, it is called a *divergent* series.

In order for an infinite geometric series to have a sum the common ratio "r" has to be between 1 and -1.

$-1 < r < 1$, $r \neq 0$

If so, we use the following formula:

$$\boxed{S_n = \frac{a}{1-r}} \quad S_\infty = \frac{a}{1-r}$$

Examples

Find S_n for the following infinite geometric series.

Recall: In order for an infinite geometric series to have a sum the common ratio "r" has to be between 1 and -1.

$$\boxed{S_n = \frac{a}{1-r}}$$

$$4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} + \ldots$$

Infinite Series

$a = 4$

$r = \frac{1}{2}$

$$S_\infty = \frac{4}{1-\frac{1}{2}} = \frac{4}{\frac{1}{2}} = 8$$

The series is converging on 8

$$3 + 6 + 12 + 24 + \ldots$$

Infinite Series

$a = 3$

$r = 2$

The Series is diverging because $r = 2$ $(-1 < r < 1)$

Infinite Series (Σ)

$$\sum_{n=1}^{\infty} (3)\left(\frac{1}{4}\right)^{n-1} = 3 + \frac{3}{4} + \frac{3}{16} + \frac{3}{64} + \ldots$$

$a = 3$

$r = \frac{1}{4}$

$$S_\infty = \frac{3}{1-\frac{1}{4}} = \frac{3}{\frac{3}{4}} = 3 \times \frac{4}{3} = 4$$

The series is converging on 4

Your Turn

You can express $0.\overline{584}$ as an infinite geometric series.

$0.\overline{584} = 0.584\ 584\ 584\ldots$

$= 0.584 + 0.000\ 584 + 0.000\ 000\ 584 + \cdots$

Determine the sum of the series.

$a = 0.584$

$r = 0.001$

$$S_\infty = \frac{0.584}{1 - 0.001} = \frac{0.584}{0.999} = \boxed{\frac{584}{999}}$$

Homework