

# The Fundamental Counting Principle

The principle which states that all possible outcomes in a *sample space* can be found by multiplying the number of ways each event can occur.

**Example 1:** A deli has a lunch special which consists of a sandwich, soup, dessert and drink for \$4.99.

They offer the following choices:

Sandwich: chicken salad, ham, and tuna, and roast beef

**Soup**: tomato, chicken noodle, vegetable

(a) Dessert: cookie and pie

Drink: tea, coffee, coke, diet coke and sprite

How many lunch specials are there?

#### Let's use the basic counting principle

There are 4 stages or events: choosing a sandwich, choosing a soup, choosing a dessert and choosing a drink.

There are 4 choices for the sandwich, 3 choices for the soup, 2 choices for the dessert and 5 choices for the drink

#### Putting that all together we get:

Sand. Soup Dessert Drink # of lunch specials  $4 \quad \mathbf{x} \quad \mathbf{3} \quad \mathbf{x} \quad \mathbf{2} \quad \mathbf{x} \quad \mathbf{5} \quad = \quad \mathbf{120}$ 

So there are 120 lunch specials possible

**Example 2:** A company places a 6-symbol code on each unit of product.

The code consists of 4 digits, the first of which is the number 5, follow by 2 letters, the first of which is NOT a vowel.

How many different codes are possible?

#### Let's use the basic counting principle

There are 6 stages or events: digit 1, digit 2, digit 3, digit 4, letter 1, and letter 2.

In general there are 10 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. The first digit is limited being the number 5, so there is only one possibility for that one. There are no restriction on digits 2 - 4, so each one of those has 10 possibilities. In general, there are 26 letters in the alphabet. The first letter, cannot be a vowel (a i, o, u), so that means there are 21 possible letters that could go there. The second letter has no restriction, so there are 26 possibilities for that one.

#### Putting that all together we get:

digit 1 digit 2 digit 3 digit 4 letter 1 letter 2 # of codes
$$1 x 10 x 10 x 10 x 21 x 26 = 546000$$

So there are 546000 different 6-symbol codes possible

## **Factorial Notation**

The product of consecutive natural numbers, in decreasing order to the number 1, can be represented using factorial notation.

The symbol for factorial is the exclamation mark, !.



Calculate the following:

- a) 5!
- b) 3!
- c) 10!

#### **Solution**

a) 5! is read as "five factorial" b) 3! is read as "three factorial"

$$5! = 5 \times 4 \times 3 \times 2 \times 1$$
  $3! = 3 \times 2 \times 1$   $= 6$ 

c) 10! is read as "ten factorial" 10! = 10 x 9 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1 = 3 628 800

Fortunately, all scientific calculators have a factoral key so we don't have to manually do the calculations.

Simplify the following:

**NOTE:** The factors 8, 7, 6, ..., 1 in the denominator of the fraction can cancel with the like factor in the numerator of the fraction. So all we are left with is:

$$14 \times 13 \times 12 \times 11 \times 10 \times 9 = 2 \times 162 \times 160$$

b) 
$$\underline{9!} = 9 \times 8 \times 7 \times 6 \times 5$$
  
4!  
= 15 120

Write  $12 \times 11 \times 10 \times 9$  as a ratio/quotient of factorials.

a divide

#### **Solution**

The expression contains the first four terms of 12! We can write an expression that would divide the remaining terms.

$$\frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$$

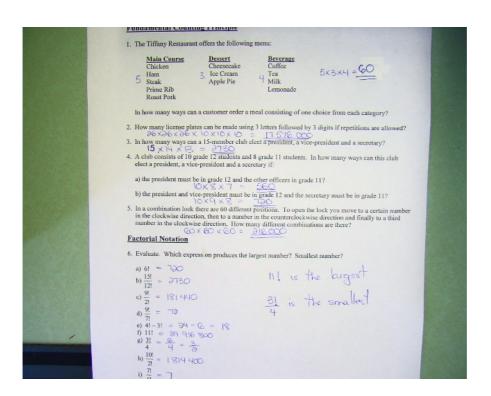
$$= \underbrace{12!}_{8!}$$

Write 10 x 9 x 8 x 3 x 2 x 1 as a ratio of factorials.

### **Solution**

$$= \frac{10!}{7!} \times 3!$$
 or  $\frac{10!}{7!} \frac{3!}{7!}$ 

**NOTE:** By definition 0! = 1



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b) the president and vice-president must be in grade 11 and the secretary must be in grade 11?

0 \times 8 \times 7 = 600

5. In a combination lock there are 60 different positions. To open the lock you move to a certain number in the elockwise direction, then to a number in the context-ockwise direction. He was marber in the elockwise direction. How many different combinations are there?

Factorial Notation

6. Evaluate. Which expression produces the largest number? Smallest number?

a) 61 = 900

b) \frac{18}{12} = 3730

c) \frac{9}{12} = 181440

31 is the largest \frac{1}{12} = \frac{1}{12}
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