

Questions from Homework

$$\textcircled{a} \text{ c) } \sqrt{x+y} + \sqrt{xy} = 4$$

$$(x+y)^{1/2} + (xy)^{1/2} = 4$$

$$\frac{1}{2}(x+y)^{-1/2}(1+y') + \frac{1}{2}(xy)^{-1/2}(xy'+y) = 0$$

$$\frac{1}{2\sqrt{x+y}}(1+y') + \frac{1}{2\sqrt{xy}}(xy'+y) = 0$$

$$\frac{1+y'}{2\sqrt{x+y}} + \frac{xy'+y}{2\sqrt{xy}} = 0 \quad \frac{4\sqrt{xy}\sqrt{x+y}}{4\sqrt{xy}\sqrt{x+y}}$$

$$2\sqrt{xy} + 2y'\sqrt{xy} + 2xy'\sqrt{x+y} + 2y\sqrt{x+y} = 0$$

$$2y'\sqrt{xy} + 2xy'\sqrt{x+y} = -2\sqrt{xy} - 2y\sqrt{x+y}$$

$$y' [2\sqrt{xy} + 2x\sqrt{x+y}] = -2\sqrt{xy} - 2y\sqrt{x+y}$$

$$y' = \frac{-2\sqrt{xy} - 2y\sqrt{x+y}}{2\sqrt{xy} + 2x\sqrt{x+y}}$$

Point

$$y' = - \frac{\sqrt{xy} + y\sqrt{x+y}}{\sqrt{xy} + x\sqrt{x+y}} \quad (2,2)$$

$$y' = - \frac{\sqrt{(2)(2)} + 2\sqrt{(2)+(2)}}{\sqrt{(2)(2)} + 2\sqrt{(2)+(2)}}$$

$$= - \frac{2+4}{2+4}$$

$$\boxed{-1}$$

Applications of Derivatives

Now that we know how to calculate derivatives, we use them in this unit to compute, *velocity, acceleration, and other rates of change.*

Suppose that an object moves along a straight line. (Think of a ball being thrown vertically upward or a car being driven along a road or a stone being dropped from a cliff.) The position function is $s = f(t)$, where s is the **displacement (directed distance)** of the object from the origin at time t . Recall that the **(instantaneous) velocity** of the object at time t is defined as the limit of average velocities over shorter and shorter time intervals.

In short, the *velocity* is the derivative of the position function and in Leibniz notation we write.

$$v = \frac{ds}{dt}$$

Velocity

If a stone is dropped from a cliff that is 122.5 m high, then its height in metres after t seconds is $h = 122.5 - 4.9t^2$

$$h' = v(t) = -9.8t$$

a) Find its velocity after 1 s and 2 s.

b) When will the stone hit the ground? (Let $h=0$)

c) With what velocity will it hit the ground? (Let $t=5$)

a) $t=1s$

$$v(1) = -9.8(1)$$

$$= -9.8 \text{ m/s}$$

$t=2s$

$$v(2) = -9.8(2)$$

$$= -19.6 \text{ m/s}$$

b) Let $h=0$

$$h = 122.5 - 4.9t^2$$

$$0 = 122.5 - 4.9t^2$$

$$4.9t^2 = 122.5$$

$$t^2 = 25$$

$$t = \pm 5s$$

\therefore In 5 seconds the stone will hit the ground.

c) Let $t=5s$

$$v(t) = -9.8t$$

$$v(5) = -9.8(5)$$

$$= -49 \text{ m/s}$$

\therefore The stone hits the ground with a velocity of -49 m/s .

The position of a particle moving on a line is given by the equation

$$s = 2t^3 - 21t^2 + 60t,$$

where t is measured in seconds and s in meters

a) Find its velocity after 3 s and 6 s.

b) When is the particle at rest? (Let $v=0$)

a) $v = 6t^2 - 42t + 60$

$$\underline{t=3s}$$

$$v = 6(3)^2 - 42(3) + 60$$

$$= 54 - 126 + 60$$

$$= -12 \text{ m/s}$$

$$\underline{t=6s}$$

$$v = 6(6)^2 - 42(6) + 60$$

$$= 216 - 252 + 60$$

$$= 24 \text{ m/s}$$

b) $v = 6t^2 - 42t + 60$

$$0 = 6(t^2 - 7t + 10)$$

$$0 = 6(t-5)(t-2)$$

$$t-5=0 \quad | \quad t-2=0$$

$$\boxed{t=5s}$$

$$\boxed{t=2s}$$

Applications of Derivatives

Now that we know how to calculate derivatives, we use them in this unit to compute, *velocity, acceleration, and other rates of change.*

If an object moves along a straight line, its **acceleration** is the rate of change of velocity with respect to time. Therefore, *the acceleration $a(t)$ at time t is the derivative of the velocity function.*

$$a(t) = v'(t) = \frac{dv}{dt}$$

Since the *velocity* is the derivative of the position function $s = f(t)$, it follows that *the acceleration is the second derivative of the position function:*

So

$$v(t) = s'(t) = \frac{ds}{dt}$$

$$a(t) = v'(t) = s''(t)$$

In Leibniz Notation,

$$a = \frac{d^2s}{dt^2}$$

Acceleration

The position function of a particle is $s = t^3 + 2t^2 + 2t$ given by where s is measured in meters and t in seconds

- a) Find the velocity and acceleration as a function of time.
b) Find the acceleration at 3 s

$$\begin{aligned} \text{a) } s &= t^3 + 2t^2 + 2t \\ v &= 3t^2 + 4t + 2 \\ a &= 6t + 4 \end{aligned}$$

$$\begin{aligned} \text{b) } t &= 3\text{ s} \\ a &= 6(3) + 4 \\ &= 22\text{ m/s}^2 \end{aligned}$$

If a ball is thrown upward with an initial velocity of 24.5 m/s, then its distance above the ground in meters after t seconds is

$$s = 24.5 - 4.9t^2$$

a) Find the acceleration of the ball.

$$s = 24.5 - 4.9t^2$$

$$v = -9.8t$$

$$a = -9.8 \quad \leftarrow \text{Constant}$$

Notice that the acceleration in *Example 2* is a constant, and is called the *acceleration due to gravity*. The fact that it is negative means that **the ball slows down as it rises and speeds up as it falls.**

- In general, a negative acceleration indicates that the velocity is decreasing
- Likewise, a positive acceleration means that the velocity is increasing

Homework

Omit 7 d
8 c, d