Warm-Up

In how many ways can a teacher seat four girls and three boys in a row of seven seats if a boy must be seated at each end of the row?

3	×	5	×	4	×	3	×	2	×	1	×	2
(Seat 1)		(Seat 2)		(Seat 3)		(Seat 4)		(Seat 5)		(Seat 6)		(Seat 7)

= 720 possible ways to seat the students

Permutations

Focus on...

- solving counting problems using the fundamental counting principle
- determining, using a variety of strategies, the number of permutations of n elements taken r at a time
- solving counting problems when two or more elements are identical
- solving an equation that involves ${}_nP_r$ notation

How safe is your password? It has been suggested that a four-character letters-only password can be hacked in under 10 s. However, an eight-character password with at least one number could take up to 7 years to crack. Why is there such a big difference?

The arrangement of objects or people in a line is called a linear **permutation**. In a permutation, the order of the objects is important. When the objects are distinguishable from one another, a new order of objects creates a new permutation.

Seven different objects can be arranged in 7! ways.

$7! = (7)(6)(5)(4)(3)(2)(1)$

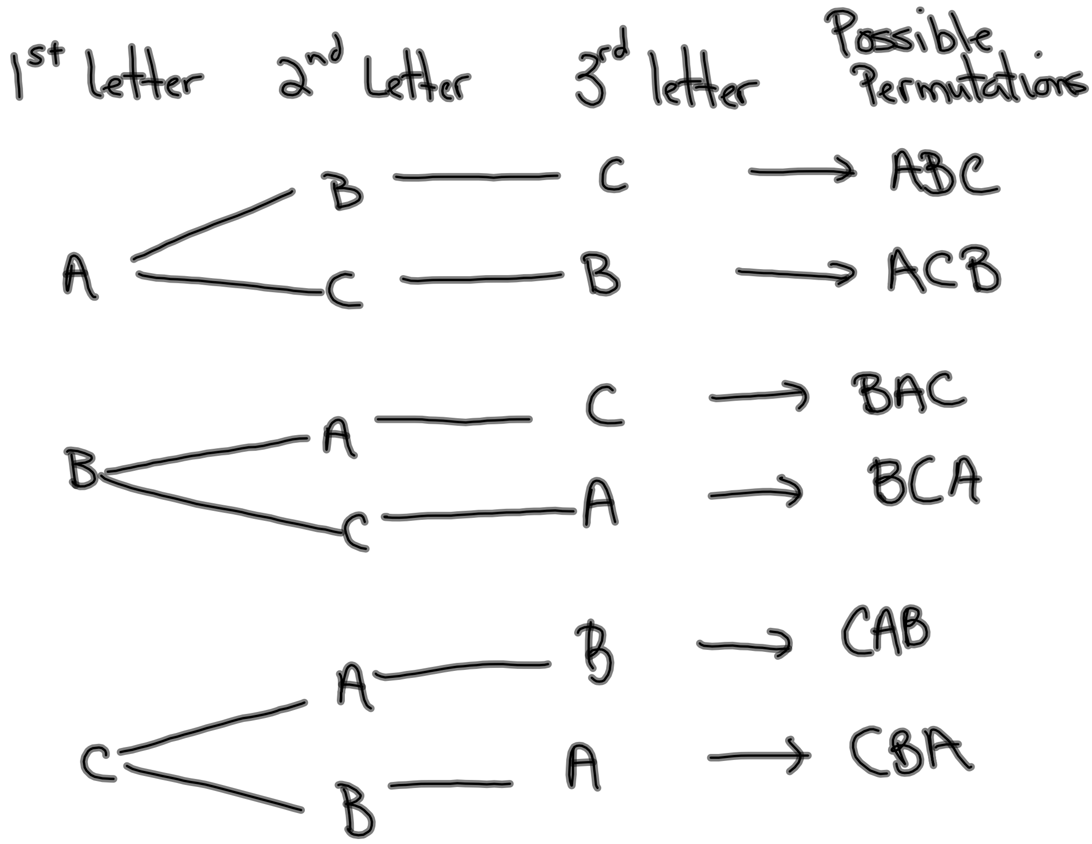
Explain why 7! is equivalent to 7(6!) or to 7(6)(5)(4!).

permutation

- an ordered arrangement or sequence of all or part of a set
- for example, the possible permutations of the letters A, B, and C are ABC, ACB, BAC, BCA, CAB, and CBA

Key Words for permutations:

- in order, in a line, in a row.
- arranged.
- 1st, 2nd, 3rd or Gold, Silver, Bronze



Example

The notation ${}_nP_r$ is used to represent the number of permutations, or arrangements in a definite order, of r items taken from a set of n distinct items. A formula for ${}_nP_r$ is ${}_nP_r = \frac{n!}{(n-r)!}$, $n \in \mathrm{N}$.

If there are seven members on the student council, in how many ways can the council select three students to be the chair, the secretary, and the treasurer of the council?

Using the fundamental counting principle, there are (7)(6)(5) possible ways to fill the three positions. Using the factorial notation,

$$\begin{aligned}\frac{7!}{4!} &= \frac{(7)(6)(5)\overset{1}{\cancel{(4)}}\overset{1}{\cancel{(3)}}\overset{1}{\cancel{(2)}}\overset{1}{\cancel{(1)}}}{\underset{1}{\cancel{(4)}}\underset{1}{\cancel{(3)}}\underset{1}{\cancel{(2)}}\underset{1}{\cancel{(1)}}}\\ &= (7)(6)(5)\\ &= 210\end{aligned}$$

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$n = 7$ students $\quad r = 3$

Using permutation notation, ${}_7P_3$ represents the number of arrangements of three objects taken from a set of seven objects.

$$\begin{aligned} {}_7P_3 &= \frac{7!}{(7-3)!} \\ &= \frac{7!}{4!} \\ &= 210 \end{aligned}$$

So, there are ways that the 3 positions can be filled from the 7-member council.

Did You Know?

The notation $n!$ was introduced in 1808 by Christian Kramp (1760–1826) as a convenience to the printer. Until then, $\underline{n|}$ had been used.

Example 2

Using Factorial Notation

a) Evaluate ${}_9P_4$ using factorial notation.
b) Show that $100! + 99! = 101(99!)$ without using technology.
c) Solve for n if ${}_nP_3 = 60$, where n is a natural number.

a) $${}_nP_r = \frac{n!}{(n-r)!}$$

$${}_9P_4 = \frac{9!}{(9-4)!} = \frac{9!}{5!} = 9\times8\times7\times6 = 3024$$

b) $100! + 99! = 101(99!)$

group like terms $100(99!) + 99!$

$101(99!)$

In general, a **permutation** is an ***arrangement*** of objects in different orders, where the **order** of the arrangement **is important**!!!

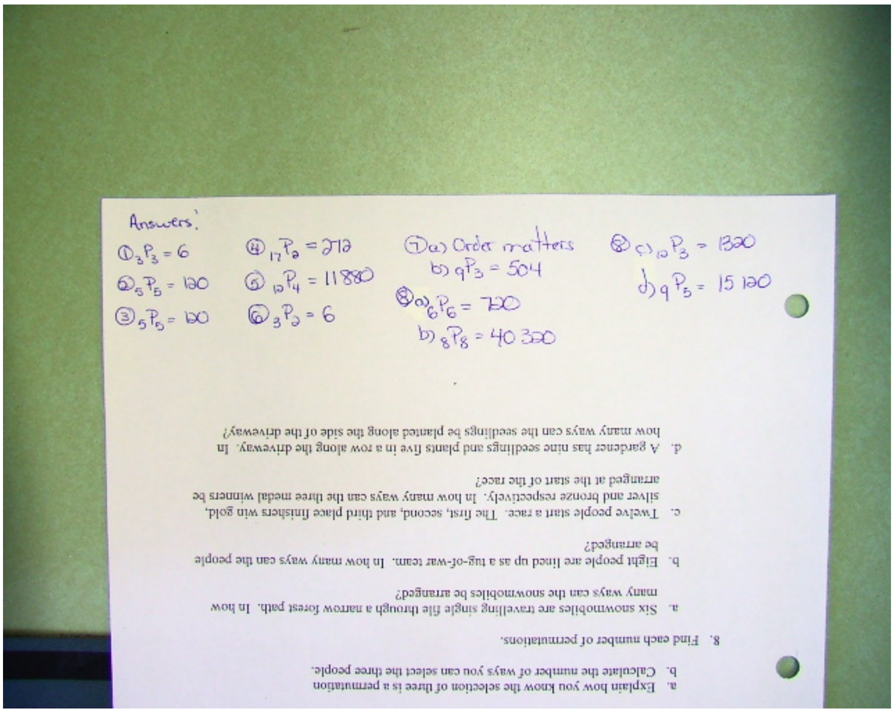
If "**n**" is the size of the sample space, and "**r**" is the number of items chosen on each trial, then the total number of **permutations** is written as:

$$_{\mathbf{n}}\mathbf{P}_{\mathbf{r}} \text{ and is calculated as } {}_{\mathbf{n}}\mathbf{P}_{\mathbf{r}} = \frac{n!}{(n-r)!}$$

Homework

Finish Permutations worksheet

Answers to Homework



Key Ideas

- The fundamental counting principle can be used to determine the number of different arrangements. If one task can be performed in a ways, a second task in b ways, and a third task in c ways, then all three tasks can be arranged in $a \times b \times c$ ways.
- Factorial notation is an abbreviation for products of successive positive integers.

  $5! = (5)(4)(3)(2)(1)$
  $(n + 1)! = (n + 1)(n)(n - 1)(n - 2)\cdots(3)(2)(1)$
- A permutation is an arrangement of objects in a definite order. The number of permutations of n different objects taken r at a time is given by ${}_nP_r = \frac{n!}{(n-r)!}$.
- A set of n objects containing a identical objects of one kind, b identical objects of another kind, and so on, can be arranged in $\frac{n!}{a!b!\ldots}$ ways.
- Some problems have more than one case. One way to solve such problems is to establish cases that together cover all of the possibilities. Calculate the number of arrangements for each case and then add the values for all cases to obtain the total number of arrangements.