

Warm Up

If the length of a square is growing at a rate of 2cm/sec, what would the original length have to be if the *area* of the square is increasing at a rate of 12cm²/sec?

$$\frac{dl}{dt} = 2\text{cm/sec}$$

$$l = ?$$

$$\frac{dA}{dt} = 12\text{cm}^2/\text{sec}$$

$$A = l^2$$

$$\frac{dA}{dt} = 2l \frac{dl}{dt}$$

$$12 = 2l(2)$$

$$12 = 4l$$

$$3\text{cm} = l$$

The original length would be 3cm.

Questions From Homework

Surface Area + Volumes

$$\textcircled{4} \frac{dA}{dt} = 150 \text{ cm}^2/\text{sec}$$

$$\frac{dr}{dt} = ?$$

$$A = 1256 \text{ cm}^2$$

find r

$$A = 4\pi r^2$$

$$1256 = 4\pi r^2$$

$$100 = r^2$$

$$\pm 10 = r$$

$$A = 4\pi r^2$$

$$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$$

$$150 = 8\pi(10) \frac{dr}{dt}$$

$$150 = 80\pi \frac{dr}{dt}$$

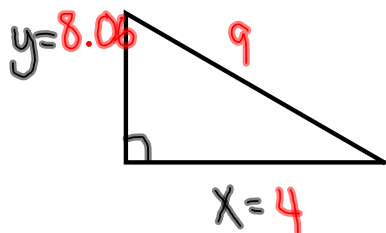
$$\frac{150}{80\pi} = \frac{dr}{dt}$$

$$0.597 \text{ cm/sec} = \frac{dr}{dt}$$

Related Rates (Lampposts and Ladders)

A ladder 9m long is set against a wall and begins to slide down. The top of the ladder slides down at a rate of 0.5m/s. How quickly is the bottom sliding away from the wall when it is 4m from the wall to begin with?

(Hint: draw a diagram)



Given:

$$\frac{dy}{dt} = -0.5 \text{ m/s}$$

$$\frac{dx}{dt} = ?$$

Find y:

$$\begin{aligned} x^2 + y^2 &= z^2 \\ (4)^2 + y^2 &= (9)^2 \\ 16 + y^2 &= 81 \\ y^2 &= 65 \\ y &= \underline{\underline{8.06}} \end{aligned}$$

$$x^2 + y^2 = 9^2$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

$$2(4) \frac{dx}{dt} + 2(8.06)(-0.5) = 0$$

$$8 \frac{dx}{dt} - 8.06 = 0$$

$$8 \frac{dx}{dt} = 8.06$$

$$\frac{dx}{dt} = 1.0075 \text{ m/s}$$

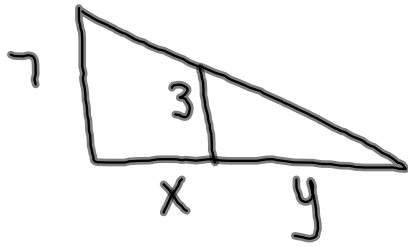
The bottom of the ladder is sliding away from the wall at a rate of 1.01m/s.

Bigfoot is 3m tall and walks curiously towards a lamppost that is 7m tall. If he walks at a rate of 2m/s, at what rate is the length of his shadow changing?

draw a diagram

Let x = distance between Bigfoot and lamppost

Let y = length of shadow



Given:

$$\frac{dx}{dt} = -2 \text{ m/s}$$

$$\frac{dy}{dt} = ?$$

$$\frac{7}{x+y} = \frac{3}{y}$$

$$7y = 3x + 3y$$

$$4y = 3x$$

$$4 \frac{dy}{dt} = 3 \frac{dx}{dt}$$

$$4 \frac{dy}{dt} = 3(-2)$$

$$4 \frac{dy}{dt} = -6$$

$$\frac{dy}{dt} = -1.5 \text{ m/s}$$

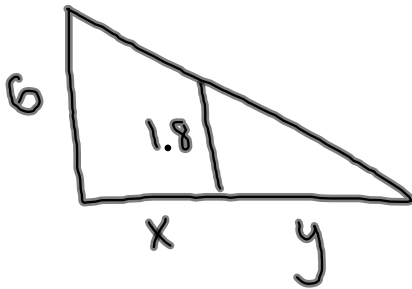
The length of his shadow is changing at a rate -1.5 m/s .

A man is 1.8m tall and walks away from a 6m lamppost at a rate of 2m/s.
How fast is his shadow changing when he is 5m from the post?

draw a diagram

Let x = distance between man and lamppost

Let y = length of shadow



Given:

$$\frac{dx}{dt} = 2 \text{ m/s}$$

$$\frac{dy}{dt} = ?$$

$$\frac{6}{x+y} = \frac{1.8}{y}$$

$$6y = 1.8x + 1.8y$$

$$4.2y = 1.8x$$

$$4.2 \frac{dy}{dt} = 1.8 \frac{dx}{dt}$$

$$4.2 \frac{dy}{dt} = 1.8(2)$$

$$4.2 \frac{dy}{dt} = 3.6$$

$$\frac{dy}{dt} = 0.857 \text{ m/s}$$

His shadow is increasing at a rate of 0.857 m/s.

Homework