Radical Functions and Transformations

Focus on...

- investigating the function $y = \sqrt{x}$ using a table of values and a graph
- graphing radical functions using transformations
- identifying the domain and range of radical functions

radical function

- a function that involves a radical with a variable in the radicand
- $y = \sqrt{3x}$ and $y = 4\sqrt[3]{5 + x}$ are radical functions.

Example 1

Graph Radical Functions Using Tables of Values

$y = a\sqrt{b(x-h)} + k$

Use a table of values to sketch the graph of each function. Then, state the domain and range of each function.

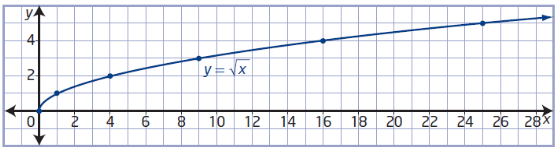
a) $y = \sqrt{x}$ **b)** $y = \sqrt{x - 2}$ **c)** $y = \sqrt{x} - 3$

(base function)

a) For the function $y = \sqrt{x}$, the radicand x must be greater than or equal to zero, $x \geq 0$. (cannot take the square root of a negative)

x	y
0	0
1	1
4	2
9	3
16	4
25	5

How can you choose values of x that allow you to complete the table without using a calculator? → Use perfect squares



The graph has an endpoint at (0, 0) and continues up and to the right. The domain is $\{x \mid x \geq 0, x \in \mathrm{R}\}$. The range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.

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b) For the function $y = \sqrt{x - 2}$, the value of the radicand must be greater than or equal to zero.

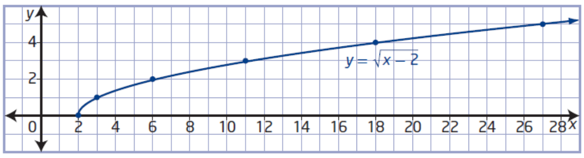
$x - 2 \geq 0$
$x \geq 2$

$h = 2$
Translated 2 units to the right

x	y
2	0
3	1
6	2
11	3
18	4
27	5

How is this table related to the table for $y = \sqrt{x}$ in part a)?

How does the graph of $y = \sqrt{x - 2}$ compare to the graph of $y = \sqrt{x}$?



The domain is $\{x \mid x \geq 2, x \in \mathrm{R}\}$. The range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.

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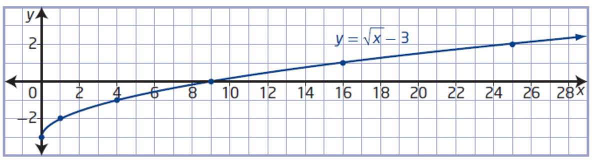
c) The radicand of $y = \sqrt{x} - 3$ must be non-negative.
$x \geq 0$

$k = -3$
Translated 3 units down

x	y
0	−3
1	−2
4	−1
9	0
16	1
25	2

How does the graph of $y = \sqrt{x} - 3$ compare to the graph of $y = \sqrt{x}$?



The domain is $\{x \mid x \geq 0, x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq -3, y \in \mathrm{R}\}$.

Graphing Radical Functions Using Transformations

You can graph a radical function of the form $y = a\sqrt{b(x - h)} + k$ by transforming the graph of $y = \sqrt{x}$ based on the values of a, b, h, and k. The effects of changing parameters in radical functions are the same as the effects of changing parameters in other types of functions.

- Parameter a results in a vertical stretch of the graph of $y = \sqrt{x}$ by a factor of $|a|$. If $a < 0$, the graph of $y = \sqrt{x}$ is reflected in the x-axis.
- Parameter b results in a horizontal stretch of the graph of $y = \sqrt{x}$ by a factor of $\frac{1}{|b|}$. If $b < 0$, the graph of $y = \sqrt{x}$ is reflected in the y-axis.
- Parameter h determines the horizontal translation. If $h > 0$, the graph of $y = \sqrt{x}$ is translated to the right h units. If $h < 0$, the graph is translated to the left $|h|$ units.
- Parameter k determines the vertical translation. If $k > 0$, the graph of $y = \sqrt{x}$ is translated up k units. If $k < 0$, the graph is translated down $|k|$ units.

Example 2

Graph Radical Functions Using Transformations

$y = a\sqrt{b(x-h)} + k$

Sketch the graph of each function using transformations. Compare the domain and range to those of $y = \sqrt{x}$ and identify any changes.

a) $y = 3\sqrt{-(x - 1)}$

b) $y - 3 = -\sqrt{2x}$

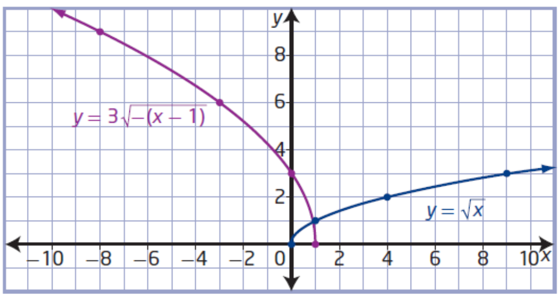
a) $y = 3\sqrt{-(x-1)}$

$a=3$ $b=-1$ $h=1$ $k=0$

x	y
0	0
1	1
4	2
9	3
16	4
25	5

$(x,y) \longrightarrow [-1x+1,\ 3y+0]$

x	y
1	0
0	3
-3	6
-8	9
-15	12
-24	15



Domain:

$\{x \mid x \leq 1, x \in R\}$

$-(x-1) \geq 0$

$-x+1 \geq 0$

$-x \geq -1$

$x \leq 1$

Range:

$\{y \mid y \geq 0, y \in R\}$

b) $y - 3 = -\sqrt{2x}$

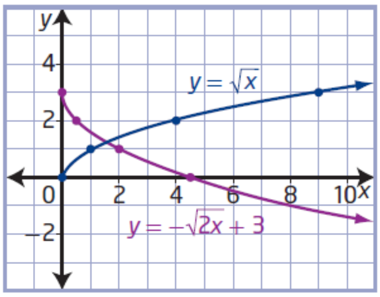
$y = -\sqrt{2x} + 3$

$a = -1 \quad b = 2 \quad h = 0 \quad k = 3$

x	y
0	0
1	1
4	2
9	3
16	4
25	5

$(x,y) \longrightarrow \left[\frac{x}{2} + 0, -1y + 3\right]$

x	y
0	3
0.5	2
2	1
4.5	0
8	-1
12.5	-2



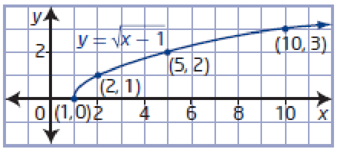
Domain: $\{x \mid x \geq 0, x \in R\}$

Range: $\{y \mid y \leq 3, y \in R\}$

#2-5 page 72

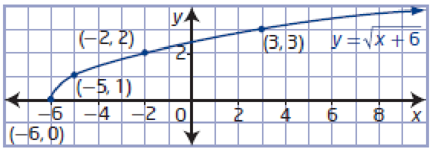
2.1 Radical Functions and Transformations, pages 72 to 77

1. a)



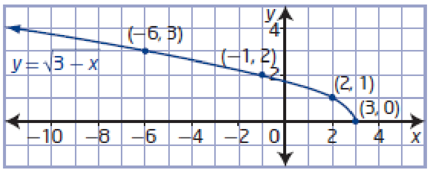
domain $\{x \mid x \geq 1, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

b)



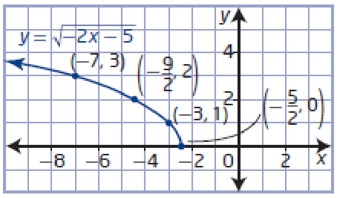
domain $\{x \mid x \geq -6, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

c)



domain $\{x \mid x \leq 3, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

d)



domain $\left\{x \mid x \leq -\frac{5}{2}, x \in \mathrm{R}\right\}$,
range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

2. a) $a = 7 \rightarrow$ vertical stretch by a factor of 7
$h = 9 \rightarrow$ horizontal translation 9 units right
domain $\{x \mid x \geq 9, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

b) $b = -1 \rightarrow$ reflected in y-axis
$k = 8 \rightarrow$ vertical translation up 8 units
domain $\{x \mid x \leq 0, x \in \mathrm{R}\}$, range $\{y \mid y \geq 8, y \in \mathrm{R}\}$

c) $a = -1 \rightarrow$ reflected in x-axis
$b = \frac{1}{5} \rightarrow$ horizontal stretch factor of 5
domain $\{x \mid x \geq 0, x \in \mathrm{R}\}$, range $\{y \mid y \leq 0, y \in \mathrm{R}\}$

d) $a = \frac{1}{3} \rightarrow$ vertical stretch factor of $\frac{1}{3}$
$h = -6 \rightarrow$ horizontal translation 6 units left
$k = -4 \rightarrow$ vertical translation 4 units down
domain $\{x \mid x \geq -6, x \in \mathrm{R}\}$,
range $\{y \mid y \geq -4, y \in \mathrm{R}\}$

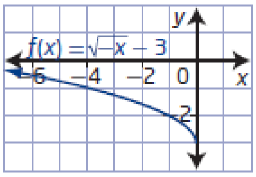
3. a) B **b)** A **c)** D **d)** C

4. a) $y = 4\sqrt{x+6}$ **b)** $y = \sqrt{8x} - 5$

c) $y = \sqrt{-(x-4)} + 11$ or $y = \sqrt{-x+4} + 11$

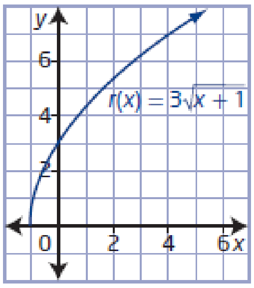
d) $y = -0.25\sqrt{0.1x}$ or $y = -\frac{1}{4}\sqrt{\frac{1}{10}x}$

5. a)



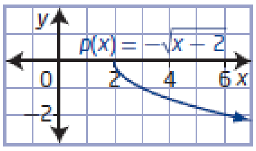
domain
$\{x \mid x \leq 0, x \in \mathbb{R}\}$,
range
$\{y \mid y \geq -3, y \in \mathbb{R}\}$

b)



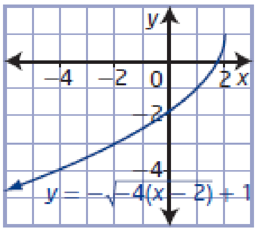
domain
$\{x \mid x \geq -1, x \in \mathbb{R}\}$,
range
$\{y \mid y \geq 0, y \in \mathbb{R}\}$

c)



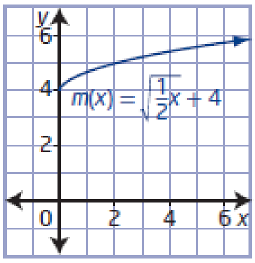
domain
$\{x \mid x \geq 2, x \in \mathbb{R}\}$,
range
$\{y \mid y \leq 0, y \in \mathbb{R}\}$

d)



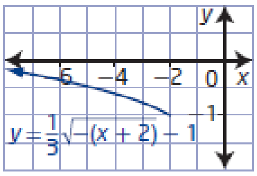
domain
$\{x \mid x \leq 2, x \in \mathbb{R}\}$,
range
$\{y \mid y \leq 1, y \in \mathbb{R}\}$

e)



domain
$\{x \mid x \geq 0, x \in \mathbb{R}\}$,
range
$\{y \mid y \geq 4, y \in \mathbb{R}\}$

f)

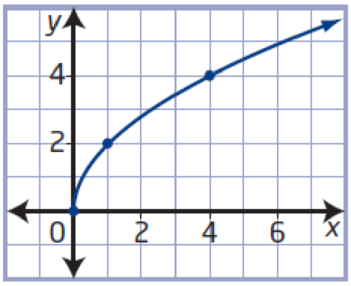


domain
$\{x \mid x \leq -2, x \in \mathbb{R}\}$
range
$\{y \mid y \geq -1, y \in \mathbb{R}\}$

Example 3

Determine a Radical Function From a Graph

Mayleen is designing a symmetrical pattern. She sketches the curve shown and wants to determine its equation and the equation of its reflection in each quadrant. The graph is a transformation of the graph of $y = \sqrt{x}$. What are the equations of the four functions Mayleen needs to work with?



A radical function that involves a stretch can be obtained from either a vertical stretch or a horizontal stretch. Use an equation of the form $y = a\sqrt{x}$ or $y = \sqrt{bx}$ to represent the image function for each type of stretch.

vertical stretch

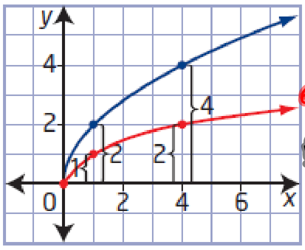
horizontal

Method 1: Compare Vertical or Horizontal Distances

Superimpose the graph of $y = \sqrt{x}$ and compare corresponding distances to determine the factor by which the function has been stretched.

View as a Vertical Stretch ($y = a\sqrt{x}$)

Each vertical distance is 2 times the corresponding distance for $y = \sqrt{x}$.



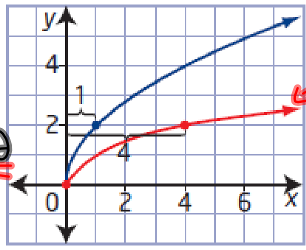
$y=\sqrt{x}$

$\frac{\text{new}}{\text{old}} = \frac{4}{2} = 2$

This represents a vertical stretch by a factor of 2, which means $a = 2$. The equation $y = 2\sqrt{x}$ represents the function.

View as a Horizontal Stretch ($y = \sqrt{bx}$)

Each horizontal distance is $\frac{1}{4}$ the corresponding distance for $y = \sqrt{x}$.



$y=\sqrt{x}$

$\frac{\text{new}}{\text{old}} = \frac{1}{4} \rightarrow b=4$

This represents a horizontal stretch by a factor of $\frac{1}{4}$, which means $b = 4$. The equation $y = \sqrt{4x}$ represents the function.

Express the equation of the function as either $y = 2\sqrt{x}$ or $y = \sqrt{4x}$.

Quad 1:	Quad 2:	Quad 3:	Quad 4:
$y = 2\sqrt{x}$	$y = 2\sqrt{-x}$	$y = -2\sqrt{-x}$	$y = -2\sqrt{x}$
$y = \sqrt{4x}$	$y = \sqrt{-4x}$	$y = -\sqrt{-4x}$	$y = -\sqrt{4x}$

Example 4

Model the Speed of Sound

Justin's physics textbook states that the speed, s, in metres per second, of sound in dry air is related to the air temperature, T, in degrees Celsius, by the function $s = 331.3\sqrt{1 + \frac{T}{273.15}}$.

a) Determine the domain and range in this context.

b) On the Internet, Justin finds another formula for the speed of sound, $s = 20\sqrt{T + 273}$. Use algebra to show that the two functions are approximately equivalent.

c) How is the graph of this function related to the graph of the base square root function? Which transformation do you predict will be the most noticeable on a graph?

d) Graph the function $s = 331.3\sqrt{1 + \frac{T}{273.15}}$ using technology.

e) Determine the speed of sound, to the nearest metre per second, at each of the following temperatures.

  i) 20 °C (normal room temperature)
  ii) 0 °C (freezing point of water)
  iii) −63 °C (coldest temperature ever recorded in Canada)
  iv) −89 °C (coldest temperature ever recorded on Earth)

a) Domain: $1 + \frac{T}{273.15} \geq 0$

$\frac{T}{273.15} \geq -1$

$T \geq -273.15°$

$\{T \mid T \geq -273.15°, T \in R\}$

$T \in [-273.15, \infty)$

Range: $\{s \mid s \geq 0, s \in R\}$

$s \in [0, \infty)$

b) $s = 331.3\sqrt{1 + \frac{T}{273.15}}$

$s = 331.3\sqrt{\frac{273.15 + T}{273.15}}$

$s = 331.3\,\frac{\sqrt{273.15 + T}}{\sqrt{273.15}}$

$s = 331.3\,\frac{\sqrt{273.15 + T}}{16.53}$

$s = 20.04\sqrt{273.15 + T} \approx 20\sqrt{T + 273}$

c) $s = 20\sqrt{T + 273}$

$a = 20$ (Vertical Stretch of 4)

$b = 1$ (No Horizontal Stretch)

* $h = -273$ (Left 273)

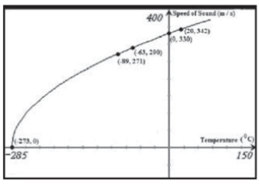
$k = 0$ (No VT)

d) $y = \sqrt{x}$ $(x, y) \rightarrow [x - 273, 20y]$

x	y
0	0
1	1
4	2
9	3

$y = 20\sqrt{T + 273}$

x	y
-273	0
-272	20
-269	40
-264	60



Are your answers to part c) confirmed by the graph?

e)

	Temperature (°C)	Approximate Speed of Sound (m/s)
i)	20	343
ii)	0	331
iii)	−63	291
iv)	−89	272

$\rightarrow s = 20\sqrt{T + 273}$

$s = 20\sqrt{20 + 273}$

$s = 20(17.1)$

$s = 342.3$ m/s

Homework
#6-12

6. a) $a = \frac{1}{4} \rightarrow$ vertical stretch factor of $\frac{1}{4}$

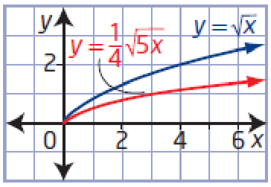
$b = 5 \rightarrow$ horizontal stretch factor of $\frac{1}{5}$

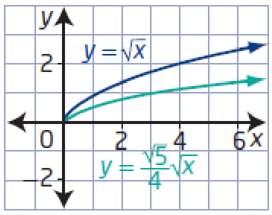
b) $y = \frac{\sqrt{5}}{4}\sqrt{x}$, $y = \sqrt{\frac{5}{16}x}$

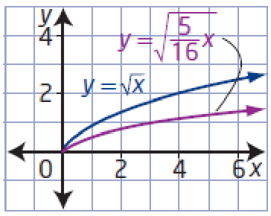
c) $a = \frac{\sqrt{5}}{4} \rightarrow$ vertical stretch factor of $\frac{\sqrt{5}}{4}$

$b = \frac{5}{16} \rightarrow$ horizontal stretch factor of $\frac{16}{5}$

d)





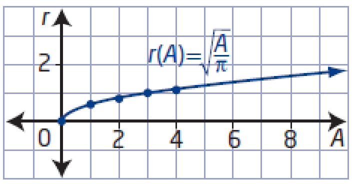


All graphs are the same.

7. a) $r(A) = \sqrt{\frac{A}{\pi}}$

b)

A	r
0	0
1	0.6
2	0.8
3	1.0
4	1.1



8. a) $b = 1.50 \rightarrow$ horizontal stretch factor of $\frac{1}{1.50}$ or $\frac{2}{3}$

b) $d \approx 1.22\sqrt{h}$ Example: I prefer the original function because the values are exact.

c) approximately 5.5 miles

9. a) domain $\{x \mid x \geq 0, x \in R\}$, range $\{y \mid y \geq -13, y \in R\}$

b) $h = 0 \rightarrow$ no horizontal translation

$k = 13 \rightarrow$ vertical translation down 13 units

10. a) $y = -\sqrt{x + 3} + 4$ **b)** $y = \frac{1}{2}\sqrt{x + 5} - 3$

c) $y = 2\sqrt{-(x - 5)} - 1$ or $y = 2\sqrt{-x + 5} - 1$

d) $y = -4\sqrt{-(x - 4)} + 5$ or $y = -4\sqrt{-x + 4} + 5$

11. Examples:

a) $y - 1 = \sqrt{x - 6}$ or $y = \sqrt{x - 6} + 1$

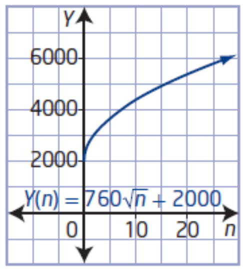
b) $y = -\sqrt{x + 7} - 9$ **c)** $y = 2\sqrt{-x + 4} - 3$

d) $y = -\sqrt{-(x + 5)} + 8$

12. a) $a = 760 \rightarrow$ vertical stretch factor of 760

$k = 2000 \rightarrow$ vertical translation up 2000

b)



c) domain

$\{n \mid n \geq 0, n \in R\}$

range

$\{Y \mid Y \geq 2000, Y \in R\}$

d) The minimum yield is 2000 kg/hectare. Example: The domain and range imply that the more nitrogen added, the greater the yield without end. This is not realistic.

① $$\frac{A_{sector}}{A_{circle}} = \frac{\theta}{2\pi}$$

② $$A_{triangle} = \frac{1}{2} r^2 \sin \theta$$