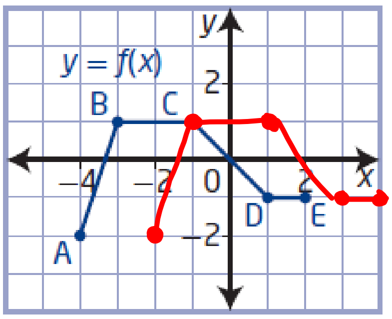
Warm-Up

8. Copy and complete the table.

Translation	Transformed Function	Transformation of Points	
vertical	$y = f(x) + 5$	$(x, y) \rightarrow (x, y + 5)$	
H	$y = f(x + \underline{7})$	$(x, y) \rightarrow (x - 7, y)$	$h = -7$
H	$y = f(x - 3)$	$(x,y) \rightarrow (x+3, y)$	$h = 3$
V	$y = f(x) - 6$	$(x,y) \rightarrow (x, y-6)$	$K = -6$
horizontal and vertical	$y + 9 = f(x + 4)$	$(x,y) \rightarrow (x-4, y-9)$	$h = -4$ $K = -9$
horizontal and vertical	$y = f(x-4) - 6$	$(x, y) \rightarrow (x + 4, y - 6)$	$h = 4$ $K = -6$
H+V	$y = f(x+2) + 3$	$(x, y) \rightarrow (x - 2, y + 3)$	$h = -2$ $K = 3$
horizontal and vertical	$y = f(x - h) + k$	$(x,y) \rightarrow (x+h, y+K)$	

Questions from Homework

②



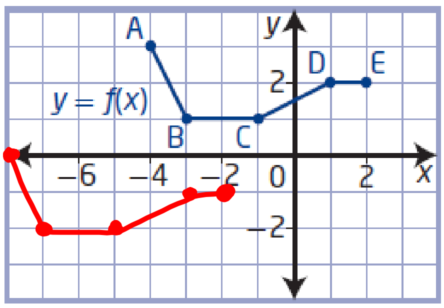
② b) $h(x) = f(x-2)$ $h=2$

$(x,y) \longrightarrow (x+2, y)$

A	(-4,-2)	A'	(-2,-2)
B	(-3,1)	B'	(-1,1)
C	(-1,1)	C'	(1,1)
D	(1,-1)	D'	(3,-1)
E	(2,-1)	E'	(4,-1)

$h=-4$ $k=-3$

④



a) $s(x) = f(x+4) - 3$

$(x,y) \longrightarrow (x-4, y-3)$

A	(-4,3)	A'	(-8,0)
B	(-3,1)	B'	(-7,-2)
C	(-1,1)	C'	(-5,-2)
D	(1,2)	D'	(-3,-1)
E	(2,2)	E'	(-2,-1)

Transformations:

New Functions From Old Functions

Translations

Stretches

Reflections

Reflections and Stretches

Focus on...

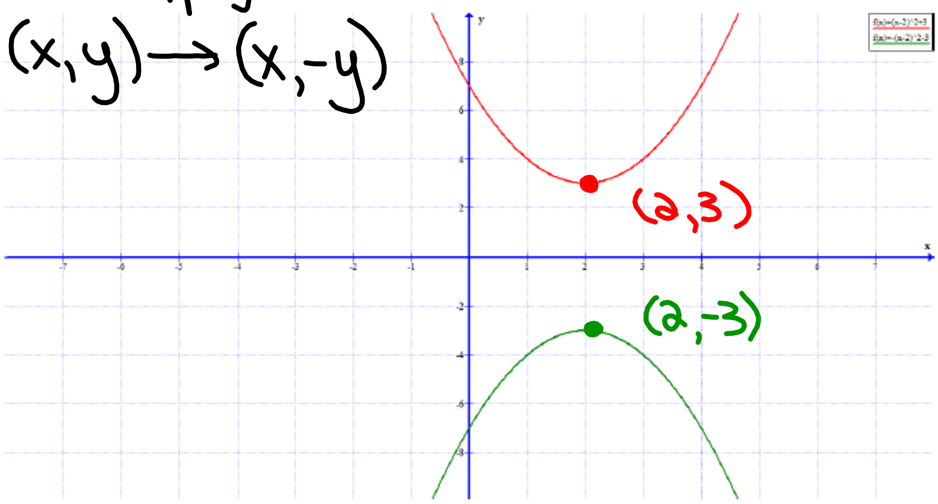
- developing an understanding of the effects of reflections on the graphs of functions and their related equations
- developing an understanding of the effects of vertical and horizontal stretches on the graphs of functions and their related equations

A **reflection** of a graph creates a mirror image in a line called the line of reflection. Reflections, like translations, do not change the shape of the graph. However, unlike translations, reflections may change the orientation of the graph.

- When the output of a function $y = f(x)$ is multiplied by -1, the result, $y = -f(x)$, is a reflection of the graph in the x-axis. (vertical reflection)

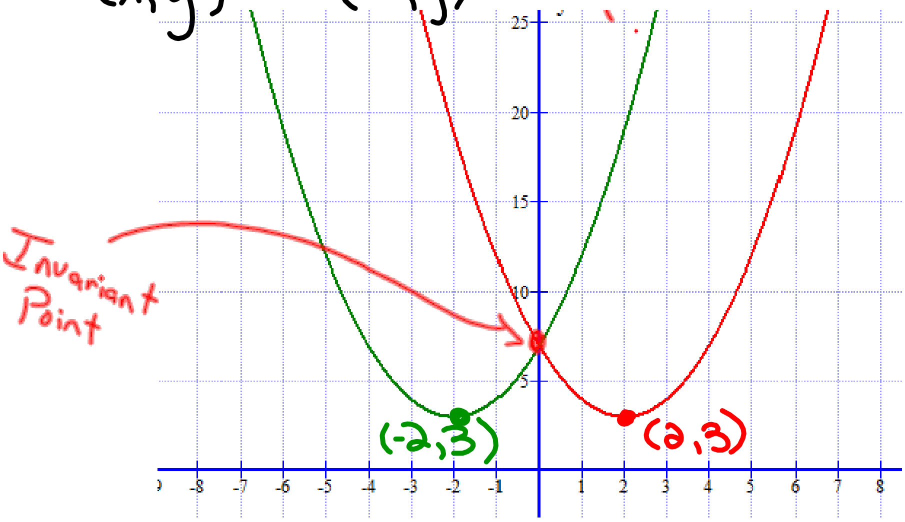
Mapping

$(x, y) \rightarrow (x, -y)$



- When the input of a function $y = f(x)$ is multiplied by -1, the result, $y = f(-x)$, is a reflection of the graph in the y-axis. (horizontal reflection)

Mapping:
$(x,y) \rightarrow (-x,y)$



invariant point

- a point on a graph that remains unchanged after a transformation is applied to it
- any point on a curve that lies on the line of reflection is an invariant point

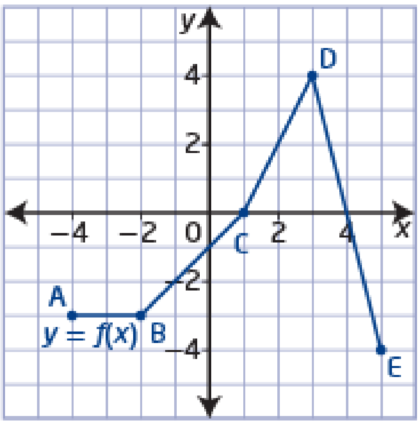
Remember...

- When the output of a function $y = f(x)$ is multiplied by -1, the result, $y = -f(x)$, is a reflection of the graph in the x-axis.

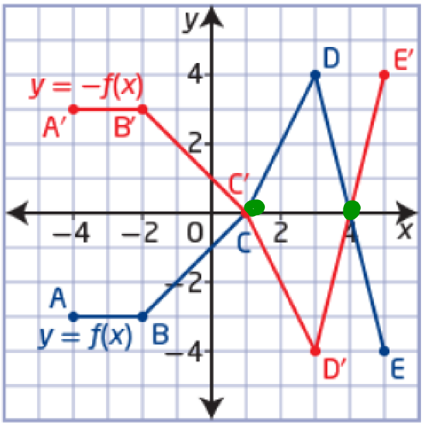
- Sketch $y = -f(x)$ on the axis below

- Invariant Points

$(x,y) \longrightarrow (x,-y)$



A(-4,-3)	A'(-4,3)
B(-2,-3)	B'(-2,3)
C(1,0)	C'(1,0)
D(3,4)	D'(3,-4)
E(5,-4)	E'(5,4)



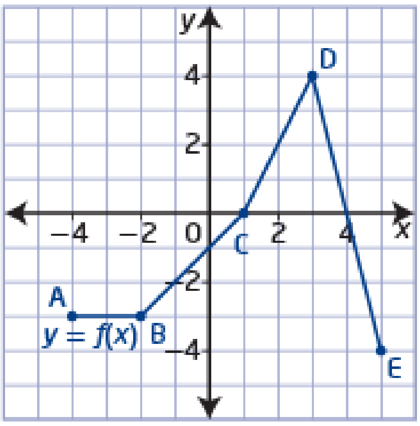
Remember...

- When the input of a function $y = f(x)$ is multiplied by -1, the result, $y = f(-x)$, is a reflection of the graph in the y-axis.

- Sketch $y = f(-x)$ on the axis below

• Invariant Point

$(x, y) \rightarrow (-x, y)$



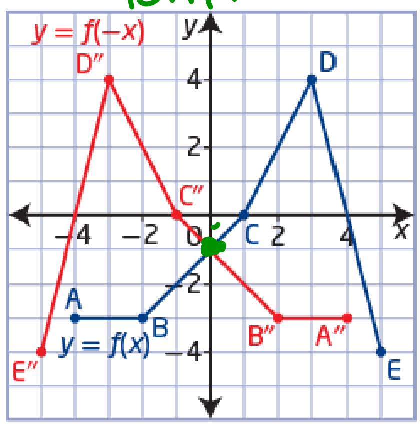
A $(-4,-3)$ A' $(4,-3)$

B $(-2,-3)$ B' $(2,-3)$

C $(1,0)$ C' $(-1,0)$

D $(3,4)$ D' $(-3,4)$

E $(5,-4)$ E' $(-5,-4)$



Questions from Homework

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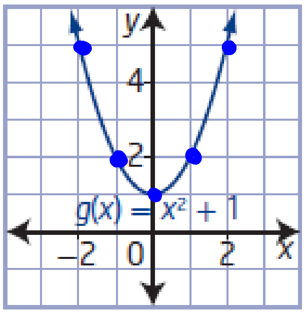
3. Consider each graph of a function.

- Copy the graph of the function and sketch its reflection in the x-axis on the same set of axes.
- State the equation of the reflected function in simplified form.
- State the domain and range of each function.

b)



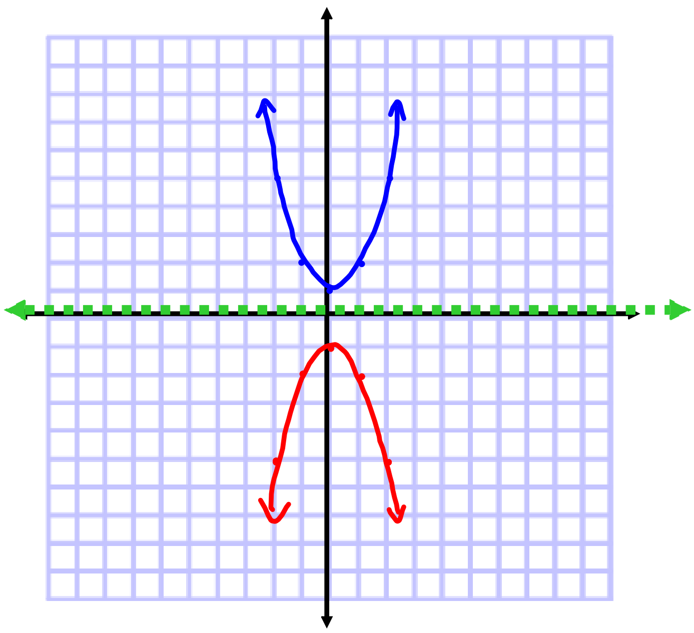
• vertical reflection

(x,y)	$\longrightarrow (x,-y)$
$(-2,5)$	$(-2,-5)$
$(-1,2)$	$(-1,-2)$
$(0,1)$	$(0,-1)$
$(1,2)$	$(1,-2)$
$(2,5)$	$(2,-5)$

$g(x) = x^2+1$

D: $\{x \mid x \in R\}$

R: $\{y \mid y \geq 1, y \in R\}$



$h(x) = -(x^2+1)$

$h(x) = -x^2-1$

D: $\{x \mid x \in R\}$

R: $\{y \mid y \leq -1, y \in R\}$

Vertical and Horizontal Stretches

A **stretch**, unlike a translation or a reflection, changes the shape of the graph. However, like translations, stretches do not change the orientation of the graph.

- When the output of a function $y = f(x)$ is multiplied by a non-zero constant a, the result, $y = af(x)$ or $\frac{y}{a} = f(x)$, is a vertical stretch of the graph about the x-axis by a factor of $|a|$. If $a < 0$, then the graph is also reflected in the x-axis.
- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b, the result, $y = f(bx)$, is a horizontal stretch of the graph about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

stretch

- a transformation in which the distance of each x-coordinate or y-coordinate from the line of reflection is multiplied by some scale factor
- scale factors between 0 and 1 result in the point moving closer to the line of reflection; scale factors greater than 1 result in the point moving farther away from the line of reflection

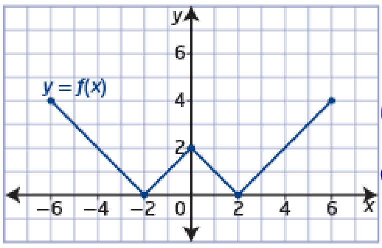
ex: $\frac{1}{2}$, 0.5, $\frac{2}{5}$

Vertical Stretch or Compression...

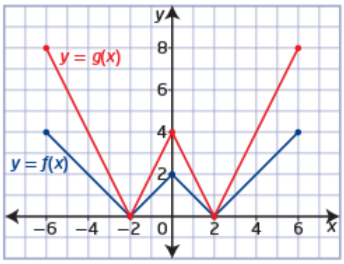
- When the output of a function $y = f(x)$ is multiplied by a non-zero constant a, the result, $y = af(x)$ or $\frac{y}{a} = f(x)$, is a vertical stretch of the graph about the x-axis by a factor of $|a|$. If $a < 0$, then the graph is also reflected in the x-axis.

a) $g(x) = 2f(x)$ $a=2 \rightarrow$ vertical stretch by a factor of 2

$(x, y) \rightarrow (x, 2y)$



(x, y)	$(x, 2y)$
(-6,4)	(-6,8)
(-2,0)	(-2,0) *
(0,2)	(0,4)
(2,0)	(2,0) *
(6,4)	(6,8)

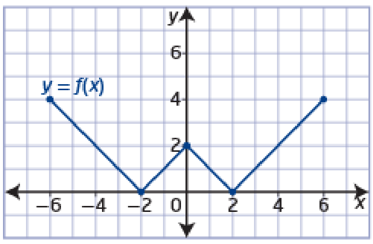


The invariant points are $(-2, 0)$ and $(2, 0)$.

For $f(x)$, the domain is
$\{x \mid -6 \le x \le 6, x \in \mathrm{R}\}$, or $[-6, 6]$,
and the range is
$\{y \mid 0 \le y \le 4, y \in \mathrm{R}\}$, or $[0, 4]$.

For $g(x)$, the domain is $\{x \mid -6 \le x \le 6, x \in \mathrm{R}\}$, or $[-6, 6]$,
and the range is $\{y \mid 0 \le y \le 8, y \in \mathrm{R}\}$, or $[0, 8]$.

b) $g(x) = \frac{1}{2}f(x)$



The invariant points are and

For $f(x)$, the domain is

and the range is

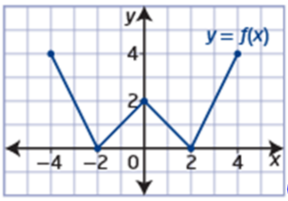
For $g(x)$, the domain is
and the range is

Horizontal Stretch or Compression...

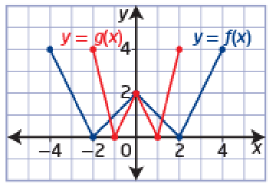
- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b, the result, $y = f(bx)$, is a horizontal stretch of the graph about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

a) $g(x) = f(2x)$ $b=2 \rightarrow$ horizontal stretch by a factor of $\frac{1}{2}$

$(x,y) \rightarrow \left(\frac{1}{2}x, y\right)$



(-4,4)	(-2,4)
(-2,0)	(-1,0)
(0,2)	(0,2) *
(2,0)	(1,0)
(4,4)	(2,4)

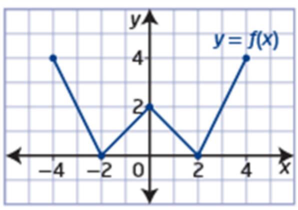


The invariant point is (0, 2).

For $f(x)$, the domain is $\{x \mid -4 \le x \le 4, x \in \mathrm{R}\}$, or $[-4, 4]$, and the range is $\{y \mid 0 \le y \le 4, y \in \mathrm{R}\}$, or $[0, 4]$.

For $g(x)$, the domain is $\{x \mid -2 \le x \le 2, x \in \mathrm{R}\}$, or $[-2, 2]$, and the range is $\{y \mid 0 \le y \le 4, y \in \mathrm{R}\}$, or $[0, 4]$.

b) $g(x) = f\left(\frac{1}{2}x\right)$ $b = \frac{1}{2} \rightarrow$ HSF of 2

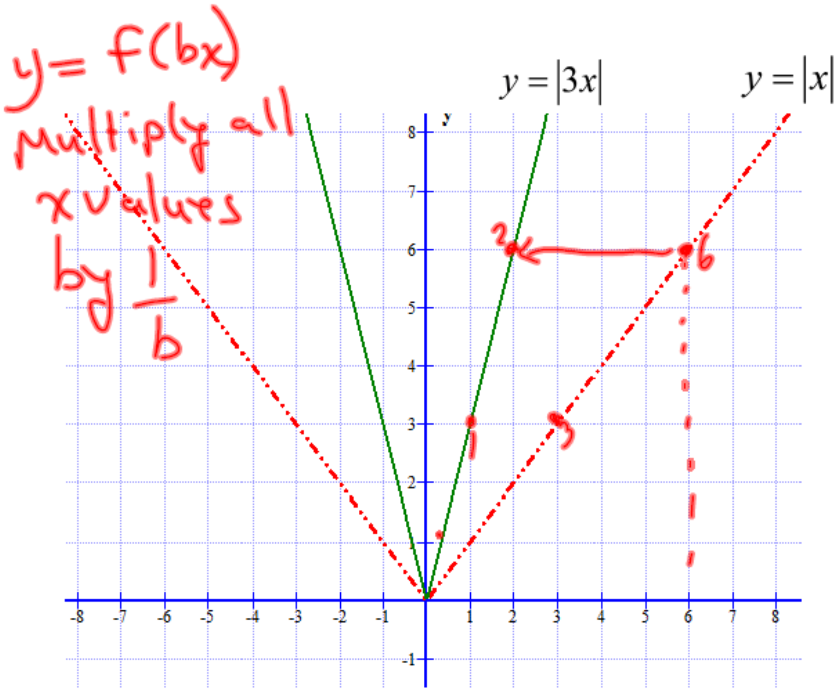


The invariant point is

For $f(x)$, the domain is
and the range is

For $g(x)$, the domain is
and the range is

Horizontal Stretch or Compression...



Horizontal Stretch or Compression...

- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b, the result, $y = f(bx)$, is a horizontal stretch of the graph about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

$y = -3f(-2x) + 7$

Homework

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