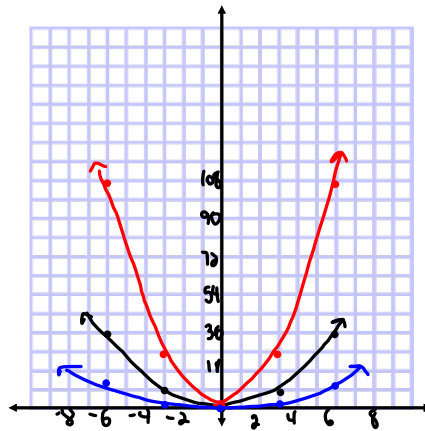


## Questions from Homework

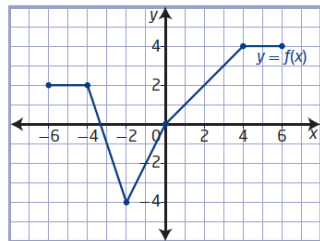
2. a) Copy and complete the table of values for the given functions.

x	$f(x) = x^2$	$g(x) = 3f(x)$	$h(x) = \frac{1}{3}f(x)$
-6	36	108	12
-3	9	27	3
0	0	0	0
3	9	27	3
6	36	108	12

$a=3$        $a=\frac{1}{3}$



6. The graph of the function  $y = f(x)$  is vertically stretched about the x-axis by a factor of 2.



Original  
 $y = f(x)$   
 $(x, y) \rightarrow$

Transformed  
 $g(x) = 2f(x)$   
 $(x, 2y)$

D:  $[-6, 6]$

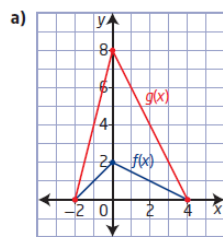
D:  $[-6, 6]$

R:  $[-4, 4]$

R:  $[-8, 8]$

b) Vertical stretch only changes the range.

7. Describe the transformation that must be applied to the graph of  $f(x)$  to obtain the graph of  $g(x)$ . Then, determine the equation of  $g(x)$  in the form  $y = af(bx)$ .



$(x, y) \rightarrow (x, 4y)$        $a=4$

$y = f(x)$

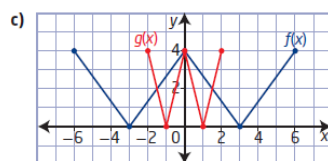
$(-2, 0) \rightarrow (-2, 0)$

$(0, 2) \rightarrow (0, 8)$

$(4, 0) \rightarrow (4, 0)$

Vertically stretched by a factor of 4

$y = 4f(x)$



$(x, y) \rightarrow (\frac{1}{3}x, y)$        $b=3$

$y = f(x)$

$(-6, 4) \rightarrow (-2, 4)$

$(-4, 0) \rightarrow (-1, 0)$

$(0, 4) \rightarrow (0, 4)$

$(3, 0) \rightarrow (1, 0)$

$(6, 4) \rightarrow (2, 4)$

horizontally compressed by a factor of  $\frac{1}{3}$

$y = f(3x)$

## Warm-Up...

Given that  $(-2, 5)$  is a point on the graph of  $y = f(x)$ , determine the coordinates of this point once the following transformations are applied...

$$(1) y = 3f(x) \quad a=3$$

$$(x, y) \rightarrow (x, 3y)$$

$$(-2, 5) \rightarrow (-2, 15)$$

$$(2) y = f\left(-\frac{1}{3}x\right) \quad b = -\frac{1}{3}$$

$$(x, y) \rightarrow (-3x, y)$$

$$(-2, 5) \rightarrow (6, 5)$$

$$(3) y = 4f\left[\frac{1}{2}(x+5)\right] - 3$$

$$a=4 \quad b=\frac{1}{2} \quad h=-5 \quad k=-3$$

$$(x, y) \rightarrow (2x-5, 4y-3)$$

$$(2, 5) \rightarrow (-9, 17)$$

$$(4) y-5 = -2f(-2x+6)$$

$$y = -2f(-2x+6) + 5$$

$$y = -2f[-2(x-3)] + 5$$

$$a=-2 \quad b=-2 \quad h=3 \quad k=5$$

$$(x, y) \rightarrow \left(-\frac{1}{2}x+3, -2y+5\right)$$

$$(-2, 5) \rightarrow (4, -5)$$

## Transformations:

2. The function  $y = f(x)$  is transformed to the function  $g(x) = -3f(4x - 16) - 10$ . Copy and complete the following statements by filling in the blanks.

The function  $f(x)$  is transformed to the function  $g(x)$  by a horizontal stretch about the **a** by a factor of **b**. It is vertically stretched about the **c** by a factor of **d**. It is reflected in the **e**, and then translated **f** units to the right and **g** units down.

## Summary of Transformations...

Transformations of the graphs of functions	
$f(x) + k$	shift $f(x)$ up $k$ units
$f(x) - k$	shift $f(x)$ down $k$ units
$f(x + h)$	shift $f(x)$ left $h$ units
$f(x - h)$	shift $f(x)$ right $h$ units
$f(-x)$	reflect $f(x)$ about the $y$ -axis
$-f(x)$	reflect $f(x)$ about the $x$ -axis
$af(x)$	When $0 < a < 1$ – vertical shrinking of $f(x)$
	When $a > 1$ – vertical stretching of $f(x)$ <b>Multiply the <math>y</math> values by <math>a</math></b>
$f(bx)$	When $0 < b < 1$ – horizontal stretching of $f(x)$
	When $b > 1$ – horizontal shrinking of $f(x)$ <b>Divide the <math>x</math> values by <math>b</math></b>

# Transformations:

$$y = f(x) \longrightarrow y = af(b(x - h)) + k$$

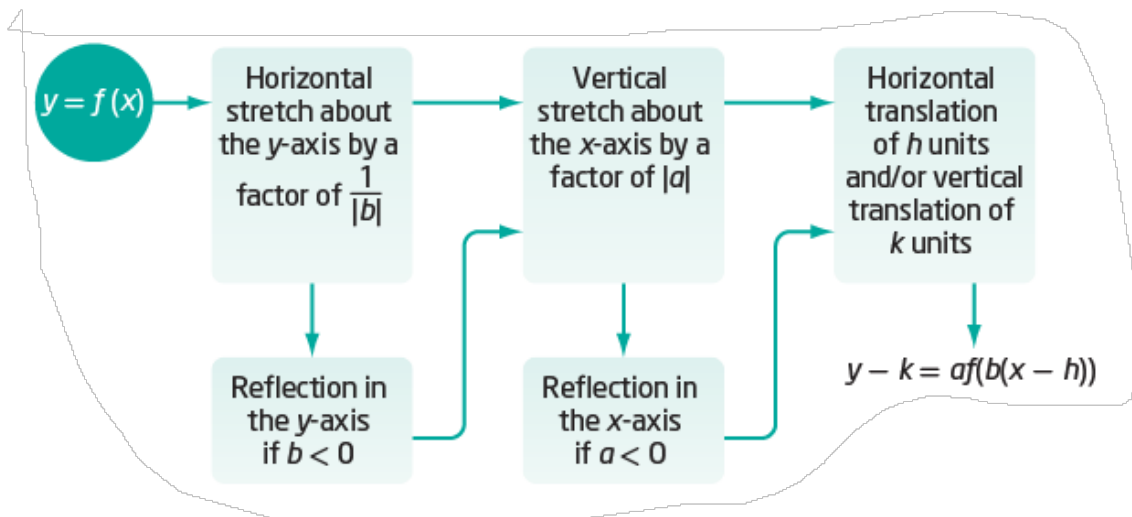
**Mapping Rule:**  $(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$

**Important note for sketching...**

Transformations should be applied in following order:

1. Reflections
2. Stretches
3. Translations

Remember...RST



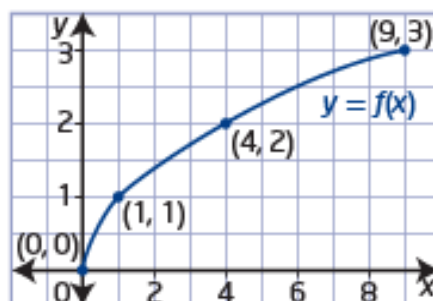
## Example 1

### Graph a Transformed Function

Describe the combination of transformations that must be applied to the function  $y = f(x)$  to obtain the transformed function. Sketch the graph, showing each step of the transformation.

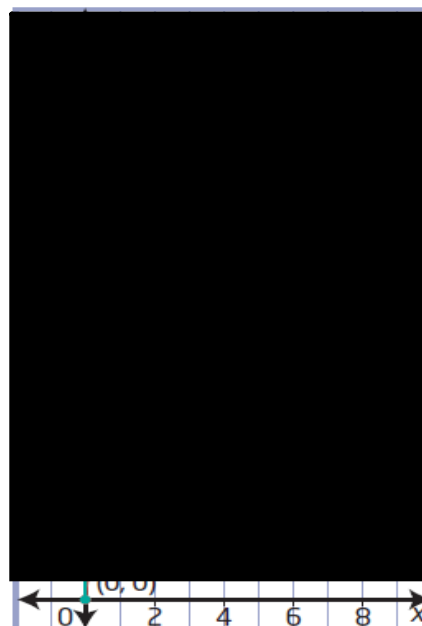
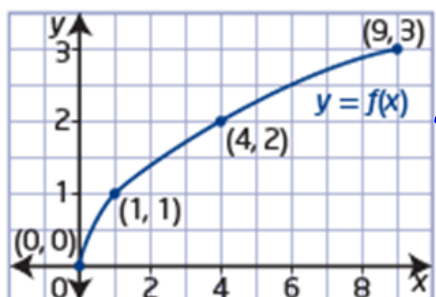
a)  $y = 3f(2x)$

b)  $y = f(3x + 6)$



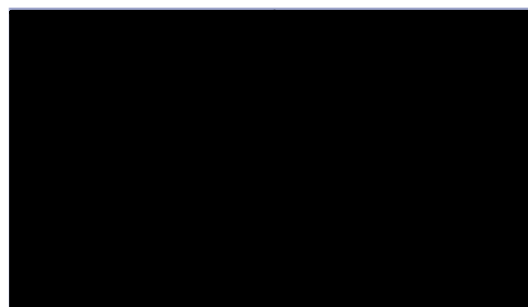
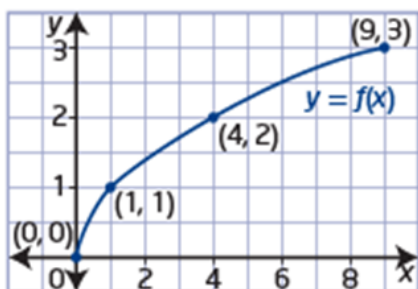
a)  $y = 3f(2x)$

The graph of  $y = f(x)$  is horizontally stretched about the  $y$ -axis by a factor of  $\frac{1}{2}$  and then vertically stretched about the  $x$ -axis by a factor of 3.



**b)  $y = f(3x + 6)$**

The graph of  $y = f(x)$  is horizontally stretched about the  $y$ -axis by a factor of  $\frac{1}{3}$  and then horizontally translated 2 units to the left.





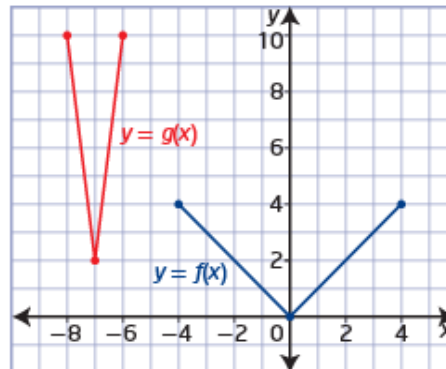
# Homework

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### Example 3

#### Write the Equation of a Transformed Function Graph

The graph of the function  $y = g(x)$  represents a transformation of the graph of  $y = f(x)$ . Determine the equation of  $g(x)$  in the form  $y = af(b(x - h)) + k$ . Explain your answer.



#### Solution

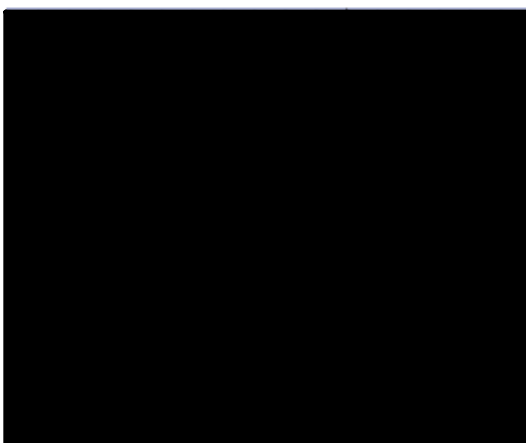
Locate key points on the graph of  $f(x)$  and their image points on the graph of  $g(x)$ .

$$(-4, 4) \rightarrow (-8, 10)$$

$$(0, 0) \rightarrow (-7, 2)$$

$$(4, 4) \rightarrow (-6, 10)$$

The equation of the transformed function is XXXXXXXXXX



How could you use the mapping  $(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$  to verify this equation?

17. The graph of the function  $y = 2x^2 + x + 1$  is stretched vertically about the  $x$ -axis by a factor of 2, stretched horizontally about the  $y$ -axis by a factor of  $\frac{1}{3}$ , and translated 2 units to the right and 4 units down. Write the equation of the transformed function.

is stretched vertically about the  $x$ -axis by a factor of 2. stretched horizontally about the  $y$ -axis by a factor of  $\frac{1}{3}$ , and translated 2 units to the right and 4 units down. Write the equation of the transformed function.