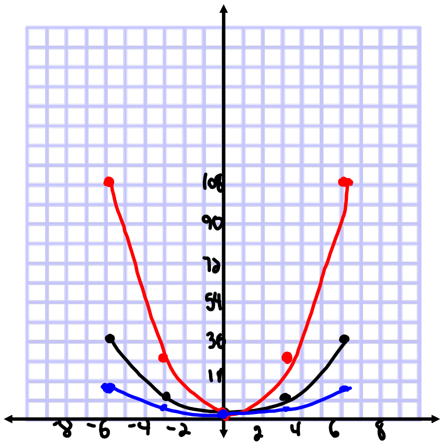
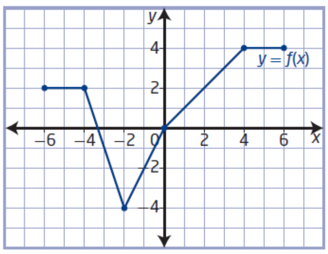
Questions from Homework

2. a) Copy and complete the table of values for the given functions.

x	$f(x) = x^2$	$g(x) = 3f(x)$ $(x, 3y)$	$h(x) = \frac{1}{3}f(x)$ $(x, \frac{1}{3}y)$
−6	36	(−6, 108)	(−6, 12)
−3	9	(−3, 27)	(−3, 3)
0	0	(0, 0)	(0, 0)
3	9	(3, 27)	(3, 3)
6	36	(6, 108)	(6, 12)



6. The graph of the function $y = f(x)$ is vertically stretched about the x-axis by a factor of 2. $a = 2$

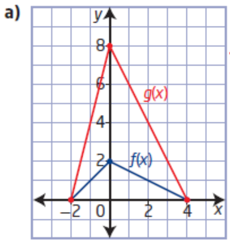


Original	Transformed
$y = f(x)$	$g(x) = 2f(x)$
(x, y) →	$(x, 2y)$
a) D: $[-6, 6]$	D: $[-6, 6]$
R: $[-4, 4]$	R: $[-8, 8]$

b) Vertical stretch only changes the range.

7. Describe the transformation that must be applied to the graph of $f(x)$ to obtain the graph of $g(x)$. Then, determine the equation of $g(x)$ in the form $y = af(bx)$.

a)



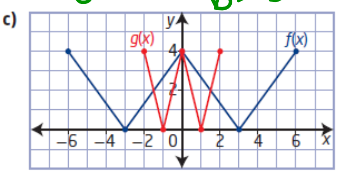
Vertical stretch factor of 4
$a = 4$

$(x, y) \rightarrow (x, 4y)$

$y = f(x)$	$g(x) = 4f(x)$
(−2, 0)	(−2, 0)
(0, 2)	(0, 8)
(4, 0)	(4, 0)

horizontal compression by a factor of $\frac{1}{3}$
$b = 3$

c)



$(x, y) \rightarrow (\frac{1}{3}x, y)$

$y = f(x)$	$g(x) = f(3x)$
(−6, 4)	(−2, 4)
(−3, 0)	(−1, 0)
(0, 4)	(0, 4)
(3, 0)	(1, 0)
(6, 4)	(2, 4)

Warm-Up...

Given that (-2 , 5) is a point on the graph of $y = f(x)$, determine the coordinates of this point once the following transformations are applied...

(1) $y = 3f(x)$

$a = 3$ $\quad (x, y) \rightarrow (x, 3y)$

$(-2, 5) \rightarrow (-2, 15)$

(2) $y = f\left(\left(-\frac{1}{3}x\right)\right)$

$b = -\frac{1}{3}$ $\quad (x, y) \rightarrow (-3x, y)$

$(-2, 5) \rightarrow (6, 5)$

(3) $y = 4f\left[\left(\frac{1}{2}\right)(x + 5)\right] - 3$

$a = 4$ $\quad (x, y) \rightarrow (2x - 5, 4y - 3)$

$b = \frac{1}{2}$ $\quad (-2, 5) \rightarrow (-9, 17)$

$h = -5$

$K = -3$

(4) $y - 5 = -2f(-2x + 6)$

Factor $\quad y = -2f(-2x + 6) + 5$

$y = -2f[-2(x - 3)] + 5$

$a = -2$ $\quad (x, y) \rightarrow \left(-\frac{1}{2}x + 3, -2y + 5\right)$

$b = -2$ $\quad (-2, 5) \rightarrow (4, -5)$

$h = 3$

$K = 5$

Transformations:

2. The function $y = f(x)$ is transformed to the function $g(x) = -3f(4x - 16) - 10$. Copy and complete the following statements by filling in the blanks.

factor

The function $f(x)$ is transformed to the function $g(x)$ by a horizontal stretch about the $\boxed{a}$ by a factor of $\boxed{b}$. It is vertically stretched about the $\boxed{c}$ by a factor of $\boxed{d}$. It is reflected in the $\boxed{e}$, and then translated $\boxed{f}$ units to the right and $\boxed{g}$ units down.

$g(x) = -3f[4(x-4)] - 10$

$a = -3$ $b = 4$ $h = 4$ $k = -10$

a) y-axis

b) $\frac{1}{4}$

c) x-axis

d) 3

e) x-axis

f) 4

g) 10

Summary of Transformations...

Transformations of the graphs of functions		
$f(x)+k$	shift $f(x)$ up k units	vertical trans. $(x,y)\to(x, y+k)$
$f(x)-k$	shift $f(x)$ down k units	
$f(x+h)$	shift $f(x)$ left h units	horizontal trans. $(x,y)\to(x+h, y)$
$f(x-h)$	shift $f(x)$ right h units	
$f(-x)$	reflect $f(x)$ about the y-axis	horizontal ref. $(x,y)\to(-x,y)$
$-f(x)$	reflect $f(x)$ about the x-axis	vertical ref. $(x,y)\to(x,-y)$
$af(x)$	When $0 < a < 1$ – vertical shrinking of $f(x)$ When $a > 1$ – vertical stretching of $f(x)$ **Multiply the y values by a**	vertical stretch $(x,y)\to(x, ay)$
$f(bx)$	When $0 < b < 1$ – horizontal stretching of $f(x)$ When $b > 1$ – horizontal shrinking of $f(x)$ **Divide the x values by b**	horizontal stretch $(x,y)\to(\frac{1}{b}x, y)$

Transformations:

$$y = f(x) \longrightarrow y = af\left(b(x-h)\right) + k$$

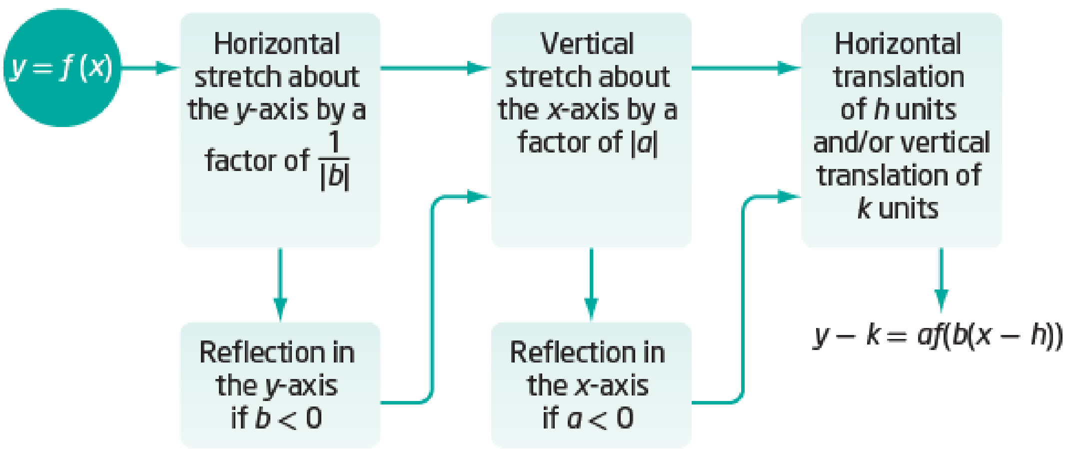
Mapping Rule: $(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$

Important note for sketching...

Transformations should be applied in following order:

1. Reflections
2. Stretches
3. Translations

Remember....RST



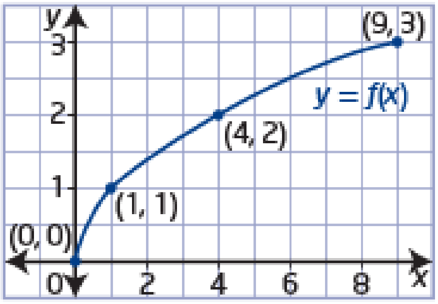
Example 1

Graph a Transformed Function

Describe the combination of transformations that must be applied to the function $y = f(x)$ to obtain the transformed function. Sketch the graph, showing each step of the transformation.

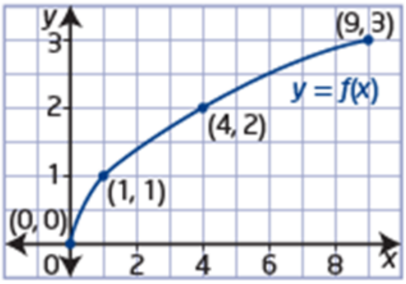
a) $y = 3f(2x)$

b) $y = f(3x + 6)$



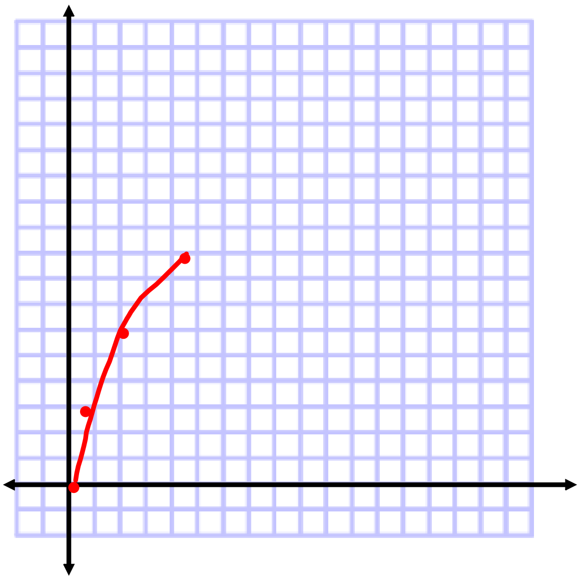
a) $y = 3f(2x)$ $a = 3$ $b = 2$ $h = 0$ $k = 0$

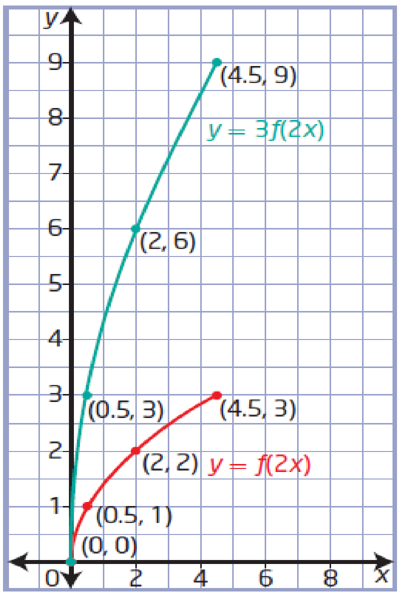
The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{2}$ and then vertically stretched about the x-axis by a factor of 3.



$(x, y) \rightarrow (\frac{1}{2}x, 3y)$

(0, 0)	$(0, 0)$
(1, 1)	$(\frac{1}{2}, 3)$
(4, 2)	$(2, 6)$
(9, 3)	$(\frac{9}{2}, 9)$





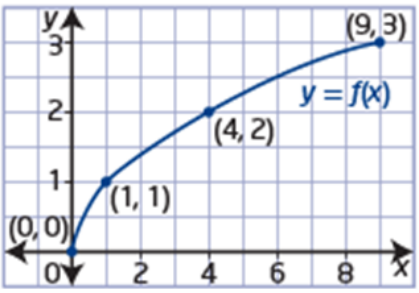
b) $y = f(3x + 6)$

$y = f[3(x+2)]$

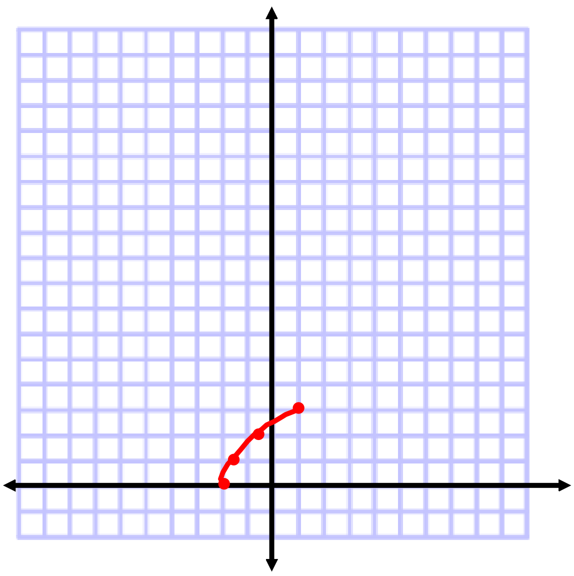
$a = 1$ $b = 3$ $h = -2$ $k = 0$

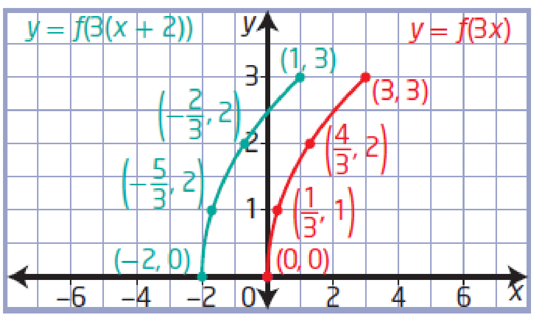
The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{3}$ and then horizontally translated 2 units to the left.

$(x, y) \longrightarrow \left(\frac{1}{3}x - 2, y\right)$



(0, 0)	$(-2, 0)$
(1, 1)	$(-\frac{5}{3}, 1)$
(4, 2)	$(-\frac{2}{3}, 2)$
(9, 3)	$(1, 3)$





Homework

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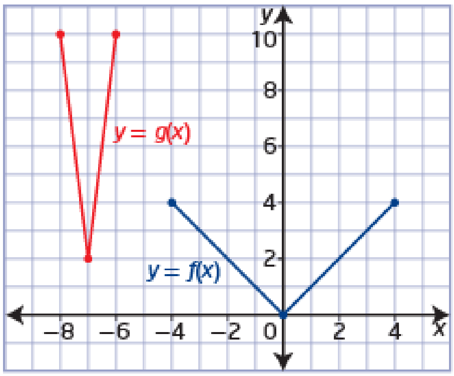
Example 3

Write the Equation of a Transformed Function Graph

The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$. Explain your answer.



Solution

Locate key points on the graph of $f(x)$ and their image points on the graph of $g(x)$.

$(-4, 4) \rightarrow (-8, 10)$
$(0, 0) \rightarrow (-7, 2)$
$(4, 4) \rightarrow (-6, 10)$

The equation of the transformed function is [redacted]

[redacted]

How could you use the mapping $(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$ to verify this equation?

17. The graph of the function $y = 2x^2 + x + 1$ is stretched vertically about the x-axis by a factor of 2, stretched horizontally about the y-axis by a factor of $\frac{1}{3}$, and translated 2 units to the right and 4 units down. Write the equation of the transformed function.

is stretched vertically about the x-axis by a factor of 2, stretched horizontally about the y-axis by a factor of $\frac{1}{3}$, and translated 2 units to the right and 4 units down. Write the equation of the transformed function.