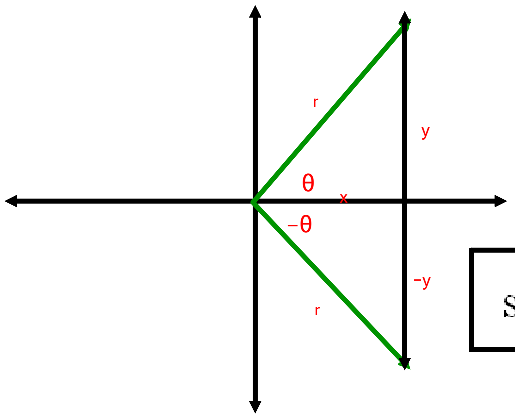
Negative Angles



$\sin\theta = \frac{y}{r}$ $\cos(\theta) = \frac{x}{r}$

$\sin(-\theta) = -\frac{y}{r}$ $\cos(-\theta) = \frac{x}{r}$

$\boxed{\sin(-\theta) = -\sin\theta}$ $\boxed{\cos(-\theta) = \cos\theta}$

Ex: 7.2

① a) $y = \cos(-4x)$

$y' = -\sin(-4x) \cdot -4$

$y' = 4\sin(-4x)$

$y' = -4\sin(4x)$

Ex: 7.3

① d) $y = -\frac{1}{4}\csc(-8x)$

$y' = -\frac{1}{4}(-\csc(-8x)\cot(-8x)) \cdot -8$

$y' = -2\csc(-8x)\cot(-8x)$

$y' = -2(-\csc 8x)(-\cot 8x)$

$y' = -2\csc(8x)\cot(8x)$

f) $y = \tan^2 x = (\tan x)^2$

$y' = 2(\tan x)(\sec^2 x)(1)$

$y' = 2\tan x \sec^2 x$

e) $y = \tan x^2$ $\quad u = x^2$, $du = 2x$

$y' = \sec^2 x^2 (2x)$

$y' = 2x \sec^2 x^2$

i) $y = \cot^3(1-2x)^2 = [\cot(1-2x)^2]^3$

$y' = 3[\cot(1-2x)^2]^2[-\csc^2(1-2x)^2][2(1-2x)(-2)]$

$y' = 12(1-2x)\cot^2(1-2x)^2\csc^2(1-2x)^2$

h) $y = (x^2)(\csc x)$

$y' = x^2(-\csc x\cot x)(1) + 2x\csc x$

$y' = 2x\csc x - x^2\csc x\cot x$

$y' = x\csc x[2 - x\cot x]$

k) $y = \dfrac{1}{\sqrt{(\sec 2x-1)^3}} = (\sec 2x-1)^{-3/2}$

$y' = -\dfrac{3}{\cancel{2}}(\sec 2x-1)^{-5/2}(\sec 2x\tan 2x)(\cancel{2})$

$y' = \dfrac{-3\sec 2x\tan 2x}{(\sec 2x-1)^{5/2}} = y' = \dfrac{-3\sec 2x\tan 2x}{\sqrt{(\sec 2x-1)^5}}$

l) $y = \dfrac{x^2\boxed{\tan x}}{\boxed{\sec x}} = x^2\left(\dfrac{\sin x}{\cancel{\cos x}}\right)\left(\dfrac{\cancel{\cos x}}{1}\right)$

$y = (x^2)(\sin x)$

$y' = x^2(\cos x)(1) + 2x\sin x$

$y' = x^2\cos x + 2x\sin x$

$y' = x(x\cos x + 2\sin x)$

$u = x^2$

Ex: $y = \sin(x^2)$

$y' = \cos(x^2) \cdot 2x$

$y' = 2x\cos(x^2)$

$u = \tan x$

① n) $y = \sin(\tan x)$

$y' = \cos(\tan x) \cdot \sec^2 x$

$y' = \sec^2 x\left[\cos(\tan x)\right]$

k) $y = \dfrac{1}{\sqrt{(\sec 2x - 1)^3}} = \dfrac{1}{(\sec 2x - 1)^{3/2}} = (\sec 2x - 1)^{-3/2}$

$y' = \frac{-3}{2}(\sec 2x - 1)^{-5/2} \cdot \sec 2x \tan 2x \cdot (2)$

$y' = \dfrac{-3\sec(2x)\tan(2x)}{(\sec 2x - 1)^{5/2}}$

$y' = -\dfrac{3\sec(2x)\tan(2x)}{\sqrt{(\sec 2x - 1)^5}}$

① a) $y = \tan^2(\cos x) = [\tan(\cos x)]^2$

$y' = 2[\tan(\cos x)] \cdot \sec^2(\cos x) \cdot (-\sin x)(1)$

$y' = -2\sin x[\tan(\cos x)][\sec^2(\cos x)]$

b) $y = \frac{x^2 \boxed{\tan x}}{\boxed{\sec x}} = x^2\left(\frac{\sin x}{\cos x}\right) \cdot \left(\frac{\cos x}{1}\right) = x^2 \sin x$

$y' = x^2(\cos x)(1) + 2x(\sin x)$

$y' = x^2\cos x + 2x\sin x$

$y' = x[x\cos x + 2\sin x]$

Final Review

① b) $f(x) = \boxed{\dfrac{2x-2}{x+3}}$ $\quad f(x+h) = \boxed{\dfrac{2x+2h-2}{x+h+3}}$

$$f'(x) = \lim_{h\to 0} \frac{(x+3)\cancel{(x+h+3)}\dfrac{2x+2h-2}{\cancel{x+h+3}} - \dfrac{2x-2}{\cancel{x+3}}\cancel{(x+3)}(x+h+3)}{h(x+3)(x+h+3)}$$

$$= \lim_{h\to 0} \frac{\cancel{2x^2}+\cancel{2xh}-\cancel{2x}+\cancel{6x}+6h-\cancel{6}-(\cancel{2x^2}+\cancel{2xh}+\cancel{6x}-\cancel{2x}-2h-\cancel{6})}{h(x+3)(x+h+3)}$$

$$= \lim_{h\to 0} \frac{8\cancel{h}}{\cancel{h}(x+3)(x+h+3)} = \boxed{\frac{8}{(x+3)^2}}$$

h) $y = (x^2)(\csc x)$

$y' = (x^2)(-\csc x \cot x)(1) + 2x\csc x$

$y' = -x^2\csc x\cot x + 2x\csc x$

$y' = x\csc x(-x\cot x + 2)$

$y' = x\csc x(2 - x\cot x)$

$u = (1-2x)^2$

$du = 2(1-2x)(-2)$

i) $y = \cot^3(1-2x)^2 = [\cot(1-2x)^2]^3$

$y' = 3[\cot(1-2x)^2]^2[-\csc^2(1-2x)^2][2(1-2x)(-2)]$

$y' = 3\cot^2(1-2x)^2[\csc^2(1-2x)^2][-4(1-2x)]$

$y' = 12(1-2x)\cot^2(1-2x)^2\csc^2(1-2x)^2$

k) $y = \dfrac{1}{\sqrt{(\sec 2x - 1)^3}} = \dfrac{1}{(\sec 2x - 1)^{3/2}} = (\sec 2x - 1)^{-3/2}$

$y' = -\dfrac{3}{\cancel{2}}(\sec 2x - 1)^{-5/2}(\sec 2x \tan 2x)(\cancel{2})$

$y' = \dfrac{-3\sec 2x \tan 2x}{(\sec 2x - 1)^{5/2}} = \dfrac{-3\sec 2x \tan 2x}{\sqrt{(\sec 2x - 1)^5}}$

l) $y = \dfrac{x^2 \boxed{\tan x}}{\boxed{\sec x}} = x^2 \cdot \dfrac{\sin x}{\cancel{\cos x}} \cdot \dfrac{\cancel{\cos x}}{1} = x^2 \sin x$

$y' = x^2(\cos x)(1) + 2x\sin x$

$y' = x^2 \cos x + 2x \sin x$

$y' = x[x\cos x + 2\sin x]$

$u = \cos x$

$du = -\sin x(1)$

o) $y = \tan^2(\cos x) = [\tan(\cos x)]^2$

$y' = 2[\tan(\cos x)][\sec^2(\cos x)][-\sin x(1)]$

$y' = -2\sin x \tan(\cos x)\sec^2(\cos x)$

⑥ Find the points on the curve $y=\frac{x}{x-1}$ where the tangent line is parallel to the line $x+4y=1$

① $x+4y=1$

$4y=-x+1$

$y=-\frac{1}{4}x+\frac{1}{4}$

$m=-\frac{1}{4}$

② $y=\frac{x}{x-1}$

$x-1 \quad - x$

$y'=\frac{(x-1)(1)-(x)(1)}{(x-1)^2}$

$y'=\frac{-1}{(x-1)^2}$

③ $\frac{-1}{(x-1)^2}=-\frac{1}{4}$

$-(x-1)^2=-4$

$(x-1)^2=4$

$x^2-2x+1=4$

$x^2-2x-3=0$

$(x-3)(x+1)=0$

$x-3=0 \quad | \quad x+1=0$

$x=3 \quad | \quad x=-1$

④ $y=\frac{x}{x-1}$

$y=\frac{3}{3-1} \quad | \quad y=\frac{-1}{-1-1}$

$y=\frac{3}{2} \quad | \quad y=\frac{1}{2}$

$\boxed{(3, 3/2)} \quad | \quad \boxed{(-1, 1/2)}$

⑥ Find the point on the curve $y = x\sqrt{x}$ where the tangent line is parallel to the line $6x - y = 4$

① $y = 6x - 4$

$m = 6$

② $y = x\sqrt{x} = x^{3/2}$

$y' = \frac{3}{2}x^{1/2} = \frac{3\sqrt{x}}{2}$

③ $\frac{3\sqrt{x}}{2} = 6$

$3\sqrt{x} = 12$

$\sqrt{x} = 4$

$\boxed{x = 16}$

④ $y = x\sqrt{x}$

$y = (16)\sqrt{(16)}$

$\boxed{y = 64}$

① a) $f(x) = \left(\frac{2x+1}{x-1}\right)^5$

$2x-2-2x-1$

$$f'(x) = 5\left(\frac{2x+1}{x-1}\right)^4\left[\frac{2(x-1)-1(2x+1)}{(x-1)^2}\right]$$

$$= 5\left[\frac{(2x+1)^4}{(x-1)^4}\right]\left[\frac{-3}{(x-1)^2}\right]$$

$$\boxed{= \frac{-15(2x+1)^4}{(x-1)^6}}$$

b) $y = (x^2-1)^3(3x-2)^2$

$$y' = \underbrace{[(x^2-1)^3]}_{f}\underbrace{[2(3x-2)(3)]}_{g'} + \underbrace{[3(x^2-1)^2(2x)]}_{f'}\underbrace{[(3x-2)^2]}_{g}$$

$$y' = 6(x^2-1)^3(3x-2) + 6x(x^2-1)^2(3x-2)^2$$

x^2-1+3x^2-2x

$$y' = 6(x^2-1)^2(3x-2)[(x^2-1)+x(3x-2)]$$

$$y' = 6(x^2-1)^2(3x-2)(4x^2-2x-1)$$

① c) $y = \frac{(2x-1)^2}{(x^4-x+1)^2}$

$$y' = \frac{(x^4-x+1)^2(2)(2x+1)(2) - (2x+1)^2(2)(x^4-x+1)(4x^3-1)}{(x^4-x+1)^4}$$

$$y' = \frac{4(2x+1)(x^4-x+1)^2 - 2(2x+1)^2(x^4-x+1)(4x^3-1)}{(x^4-x+1)^4}$$

$2x^4-2x+2-(8x^4-2x+4x^3-1)$

$$y' = \frac{2(2x+1)(x^4-x+1)[2(x^4-x+1)-(2x+1)(4x^3-1)]}{(x^4-x+1)^4}$$

$$y' = \frac{2(2x+1)\cancel{(x^4-x+1)}(-6x^4-4x^3+3)}{(x^4-x+1)^{\cancel{4}\,3}}$$

$$y' = \frac{2(2x+1)(-6x^4-4x^3+3)}{(x^4-x+1)^3}$$

(5) $y=(x^2-3)^8$ at $x=2$ Point $(2,1)$

(1) Find y

$y=((2)^2-3)^8$

$y=(4-3)^8=1$

(2) $y'=8(x^2-3)^7(2x)$

$y'=16x(x^2-3)^7$

(3) $y'(2)=16(2)((2)^2-3)^7$

$=32(1)$

$=32$

m

(4) $y-y_1=m(x-x_1)$

$y-1=32(x-2)$

$y-1=32x-64$

$\boxed{0=32x-y-63}$

4 b) $y = \dfrac{\sqrt{x}}{3+x^2}$

$$y' = \frac{\frac{1}{2}x^{-1/2}(3+x^2) - x^{1/2}(2x)}{(3+x^2)^2}$$

$$y' = \frac{\frac{3}{2}x^{-1/2} + \frac{1}{2}x^{3/2} - 2x^{3/2}}{2(3+x^2)^2}$$

$$y' = \frac{3x^{-1/2} + x^{3/2} - 4x^{3/2}}{2(3+x^2)^2}$$

$$y' = \frac{3x^{-1/2} - 3x^{3/2}}{2(3+x^2)^2}$$ ← Factor out $3x^{-1/2}$

$$y' = \frac{3x^{-1/2}(1-x^2)}{2(3+x^2)^2}$$

$$y' = \frac{3(1-x^2)}{2\sqrt{x}(3+x^2)^2}$$

8 a) $y = \sin^3 x + \cos^3 x = (\sin x)^3 + (\cos x)^3$

$$y' = 3(\sin x)^2(\cos x)(1) + 3(\cos x)^2(-\sin x)(1)$$

$$y' = 3\sin^2 x\cos x - 3\sin x\cos^2 x$$

$$y' = 3\sin x\cos x(\sin x - \cos x)$$

Do Ex. 2.9 on page 112 of Text
Questions 1, 4, 6, 9, 11, 12, 14

Section 4.1

(5) $y = \left(\dfrac{\sin x}{1+\cos x}\right)^2$

$y' = 2\left(\dfrac{\sin x}{1+\cos x}\right)^1 \left[\dfrac{\cos x(1+\cos x) - \sin x(-\sin x)}{(1+\cos x)^2}\right]$

$y' = \left(\dfrac{2\sin x}{1+\cos x}\right)\left(\dfrac{\cos x + \boxed{\cos^2 x + \sin^2 x}}{(1+\cos x)^2}\right)$ ← Pythagore

$y' = \left(\dfrac{2\sin x}{\cancel{1+\cos x}_{\,1}}\right)\left(\dfrac{\cancel{1+\cos x}^{\,1}}{(1+\cos x)^2}\right) = \dfrac{2\sin x}{(1+\cos x)^2}$

Exercise 2.9 Page 113

Point $\left(2, \frac{1}{4\sqrt{2}}\right)$

x_1 y_1

9 c) $f(x) = \frac{1}{\sqrt{x^5}} = \frac{1}{x^{5/2}} = x^{-5/2}$

① find $f'(x)$

$f'(x) = -\frac{5}{2}x^{-7/2} = \frac{-5}{2\sqrt{x^7}}$

② find $f'(2)$ (slope)

$f'(2) = \frac{-5}{2\sqrt{(2)^7}}$

$f'(2) = \frac{-5}{2\sqrt{128}}$

$f'(2) = \frac{-5}{16\sqrt{2}}$

③ Equation:

$y - y_1 = m(x - x_1)$

$y - \frac{1}{4\sqrt{2}} = \frac{-5}{16\sqrt{2}}(x-2)$

$16y\sqrt{2} - 4 = -5(x-2)$

$16y\sqrt{2} - 4 = -5x + 10$

$5x + 16y\sqrt{2} - 14 = 0$