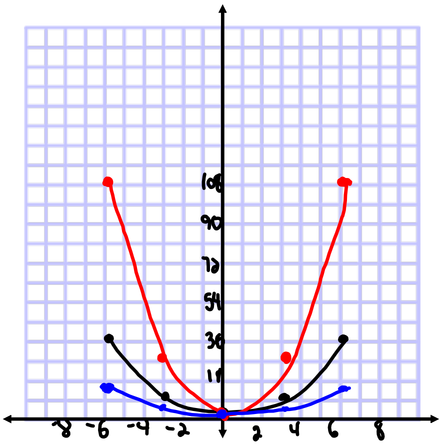
Questions from Homework

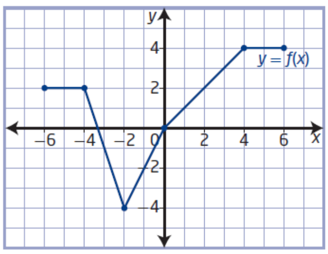
2. a) Copy and complete the table of values for the given functions.

$(x, 3y)$ $\left(x, \frac{1}{3}y\right)$

x	$f(x) = x^2$	$g(x) = 3f(x)$	$h(x) = \frac{1}{3}f(x)$
−6	36	(−6, 108)	(−6, 12)
−3	9	(−3, 27)	(−3, 3)
0	0	(0, 0)	(0, 0)
3	9	(3, 27)	(3, 3)
6	36	(6, 108)	(6, 12)



6. The graph of the function $y = f(x)$ is vertically stretched about the x-axis by a factor of 2. $a = 2$



Original $y = f(x)$ Transformed $g(x) = 2f(x)$

$(x, y) \longrightarrow (x, 2y)$

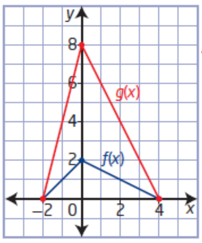
a) D: $[-6, 6]$ D: $[-6, 6]$

R: $[-4, 4]$ R: $[-8, 8]$

b) Vertical stretch only changes the range.

7. Describe the transformation that must be applied to the graph of $f(x)$ to obtain the graph of $g(x)$. Then, determine the equation of $g(x)$ in the form $y = af(bx)$.

a)



Vertical stretch factor of 4
$a = 4$

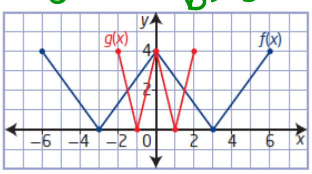
$(x, y) \longrightarrow (x, 4y)$

$y = f(x)$	$g(x) = 4f(x)$
(−2, 0)	(−2, 0)
(0, 2)	(0, 8)
(4, 0)	(4, 0)

horizontal compression by a factor of $\frac{1}{3}$
$b = 3$

$(x, y) \longrightarrow \left(\frac{1}{3}x, y\right)$

c)



$y = f(x)$	$g(x) = f(3x)$
(−6, 4)	(−2, 4)
(−3, 0)	(−1, 0)
(0, 4)	(0, 4)
(3, 0)	(1, 0)
(6, 4)	(2, 4)

Warm-Up...

Given that (-2 , 5) is a point on the graph of $y = f(x)$, determine the coordinates of this point once the following transformations are applied...

(1) $y = 3f(x)$

$a=3$ $\quad (x,y) \rightarrow (x, 3y)$

$(-2,5) \rightarrow (-2,15)$

(2) $y = f\left(-\frac{1}{3}x\right)$

$b = -\frac{1}{3}$ $\quad (x,y) \rightarrow (-3x, y)$

$(-2,5) \rightarrow (6,5)$

(3) $y = 4f\left[\frac{1}{2}(x+5)\right] - 3$

$a=4$ $\quad (x,y) \rightarrow (2x-5, 4y-3)$

$b=\frac{1}{2}$ $\quad (-2,5) \rightarrow (-9,17)$

$h=-5$

$K=-3$

(4) $y - 5 = -2f(-2x+6)$

Factor $\quad y = -2f(-2x+6) + 5$

$y = -2f[-2(x-3)] + 5$

$a=-2$ $\quad (x,y) \rightarrow \left(-\frac{1}{2}x+3, -2y+5\right)$

$b=-2$ $\quad (-2,5) \rightarrow (4,-5)$

$h=3$

$K=5$

Transformations:

2. The function $y = f(x)$ is transformed to the function $g(x) = -3f(4x - 16) - 10$. Copy and complete the following statements by filling in the blanks.

factor

$g(x) = -3f[4(x-4)] - 10$

$a = -3$
$b = 4$
$h = 4$
$k = -10$

The function $f(x)$ is transformed to the function $g(x)$ by a horizontal stretch about the **a** by a factor of **b**. It is vertically stretched about the **c** by a factor of **d**. It is reflected in the **e**, and then translated **f** units to the right and **g** units down.

a) y-axis

b) $\frac{1}{4}$

c) x-axis

d) 3

e) x-axis

f) 4

g) 10

Summary of Transformations...

Transformations of the graphs of functions		
$f(x)+k$	shift $f(x)$ up k units	vertical translation $(x,y)\to(x,y+k)$
$f(x)-k$	shift $f(x)$ down k units	
$f(x+h)$	shift $f(x)$ left h units	horizontal translation $(x,y)\to(x+h,y)$
$f(x-h)$	shift $f(x)$ right h units	
$f(-x)$	reflect $f(x)$ about the y-axis	horizontal $(x,y)\to(-x,y)$
$-f(x)$	reflect $f(x)$ about the x-axis	vertical $(x,y)\to(x,-y)$
$af(x)$	When $0<a<1$ – vertical shrinking of $f(x)$ When $a>1$ – vertical stretching of $f(x)$ **Multiply the y values by a**	vertical stretch $(x,y)\to(x,ay)$
$f(bx)$	When $0<b<1$ – horizontal stretching of $f(x)$ When $b>1$ – horizontal shrinking of $f(x)$ **Divide the x values by b**	horizontal stretch $(x,y)\to(\frac{1}{b}x,y)$

Transformations:

$$y = f(x) \longrightarrow y = af\left(b(x-h)\right)+k$$

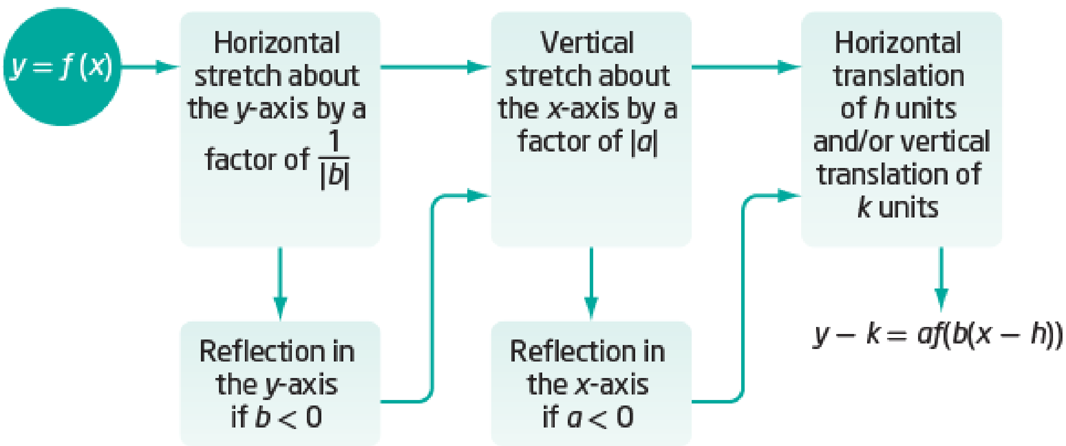
Mapping Rule: $(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$

Important note for sketching...

Transformations should be applied in following order:

1. Reflections
2. Stretches
3. Translations

Remember....RST



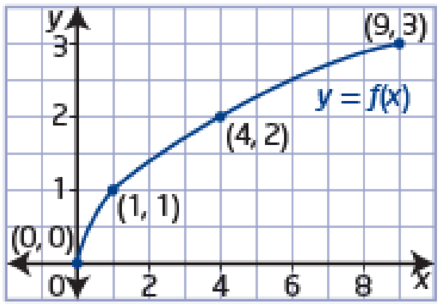
Example 1

Graph a Transformed Function

Describe the combination of transformations that must be applied to the function $y = f(x)$ to obtain the transformed function. Sketch the graph, showing each step of the transformation.

a) $y = 3f(2x)$

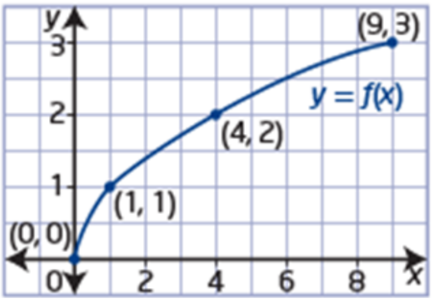
b) $y = f(3x + 6)$



a) $y = 3f(2x)$ $a=3$ $b=2$ $h=0$ $k=0$

The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{2}$ and then vertically stretched about the x-axis by a factor of 3.

$(x, y) \longrightarrow \left[\frac{1}{2}x, 3y\right]$



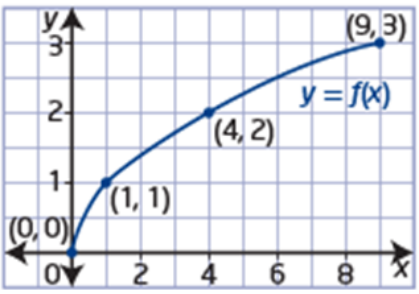
$y=f(x)$	$y=3f(2x)$
(0, 0)	(0,0)
(1, 1)	(½,3)
(4, 2)	(2,6)
(9, 3)	(9/2,9)

b) $y = f(3x + 6)$ (Factor out a 3)

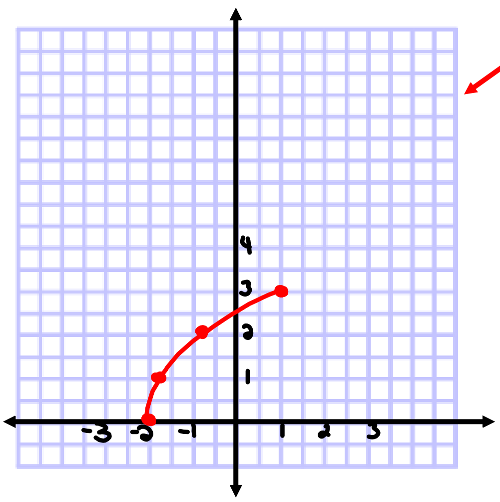
$y = 1f[3(x+2)] + 0$ $\quad a=1 \quad b=3 \quad h=-2 \quad k=0$

The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{3}$ and then horizontally translated 2 units to the left.

$(x,y) \longrightarrow [\frac{1}{3}x - 2, y]$



$y=f(x)$	$y=f[3(x-2)]$
(0, 0)	$(-2, 0)$
(1, 1)	$(-\frac{5}{3}, 1)$
(4, 2)	$(-\frac{2}{3}, 2)$
(9, 3)	$(1, 3)$



Questions From Homework

3. Copy and complete the table by describing the transformations of the given functions, compared to the function $y = f(x)$.

Function	Reflections	Vertical Stretch Factor	Horizontal Stretch Factor	Vertical Translation	Horizontal Translation
$y - 4 = f(x - 5)$	No			4	5
$y + 5 = 2f(3x)$	No	2	$\frac{1}{3}$	-5	0
$y = \frac{1}{2}f\left(\frac{1}{2}(x - 4)\right)$	No	$\frac{1}{2}$	2	0	4
$y + 2 = -3f(2(x + 2))$	in x-axis	3	$\frac{1}{2}$	-2	-2

6. The key point $(-12, 18)$ is on the graph of $y = f(x)$. What is its image point under each transformation of the graph of $f(x)$?

d) $y = -2f(-\frac{2}{3}x - 6) + 4$

$y = -2f\left[-\frac{2}{3}(x + 9)\right] + 4$

$a = -2$ Vertical stretch by a factor of 2 and a reflection in the x-axis

$b = -\frac{2}{3}$ horizontal stretch by a factor of $\frac{3}{2}$ and a reflection in the y-axis

$h = -9$ translated 9 units left

$k = 4$ translated 4 units up.

$(x, y) \rightarrow \left[-\frac{3}{2}x - 9, -2y + 4\right]$

$(-12, 18) \rightarrow \boxed{(9, -32)}$

Ex: $2y + 5 = 4f(3x + 12) - 7$

$\frac{2y}{2} = \frac{4f(3x+12)}{2} - \frac{12}{2}$

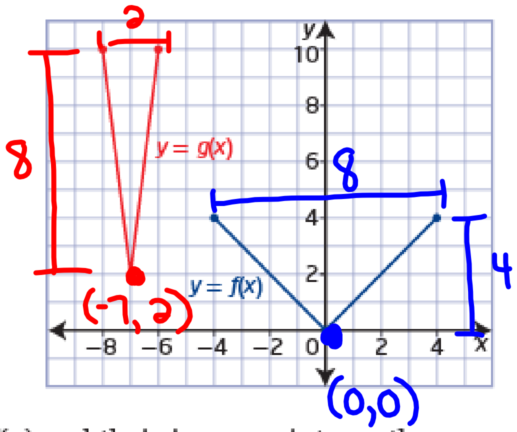
$y = 2f(3x + 12) - 6$

$y = 2f[3(x + 4)] - 6$

Example 3

Write the Equation of a Transformed Function Graph

The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$. Explain your answer.



Solution

Locate key points on the graph of $f(x)$ and their image points on the graph of $g(x)$.

$(-4,4) \rightarrow (-8,10)$ $\quad (x,y) \rightarrow \left[\frac{1}{4}x - 7, 2y+2\right]$

$(0,0) \rightarrow (-7,2)$

$(4,4) \rightarrow (-6,10)$

The equation of the transformed function is $g(x) = 2f(4(x + 7)) + 2$.

① Reflections: No reflections

② Vertical Stretch $\left(\frac{\text{New}}{\text{Old}}\right)$ $\frac{8}{4} = 2$ $\quad a = 2$

③ Horizontal Stretch $\left(\frac{\text{New}}{\text{Old}}\right)$ $\frac{2}{8} = \frac{1}{4}$ $\quad b = 4$

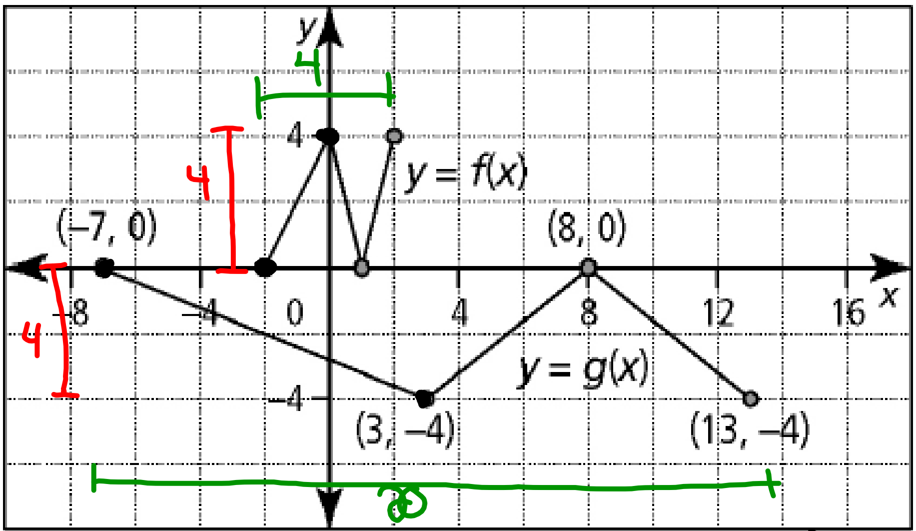
④ Horizontal Translation: $(\underline{0},0) \rightarrow (\underline{-7},2)$ Left 7 $\quad h = -7$
(Pick a point where x=0)

⑤ Vertical Translation: $(0,\underline{0}) \rightarrow (-7,\underline{2})$ Up 2 $\quad k = 2$
(Pick a point where y=0)

⑥ Equation: $y = 2f[4(x+7)] + 2$

The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$.

$y = -f\left(\frac{1}{5}(x-3)\right)$



① Reflections: Vertical (in x-axis)*

② VSF: $\frac{4}{4} = 1$ $a = -1$

③ HSF: $\frac{20}{4} = 5$ $b = \frac{1}{5}$

④ HT: $(\underline{0},4) \rightarrow (\underline{3},-4)$ Right 3 $h=3$

⑤ VT: $(-2,\underline{0}) \rightarrow (-7,\underline{0})$ $k=0$

⑥ Equation: $y = -1f[\frac{1}{5}(x-3)]$

Homework

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Plus 7, 8, 9 (a, c, e) and 10