

$$\textcircled{a} \left(x^2 - \frac{x}{2}\right)^5$$

$${}_5C_0(x^2)^5\left(\frac{-x}{2}\right)^0 + {}_5C_1(x^2)^4\left(\frac{-x}{2}\right)^1 + {}_5C_2(x^2)^3\left(\frac{-x}{2}\right)^2 + {}_5C_3(x^2)^2\left(\frac{-x}{2}\right)^3 + {}_5C_4(x^2)^1\left(\frac{-x}{2}\right)^4 + {}_5C_5(x^2)^0\left(\frac{-x}{2}\right)^5$$

$$1(x^{10})(1) + 5(x^8)\left(\frac{-x}{2}\right) + 10(x^6)\left(\frac{x^2}{4}\right) + 10(x^4)\left(\frac{-x^3}{8}\right) + 5(x^2)\left(\frac{x^4}{16}\right) + 1(1)\left(\frac{-x^5}{32}\right)$$

$$x^{10} - \frac{5x^9}{2} + \frac{5x^8}{2} - \frac{5x^7}{4} + \frac{5x^6}{16} - \frac{x^5}{32}$$

④ c) Case 1 (All Black)

$${}_{26}C_5 = \boxed{65\,780}$$

Case 2 (4 Black + 1 Red)

$${}_{26}C_4 \times {}_{26}C_1 = 14\,950 \times 26 = \boxed{388\,700}$$

Case 3 (3 black + 2 red)

$${}_{26}C_3 \times {}_{26}C_2 = 2600 \times 325 = \boxed{845\,000}$$

$$\text{Total} = 845\,000 + 388\,700 + 65\,780 = \boxed{1\,299\,480}$$

⑤ 14 Letters

- 5 are burnt
- 9 are good

3 good + 2 bad

$${}^9C_3 \times {}^5C_2$$

$$84 \times 10$$

$$840$$

Derivatives Exam Review!

$$\textcircled{4} \text{ b) } y = \frac{\sqrt{x}}{3+x^2} = \frac{x^{1/2}}{(3+x^2)}$$

$$y' = \frac{(3+x^2)\left(\frac{1}{2}x^{-1/2}\right) - x^{1/2}(2x)}{(3+x^2)^2}$$

$$y' = \frac{\frac{3}{2}x^{-1/2} + \frac{1}{2}x^{3/2} - 2x^{3/2}}{2(3+x^2)^2}$$

$$y' = \frac{3x^{-1/2} + x^{3/2} - 4x^{3/2}}{2(3+x^2)^2}$$

$$y' = \frac{x^{-1/2}(3+x^2-4x^2)}{2(3+x^2)^2}$$

$$y' = \frac{3-3x^2}{2x^{1/2}(3+x^2)^2} = \frac{3(1-x^2)}{2\sqrt{x}(3+x^2)^2}$$

$$\textcircled{5} \text{ c) } \underline{1} + \underline{5} + \underline{9} + \dots + \underline{77}$$

$\begin{array}{cc} \vee & \vee \\ 4 & 4 \end{array}$

$$a = 1$$

$$t_n = 77$$

$$d = 4$$

① Find n :

$$t_n = a + (n-1)d$$

$$77 = 1 + (n-1)4$$

$$76 = 4(n-1)$$

$$19 = n-1$$

$$\boxed{20 = n}$$

② Find S_{20}

$$S_{20} = \frac{20}{2} (1 + 77)$$

$$= 10(78)$$

$$\boxed{= 780}$$

$$\textcircled{5} \text{ b) } \sum_{n=1}^{\infty} \underline{3} \left(\frac{1}{2} \right)^{n-1} = \underline{3} + \frac{3}{2} + \frac{3}{4} + \dots$$

$\begin{array}{cc} \vee & \vee \\ 2 & 4 \\ \frac{1}{2} & \frac{1}{2} \end{array}$

$$a = 3$$

$$r = \frac{1}{2}$$

$$S_n = \frac{3}{1 - \frac{1}{2}} = \frac{3}{\frac{1}{2}} = 3 \cdot 2 = \textcircled{6}$$

$$\textcircled{4} \text{ c) } \lim_{n \rightarrow \infty} (-1)^{n+1} n^2$$

$$= 1, -4, 9, -16, 25, -36, 49, -64, 81, -100$$

= Diverging (has no limit)

$$- , \frac{1}{5} , - , - , \frac{25}{5} , - , -$$

$$t_2 = \frac{1}{5} , t_5 = 25$$

$$t_2 = ar^{2-1} \quad t_5 = ar^{5-1}$$

$$t_2 = ar^1 \quad t_5 = ar^4$$

$$\frac{1}{5} = ar \quad 25 = ar^4$$

$$ar = \frac{1}{5} \quad ar^4 = 25$$

$$\frac{ar^4 = 25}{ar = \frac{1}{5}}$$

$$r^3 = 125$$

$$r = 5$$

$$ar = \frac{1}{5}$$

$$a(5) = \frac{1}{5}$$

$$5a = \frac{1}{5}$$

$$a = \frac{1}{25}$$

Expand: $(\underline{x} - \underline{\frac{1}{2}})^{\textcircled{4}}$

$${}^4C_0(\cancel{x})^4\left(\frac{1}{2}\right)^0 + {}^4C_1(\cancel{x})^3\left(\frac{1}{2}\right)^1 + {}^4C_2(\cancel{x})^2\left(\frac{1}{2}\right)^2 + {}^4C_3(\cancel{x})^1\left(\frac{1}{2}\right)^3 + {}^4C_4(\cancel{x})^0\left(\frac{1}{2}\right)^4$$

$$(1)(x^4)(1) + (4)(x^3)\left(-\frac{1}{2}\right) + (6)(x^2)\left(\frac{1}{4}\right) + (4)(x)\left(-\frac{1}{8}\right) + (1)(1)\left(\frac{1}{16}\right)$$

$$x^4 - \frac{4x^3}{2} + \frac{6x^2}{4} - \frac{4x}{8} + \frac{1}{16}$$

$$x^4 - 2x^3 + \frac{3x^2}{2} - \frac{1}{2}x + \frac{1}{16}$$