

Questions From Homework

⑥ Given:

$$\frac{dV}{dt} = 8 \text{ m}^3/\text{min}$$

$$\frac{dr}{dt} = ?$$

$$d = 2 \text{ m}$$

$$* r = 1 \text{ m}$$

$$V = \frac{4}{3} \pi r^3$$

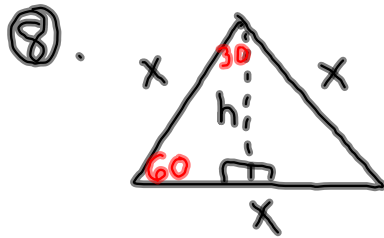
$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$8 = 4\pi(1)^2 \frac{dr}{dt}$$

$$\frac{8}{4\pi} = \frac{dr}{dt}$$

$$0.64 \text{ m/min} = \frac{dr}{dt}$$

Questions From Homework



$$\frac{x}{h} = \frac{2}{\sqrt{3}}$$

$$2h = \sqrt{3}x$$

$$h = \frac{\sqrt{3}x}{2}$$

Let base = x

$$\frac{dx}{dt} = -2 \text{ cm/s}$$

$$\frac{dA}{dt} = ?$$

$$A = 100 \text{ cm}^2$$

Find x :

$$100 = \frac{\sqrt{3}}{4} x^2$$

$$400 = \sqrt{3} x^2$$

$$230.94 = x^2$$

$$15.196 = x$$

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}x \frac{\sqrt{3}}{2}x$$

$$A = \frac{\sqrt{3}}{4}x^2$$

$$\frac{dA}{dt} = \frac{\sqrt{3}}{2}x \frac{dx}{dt}$$

$$\frac{dA}{dt} = \frac{\sqrt{3}}{2} (15.196) (-2)$$

$$\frac{dA}{dt} = -26.3 \text{ cm}^2/\text{s}$$

Questions From Homework

⑨ Given:

$$\frac{dA}{dt} = 4 \text{ cm}^2/\text{min}$$

$$h = 20 \text{ cm}$$

$$\frac{db}{dt} = 1 \text{ cm}/\text{min}$$

$$A = 80 \text{ cm}^2$$

$$\frac{dh}{dt} = ?$$

$$\begin{aligned} \text{Find } b: & * 80 = \frac{1}{2} b(20) \\ & 80 = 10b \\ & 8 \text{ cm} = b \end{aligned}$$

$$A = \frac{1}{2} b h$$

$$\frac{dA}{dt} = \frac{1}{2} b \frac{dh}{dt} + \frac{1}{2} \frac{db}{dt} h$$

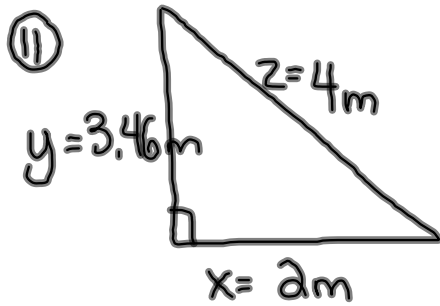
$$4 = \frac{1}{2}(8) \frac{dh}{dt} + \frac{1}{2}(1)(20)$$

$$4 = 4 \frac{dh}{dt} + 10$$

$$-6 = 4 \frac{dh}{dt}$$

\therefore The height is decreasing at a rate of 1.5 cm/min

$$\boxed{-1.5 \text{ cm}/\text{min} = \frac{dh}{dt}}$$



Given:

$$\frac{dx}{dt} = 0.3 \text{ m/s}$$

$$x = 2$$

$$z = 4$$

$$\frac{dy}{dt} = ?$$

$$y = \sqrt{z^2 - x^2}$$

$$y = \sqrt{16 - 4}$$

$$y = \sqrt{12}$$

$$y = 3.46 \text{ m}$$

$$x^2 + y^2 = z^2$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

$$2(2)(0.3) + 2(3.46) \frac{dy}{dt} = 0$$

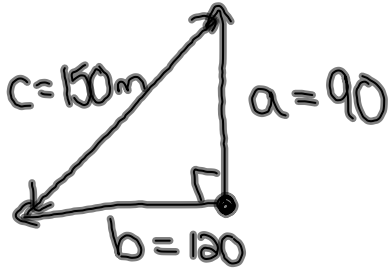
$$6.92 \frac{dy}{dt} = -1.2$$

$$\frac{dy}{dt} = -0.173 \text{ m/s}$$

Related Rates

A man starts jogging north at a rate of 1.5 m/s and a woman starts at the same point P , at the same time jogging west at a rate of 2 m/s. At what rate is the distance between the man and the woman increasing one minute later?

(Hint: draw a diagram)



Given:

$$\frac{da}{dt} = 1.5 \text{ m/s} \quad a = 90 \text{ m}$$

$$\frac{db}{dt} = 2 \text{ m/s} \quad b = 120 \text{ m}$$

$$\frac{dc}{dt} = ?$$

$$c^2 = a^2 + b^2$$

$$c^2 = 8100 + 14400$$

$$c^2 = 22500$$

$$c = 150 \text{ m}$$

$$a^2 + b^2 = c^2$$

$$2a \frac{da}{dt} + 2b \frac{db}{dt} = 2c \frac{dc}{dt}$$

$$2(90)(1.5) + 2(120)(2) = 2(150) \frac{dc}{dt}$$

$$270 + 480 = 300 \frac{dc}{dt}$$

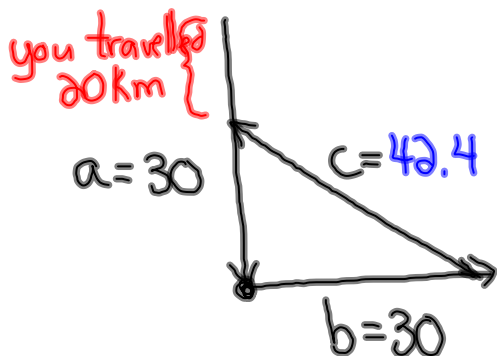
$$750 = 300 \frac{dc}{dt}$$

$$2.5 \text{ m/s} = \frac{dc}{dt}$$

The distance between the two is increasing at a rate of 2.5 m/s.

Ship A is 50 km north of Hawaii and sailing towards it (south) at a 10 km/h. At the same time Ship B leaves Hawaii and sails east at 15 km/h. How fast is the distance between the ships changing 2 hours later?

(Hint: draw a diagram)



Given:

$$\frac{da}{dt} = -10 \text{ km/h} \quad a = 30 \text{ km}$$

$$\frac{db}{dt} = 15 \text{ km/h} \quad b = 30 \text{ km}$$

$$\frac{dc}{dt} = ?$$

$$a^2 + b^2 = c^2$$

$$900 + 900 = c^2$$

$$1800 = c^2$$

$$\underline{42.4 = c}$$

$$a^2 + b^2 = c^2$$

$$2a \frac{da}{dt} + 2b \frac{db}{dt} = 2c \frac{dc}{dt}$$

$$2(30)(-10) + 2(30)(15) = 2(42.4) \frac{dc}{dt}$$

$$-600 + 900 = 84.8 \frac{dc}{dt}$$

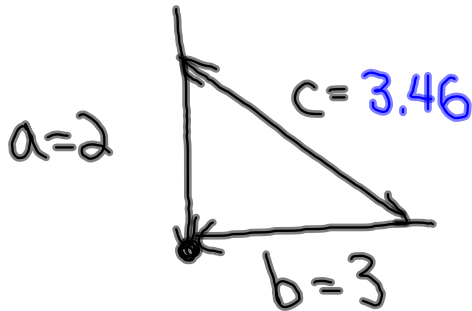
$$300 = 84.8 \frac{dc}{dt}$$

$$3.5 \text{ km/h} = \frac{dc}{dt}$$

The distance between the ships is increasing at a rate of 3.5 km/h.

Jack is headed south at 60 km/h towards JMH and Jill is headed west towards the school at 50 km/h. At what rate is the distance between them closing when Jack is 2 km and Jill is 3 km from the school?

(Hint: draw a diagram)



Given

$$\frac{da}{dt} = -60 \text{ km/h} \quad a = 2 \text{ km}$$

$$\frac{db}{dt} = -50 \text{ km/h} \quad b = 3 \text{ km}$$

$$\frac{dc}{dt} = ?$$

$$* c = \underline{\underline{3.46}}$$

$$a^2 + b^2 = c^2$$

$$2a \frac{da}{dt} + 2b \frac{db}{dt} = 2c \frac{dc}{dt}$$

$$2(2)(-60) + 2(3)(-50) = 2(3.46) \frac{dc}{dt}$$

$$-240 - 300 = 6.92 \frac{dc}{dt}$$

$$-540 = 6.92 \frac{dc}{dt}$$

$$-78 \text{ km/h} = \frac{dc}{dt}$$

∴ The distance between the ships is decreasing at a rate of 78 km/h.

Homework