Questions From Homework

Thomas 1

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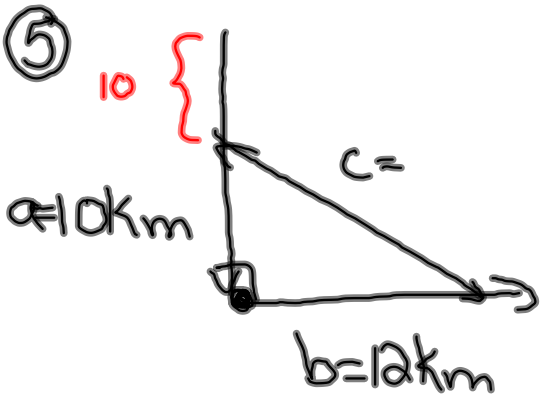

Given:

$\frac{da}{dt} = -5\text{km/h}$ $a = 10\text{km}$

$\frac{db}{dt} = 6\text{km/h}$ $b = 12\text{km}$

$\frac{dc}{dt} = ?$ $c = 15.6\text{km}$

$$a^2 + b^2 = c^2$$

$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

$$2(10)(-5) + 2(12)(6) = 2(15.6)\frac{dc}{dt}$$

$$-100 + 144 = 31.2\frac{dc}{dt}$$

$$44 = 31.2\frac{dc}{dt}$$

$$1.41\text{ km/h} = \frac{dc}{dt}$$

Jack is headed south at 60 km/h towards JMH and Jill is headed west towards the school at 50 km/h. At what rate is the distance between them closing when Jack is 2 km and Jill is 3 km from the school?

(Hint: draw a diagram)

A water tank is built in the shape of a circular cone with height 5 m and diameter 6 m at the top. Water is being pumped into the tank at a rate of 1.6 m^3/min. Find the rate at which the water level is rising when the water is 2 m deep?

Let V be the volume of the water and let r and h be the radius of the surface and the height at time t, where t is measured in minutes. We are given the rate of increase of V, that is:

$$\frac{dV}{dt} = 1.6m^3/\text{min}$$

We are asked to find $\frac{dh}{dt}$ when $h = 2m$.

The quantities V and h are related by the equation:

$$V = \frac{1}{3}\pi r^2 h$$

But we have to express V as a function of h alone. To eliminate r we look for a relationship between r and h. We use similar triangles in the figure to write.

$\frac{r}{h} = \frac{3}{5}$ Thus $r = \frac{3}{5}h$ and we have:

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi\left(\frac{3h}{5}\right)^2 h$$

$$V = \frac{1}{3}\pi\left(\frac{9h^2}{25}\right)h$$

$$V = \frac{9\pi h^3}{75}$$

$$V = \frac{3}{25}\pi h^3$$

$$\frac{dV}{dt} = \frac{9\pi}{25}h^2\frac{dh}{dt}$$

$$1.6 = \frac{9\pi}{25}(2)^2\frac{dh}{dt}$$

$$1.6 = \frac{36}{25}\pi\left(\frac{dh}{dt}\right) \quad \text{or} \rightarrow \quad 1.6 = 4.52\frac{dh}{dt}$$

$$40 = 36\pi\frac{dh}{dt}$$

$$0.354\,m/\text{min} = \frac{dh}{dt}$$

$$\frac{10}{9\pi} = \frac{dh}{dt}$$

$$0.354\,m/\text{min} = \frac{dh}{dt}$$

Homework