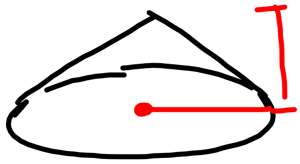
③ Given:

$\frac{dV}{dt} = 2\,m^3/min$

$h = 4m$

$\frac{dh}{dt} = ?$



$r = 2h$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi (2h)^2 h$

$V = \frac{4}{3}\pi h^3$

$\frac{dV}{dt} = 4\pi h^2 \frac{dh}{dt}$

$2 = 4\pi(4)^2 \frac{dh}{dt}$

$2 = 64\pi \frac{dh}{dt}$

$\frac{2}{64\pi} = \frac{dh}{dt}$

$\frac{1}{32\pi}\, m/min = \frac{dh}{dt}$

$A = l^2$ (Square)

$V = l^3$ (Cube)

$A = \pi r^2$ (Circle)

$A = 4\pi r^2$ (Sphere)

$V = \frac{4}{3}\pi r^3$ (Sphere)

$A = \frac{1}{2}bh$ (Triangle)

$V = \frac{1}{3}\pi r^2 h$ (Cone)

$x^2 + y^2 = z^2$ ← z is constant (Ladder)

$x^2 + y^2 = z^2$ (Intersection)

$\frac{h^{\text{of post}}}{x+y} = \frac{h^{\text{of man}}}{y}$ (Lamppost)

$$x = y^3 + y$$

$$1 = 3y^2\frac{dy}{dx} + \frac{dy}{dx}$$

$$1 = \frac{dy}{dx}(3y^2+1)$$

$$\frac{dy}{dx} = \boxed{\frac{1}{3y^2+1}}$$

$$y' = \frac{1}{3y^2+1} = (3y^2+1)^{-1}$$

$$y'' = -(3y^2+1)^{-2}\left(6y\frac{dy}{dx}\right)$$

$$y'' = \frac{-6y\boxed{\frac{dy}{dx}}}{(3y^2+1)^2}$$

$$y'' = \frac{-6y\left[\frac{1}{3y^2+1}\right]}{(3y^2+1)^2}$$

$$y'' = \frac{-6y}{3y^2+1} \cdot \frac{1}{(3y^2+1)^2}$$

$$y'' = \frac{-6y}{(3y^2+1)^3}$$

6

$\frac{dV}{dt} = 6m^3/min$

$V = \frac{4}{3}\pi r^3$

$\frac{dr}{dt} = ?$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

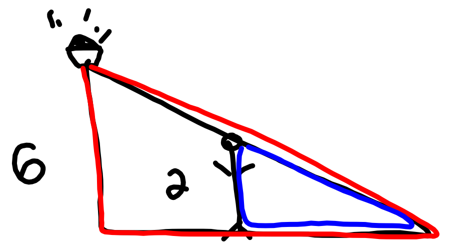
$r = 2m$

$6 = 4\pi(2)^2 \frac{dr}{dt}$

$6 = 16\pi \frac{dr}{dt}$

$\frac{3}{8\pi}\ m/min = \frac{dr}{dt}$

1



Let x = distance from post

Let y = shadow length

Given:

$\frac{dy}{dt} = ?$

$\frac{dx}{dt} = -1.5 m/s$

Similar △'s

$\frac{6}{x+y} = \frac{2}{y}$

$2x + 2y = 6y$

$2x = 4y$

$2\frac{dx}{dt} = 4\frac{dy}{dt}$

$2(-1.5) = 4\frac{dy}{dt}$

$-3 = 4\frac{dy}{dt}$

$\boxed{-0.75 m/s = \frac{dy}{dt}}$

8) $x^2 y + y^2 = 22$ find $\frac{dx}{dt}$ when $y = 2$ and $\frac{dy}{dt} = 4$

$x^2 y' + 2x x' y + 2y y' = 0$

$(3)^2(4) + 2(2)x'(3) + 2(2)(4) = 0$

$36 + 12x' + 16 = 0$

$12x' = -52$

$\boxed{x' = \pm 4.\overline{3}}$

To find x:

$x^2 y + y^2 = 22$

$x^2(2) + (2)^2 = 22$

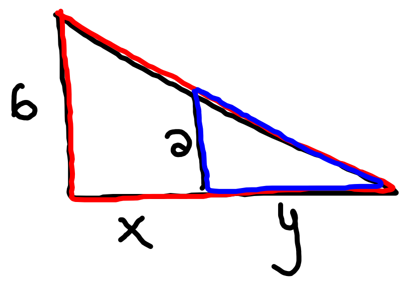
$2x^2 + 4 = 22$

$2x^2 = 18$

$x^2 = 9$

$x = \pm 3$

7



Given:

$\frac{dx}{dt} = -1.5 m/s$

$x = 5m$

$$\frac{6}{x+y} = \frac{2}{y}$$

$$2x + 2y = 6y$$

$$2x = 4y$$

$$2\frac{dx}{dt} = 4\frac{dy}{dt}$$

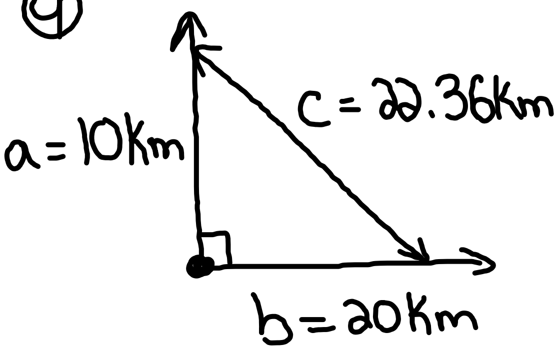
$$2(-1.5) = 4\frac{dy}{dt}$$

$$-3 = 4\frac{dy}{dt}$$

$$-0.75 m/s = \frac{dy}{dt}$$

$\therefore$ His shadow is decreasing at a rate of 0.75m/s

9)



Given:

$\frac{da}{dt} = 5\text{km/h}$ $\quad a = 10\text{Km}$

$\frac{db}{dt} = 10\text{Km/h}$ $\quad b = 20\text{Km}$

$\frac{dc}{dt} = ?$

$c = \sqrt{a^2+b^2}$

$c = \sqrt{100+400}$

$c = 22.36$

$$a^2+b^2=c^2$$

$$2a\frac{da}{dt}+2b\frac{db}{dt}=2c\frac{dc}{dt}$$

$$2(10)(5)+2(20)(10)=2(22.36)\frac{dc}{dt}$$

$$100+400=44.72\frac{dc}{dt}$$

$$11.18\text{Km/h}=\frac{dc}{dt}$$

∴ The distance between the two is increasing at a rate of 11.18 Km/h.

6 Given:

$\frac{dV}{dt} = 6m^3/\text{min}$

$\frac{dr}{dt} = ?$

$r = 2m$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$6 = 4\pi(2)^2 \frac{dr}{dt}$

$6 = 16\pi \frac{dr}{dt}$

$0.119 m/\text{min} = \frac{dr}{dt}$

Square: $A = l^2 \rightarrow \frac{dA}{dt} = 2l\frac{dl}{dt}$

Cube: $V = l^3 \rightarrow \frac{dV}{dt} = 3l^2\frac{dl}{dt}$

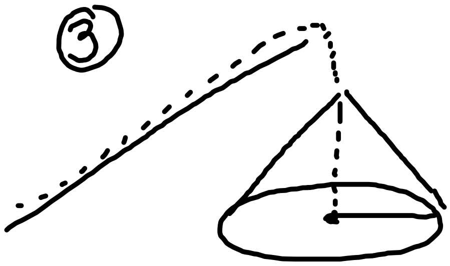
Circle: $A = \pi r^2 \rightarrow \frac{dA}{dt} = 2\pi r\frac{dr}{dt}$

Cone: $V = \frac{1}{3}\pi r^2 h$

Sphere: $V = \frac{4}{3}\pi r^3 \longrightarrow \frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$

$A = 4\pi r^2 \longrightarrow \frac{dA}{dt} = 8\pi r\frac{dr}{dt}$

3



$r = \boxed{2h}$

Given:

$\frac{dV}{dt} = 2\,m^3/min$

$\frac{dh}{dt} = ?$

$h = 4m$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi (2h)^2 h$

$V = \frac{4}{3}\pi h^3$

$\frac{dV}{dt} = 4\pi h^2 \frac{dh}{dt}$

$2 = 4\pi (4)^2 \frac{dh}{dt}$

$2 = 64\pi \frac{dh}{dt}$

$\frac{1}{32\pi}\, m/min = \frac{dh}{dt}$

Related Rates Review #2

(8) $x = y^3 + y$ * Use Implicit Diff.

$1 = 3y^2y' + y'$

$1 = y'(3y^2+1)$

$\frac{1}{3y^2+1} = y'$

$\boxed{y' = \frac{1}{3y^2+1}}$

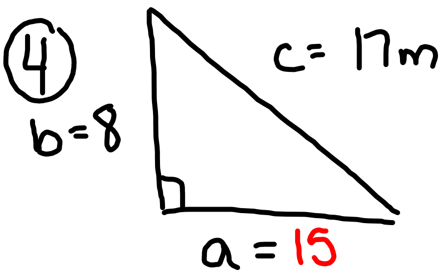
$y'' = \frac{\cancel{(3y^2+1)(0)} - 1(6yy')}{(3y^2+1)^2}$

$y'' = \frac{-6y\boxed{y'}}{(3y^2+1)^2}$

$y'' = \frac{-6y\left(\frac{1}{3y^2+1}\right)}{(3y^2+1)^2}$

$y'' = \frac{\frac{-6y}{3y^2+1}}{(3y^2+1)^2} = \frac{-6y}{3y^2+1} \times \frac{1}{(3y^2+1)^2} \boxed{= \frac{-6y}{(3y^2+1)^3}}$

4



Given

$\frac{db}{dt} = -4\,m/s$ $\quad \frac{da}{dt} = ?$

$b=8 \quad a=15$

$$a^2 + b^2 = c^2$$

$$a^2 + b^2 = 17^2$$

$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 0$$

$$2(15)\frac{da}{dt} + 2(8)(-4) = 0$$

$$30\frac{da}{dt} = 64$$

$$\frac{da}{dt} = 2.13\ m/s$$

⑤ $\frac{dV}{dt} = \pi \; m^3/min$

$\frac{dh}{dt} = ?$

$h = 3$

$* \; \frac{r}{h} = \frac{3}{6}$

$6r = 3h$

$r = \frac{3}{6}h$

$\boxed{r = \frac{1}{2}h}$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \left(\frac{1}{2}h\right)^2 h$

$V = \frac{1}{3}\pi \left(\frac{1}{4}h^2\right) h$

$V = \frac{1}{12}\pi h^3$

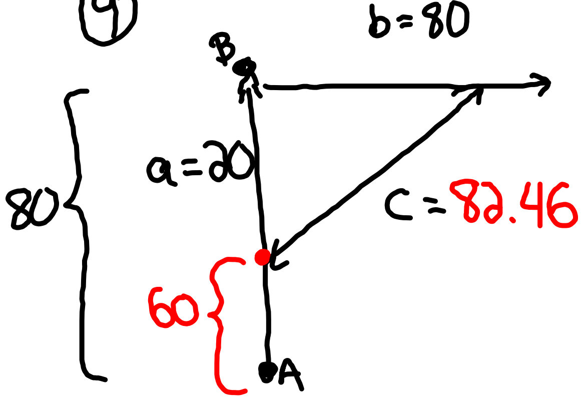
$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$

$\pi = \frac{1}{4}\pi (3)^2 \frac{dh}{dt}$

$\pi = \frac{9\pi}{4} \frac{dh}{dt}$

$\boxed{0.\overline{4} \; m/min = \frac{dh}{dt}}$

9



Given:

$\frac{da}{dt} = -30 km/h$ $a = 20$

$\frac{db}{dt} = 40 km/h$ $b = 80 km$

$\frac{dc}{dt} = ?$ $c = 82.46 km$

$$a^2 + b^2 = c^2$$

$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

$$2(20)(-30) + 2(80)(40) = 2(82.46)\frac{dc}{dt}$$

$$-1200 + 6400 = 164.92\frac{dc}{dt}$$

$$5600 = 164.92\frac{dc}{dt}$$

$$31.5 km/h = \frac{dc}{dt}$$