Questions from homework

③ a) $f(x) = 2x^3 - 3x^2 \qquad -2 \le x \le 2$

$f'(x) = 6x^2 - 6x$

$f'(x) = 6x(x-1)$

CV: $x = 0, 1$

$f(0) = 0 \qquad (0,0)$

$f(1) = -1 \qquad (1,-1)$

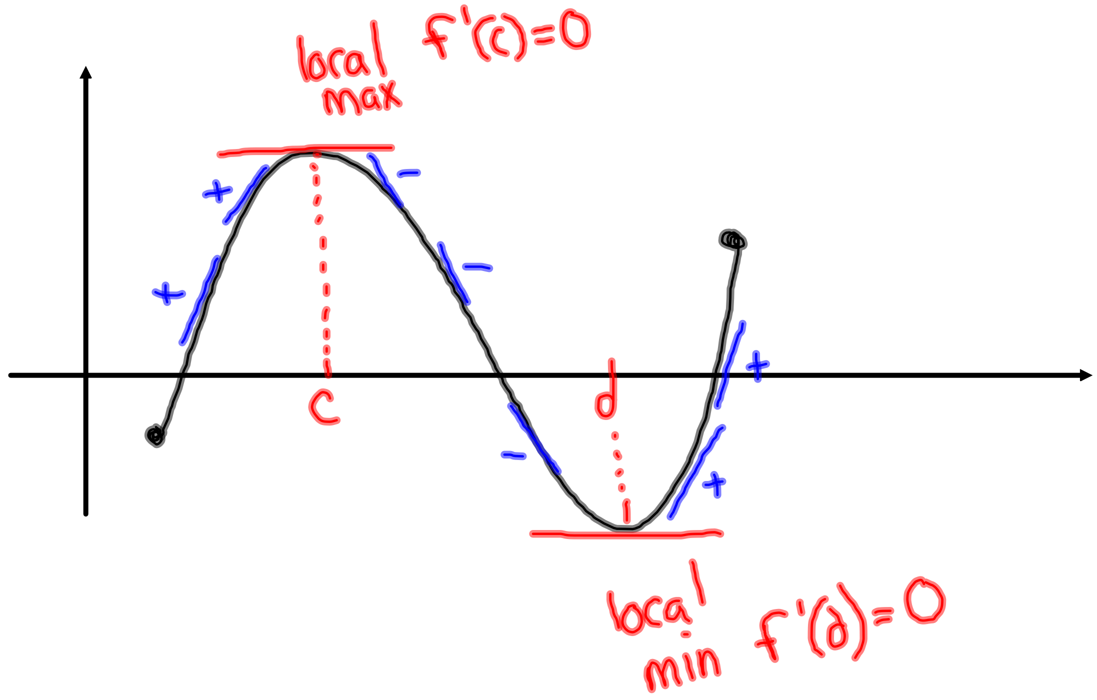
$f(-2) = -16 - 12 = -28 \qquad (-2,-28)$ abs min

$f(2) = 16 - 12 = 4 \qquad (2,4)$ abs max

The First Derivative Test

If f has a local maximum or minimum at c, then c must be a critical value of f (Fermat's Theorem), but not all critical numbers give rise to a maximum or minimum. For instance, recall that 0 is a critical number of the function $y = x^3$ but this function has no maximum of minimum at a critical number.

One way of solving this is suggested by the figure below.
If f is increasing to the left of a critical number c and decreasing to the right of c, then f has a local max at c.
If f is decreasing to the left of a critical number c and increasing to the right of c, then f has a local min at c.



The First Derivative Test

Let c be a critical number of a continuous function f.

1. If $f'(x)$ changes from positive to negative at c, then f has a local max at c.

2. If $f'(x)$ changes from negative to positive at c, then f has a local min at c.

3 If $f'(x)$ does not change signs at c, then f has no max or min at c.

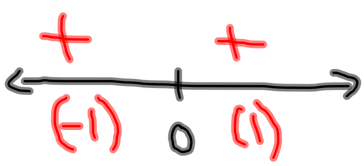
$f(x) = x^3$

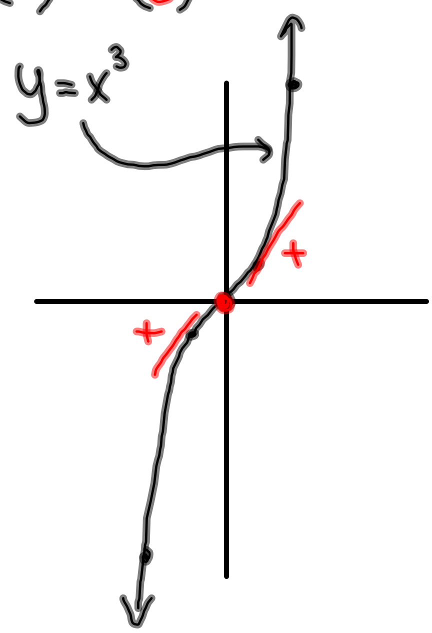
$f(0) = (0)^3 = 0$

$(0,0)$ no max or min

$f'(x) = 3x^2$

Cr: $x=0$





Example 1

Find the local maximum and minimum values of
f(x) = x³ - 3x + 1

$f'(x) = 3x^2 - 3$

$f'(x) = 3(x^2 - 1)$

$f'(x) = 3(x+1)(x-1)$

CV: $x = \pm 1$

$+ \quad\quad - \quad\quad +$

$(-2) \quad -1 \quad (0) \quad 1 \quad (2)$

$f(-1) = -1 + 3 + 1 = 3$

$(-1, 3)$ local max

$f(1) = 1 - 3 + 1 = -1$

$(1, -1)$ local min

Example 2

Find the local maximum and minimum values of
g(x) = x⁴ - 4x³ - 8x² - 1. Use this information to sketch the graph of *g*.

$g'(x) = 4x^3 - 12x^2 - 16x$

$g'(x) = 4x(x^2 - 3x - 4)$

$g'(x) = 4x(x+1)(x-4)$

CV: $x = -1, 0, 4$

$-\quad +\quad -\quad +$

$(-2)\ -1\ (-\tfrac{1}{2})\ 0\ (1)\ 4\ (5)$

$g(-1) = 1 + 4 - 8 - 1 = -4$

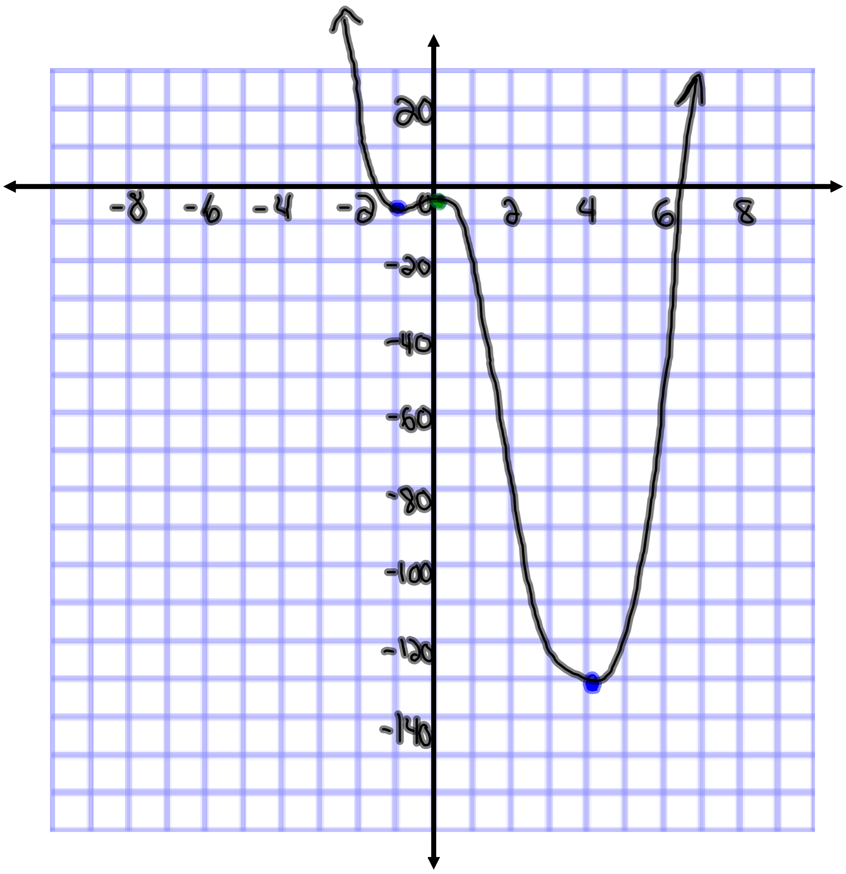
$(-1, -4)$ local min

$g(0) = -1$

$(0, -1)$ local max

$g(4) = 256 - 256 - 128 - 1 = -129$

$(4, -129)$ local min



The First Derivative Test

(for absolute extreme values)

Let c be a critical number of a continuous function f.

1. If $f'(x)$ is positive for all $x < c$ and $f'(x)$ is negative for all $x > c$, then $f(c)$ is the absolute maximum value.

2. If $f'(x)$ is negative for all $x < c$ and $f'(x)$ is positive for all $x > c$, then $f(c)$ is the absolute minimum value.

Homework