Questions From Homework

③ a) $f(x) = 2x^{2/3}(3-4x^{1/3})$ $\qquad f(0)=0 \quad (0,0)$ min

$= 6x^{2/3} - 8x$ $\qquad f\left(\frac{1}{8}\right) = \frac{1}{2} \quad \left(\frac{1}{8}, \frac{1}{2}\right)$ max

$f'(x) = 4x^{-1/3} - 8$

$= \dfrac{4}{\sqrt[3]{x}} - 8$

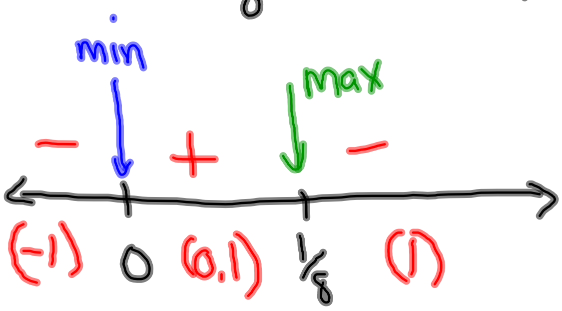
$= \dfrac{4 - 8\sqrt[3]{x}}{\sqrt[3]{x}}$

CV: $4 - 8\sqrt[3]{x} = 0$ $\qquad \sqrt[3]{x} = 0$

$4 = 8\sqrt[3]{x}$ $\qquad x = 0$

$\frac{1}{2} = \sqrt[3]{x}$

$\frac{1}{8} = x$



Questions From Homework

③ b) $f(x) = \dfrac{x^2}{x^2-1}$ $\qquad f(0) = 0 \quad \overset{\text{max}}{(0,0)}$

$$f'(x) = \frac{(x^2-1)(2x) - x^2(2x)}{(x^2-1)^2}$$

$$f'(x) = \frac{2x^3 - 2x - 2x^3}{(x^2-1)^2} = \frac{-2x}{(x^2-1)^2}$$

CV: $-2x = 0$ | $(x^2-1)^2 = 0$

$x = 0$ | $x^2 - 1 = 0$

$x^2 = 1$

$x = \pm 1$

neither, max, neither

$+ \quad + \quad - \quad -$

$(-2) \quad -1 \quad (-0.5) \quad 0 \quad (0.5) \quad 1 \quad (2)$

④ d) $g(x) = \dfrac{x^2 - x + 1}{x^2+1}$, $\underline{x \geq 0}$

$$g'(x) = \frac{(x^2+1)(2x-1) - (x^2-x+1)(2x)}{(x^2+1)^2}$$

$$g'(x) = \frac{\cancel{2x^3} - x^2 + \cancel{2x} - 1 - \cancel{2x^3} + 2x^2 - \cancel{2x}}{(x^2+1)^2}$$

$$g'(x) = \frac{x^2-1}{(x^2+1)^2}$$ ← always "+"

CV: $x = \pm 1$ $\qquad g(1) = \frac{1}{2} \quad \boxed{(1, \tfrac{1}{2})}$ ← absolute min

abs min

$- \quad +$

$-1 \quad (0) \quad 1 \quad (2)$

↑ is outside the domain $x \geq 0$

③c) $f(x) = x\sqrt{4-x}$

$= x(4-x)^{1/2}$

$f'(x) = x\left(\frac{1}{2}\right)(4-x)^{-1/2}(-1) + (4-x)^{1/2}$

$= -\frac{x}{2}(4-x)^{-1/2} + (4-x)^{1/2}$

$= (4-x)^{-1/2}\left[-\frac{x}{2} + (4-x)\right]$

$= \dfrac{2\cdot -\frac{3x}{2} + 4\cdot 2}{2(4-x)^{1/2}}$

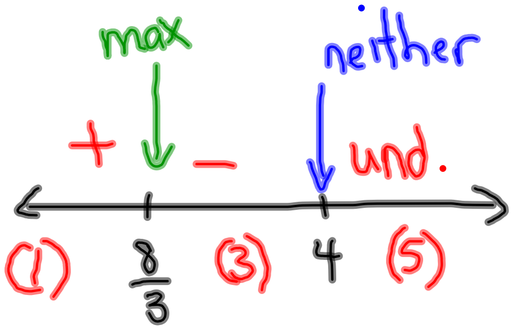
$= \dfrac{8-3x}{2\sqrt{4-x}}$

CV: $8-3x=0$ | $2\sqrt{4-x}=0$

$8 = 3x$ | $\sqrt{4-x}=0$

$\frac{8}{3} = x$ | $4-x=0$

$4 = x$



$f\left(\frac{8}{3}\right) = \frac{16}{9}\sqrt{3}$ max

Applied Max and Min Problems

One of the most important applications of derivatives occurs in the solution of "*optimization*" problems, in which a quantity must be maximized or minimized. In this section we will look at maximizing areas, volumes, and profits, and minimizing distances, times, and costs.

In solving these problems, the first step is to express the problem in a mathematical language by setting up the function that is to be maximized or minimized. Then we use the methods of this chapter to find the extreme value.

Example

Find two positive numbers whose product is 10 000 and whose sum is a minimum.

Let x be the first number and y be the second number. We wish to minimize the sum $S = x + y$ but first we express S in terms of only one variable. To eliminate one of the variables we use the given condition that the product of the number is 10 000.

$xy = 10\,000$

Solving for y, we get

$$y = \boxed{\frac{10000}{x}}$$

$$y = \frac{10000}{100}$$

$$y = 100$$

The two #'s are both 100.

$$S = x + \boxed{y}$$

$$S = x + \frac{10000}{x}$$

$$S = x + 10000x^{-1}$$

$$S' = 1 - 10000x^{-2}$$

$$S' = 1 - \frac{10000}{x^2}$$

$$S' = \frac{x^2 - 10000}{x^2}$$

CV: $x = \pm 100, 0$

max, min

$+ \quad - \quad - \quad +$

$(-101) \quad -100 \quad (-1) \quad 0 \quad (1) \quad 100 \quad (101)$

$$x = 100$$

Find two numbers whose difference is 30 and whose product is a minimum.

Let x = first #

Let y = second #

$x - y = 30$

$-y = 30 - x$

$y = \boxed{x - 30}$

$y = 15 - 30$

$y = -15$

$P = x\boxed{y}$

$P = x(x-30)$

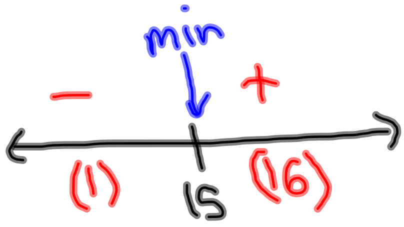
$P = x^2 - 30x$

$P' = 2x - 30$

$0 = 2x - 30$

$30 = 2x$

CV: $15 = x$



The two #'s are 15 and -15.

A rectangle has a perimeter of 200cm. What length and width should it have so that its area is a maximum?

Let l = length
Let w = width

$$2l + 2w = 200$$
$$2w = 200 - 2l$$
$$w = \boxed{100 - l}$$

$$A = l \times \boxed{w}$$
$$A = l(100 - l)$$
$$A = 100l - l^2$$
$$A' = 100 - 2l$$
$$0 = 100 - 2l$$
$$2l = 100$$
$$l = 50\text{cm}$$

max

$+$ ↓ $-$

(1) 50 (51)

$$w = 100 - 50$$
$$w = 50\text{cm}$$

The length and width are both 50cm.

Homework