Questions from Homework

② c) $f(x) = \frac{-3}{x} + 5x^{-2}$

$$F(x) = -3\ln|x| + \frac{5x^{-1}}{-1} + C$$

$$\boxed{F(x) = -3\ln|x| - \frac{5}{x} + C}$$

② d) $f(x) = x^{-7} + x^{-5} + x^{-3} + \frac{1}{x}$

$$F(x) = \frac{x^{-6}}{-6} + \frac{x^{-4}}{-4} + \frac{x^{-2}}{-2} + \ln|x| + C$$

$$\boxed{F(x) = -\frac{1}{6x^6} - \frac{1}{4x^4} - \frac{1}{2x^2} + \ln|x| + C}$$

* ③ c) $f(x) = \sqrt{-x} = (-x)^{1/2}$

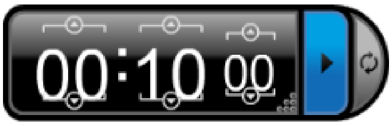
$$F(x) = \frac{(-x)^{3/2}}{\frac{3}{2}}(-1) + C$$

when you have a negative x under a radical sign!

$$F(x) = -\frac{2}{3}(-x)^{3/2} + C$$

$$F'(x) = -(-x)^{1/2}(-1)$$
$$= \sqrt{-x}$$

Warm Up



Determine the general antiderivative for the following:

- What would you differentiate that would give the function below?
- Remember add 1 to the exponent, then divide by this exponent.

Find the most general antiderivative of:

$$f'(x) = 7x^3 + 9x^2 + 8x - 1$$

$$f(x) = \frac{7x^4}{4} + \frac{9x^3}{3} + \frac{8x^2}{2} - \frac{1x}{1} + C$$

$$\boxed{f(x) = \frac{7}{4}x^4 + 3x^3 + 4x^2 - x + C}$$

Antiderivatives

This operation of determining the original function from its derivative is the inverse operation of **differentiation** and we call it **antidifferentiation**.

Definition: A function F is called an antiderivative of f on an interval I if $F'(x) = f(x)$ for all x in I.

"$F(x)$ is an antiderivative of $f(x)$"

It should be emphasized that if $F(x)$ is an antiderivative of $f(x)$, then $F(x) + C$ **(C is any constant)** is also an antiderivative of $f(x)$.

Indefinite Integration

The process of antidifferentiation is often called **integration or indefinite integration.** To indicate that the antiderivative of $f(x) = 3x^2$ is $F(x) = x^3 + C$, we write

$$\int 3x^2 dx = x^3 + C$$

We say that the **antiderivative or indefinite integral** of $3x^2$ with respect to x equals $x^3 + C$.

In general, $$\int f(x)dx = F(x) + C$$

Integral Sign — Integrand — Variable of Intergration — Constant of Integration

Examples:

Determine the general antiderivative:

$$f'(x) = 8x^{\frac{1}{2}} + 2x^{-3} + 5x - 1$$

$$f(x) = \frac{8x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{2x^{-2}}{-2} + \frac{5x^2}{2} - \frac{1x}{1} + C$$

$$f(x) = \frac{16}{3}x^{\frac{3}{2}} - \frac{1}{x^2} + \frac{5}{2}x^2 - x + C$$

$$\int (x^{\frac{5}{6}} - 3x^{\frac{9}{2}} + x^{-6} - 3x^{-\frac{1}{2}})\,dx$$

$$= \frac{x^{\frac{11}{6}}}{\frac{11}{6}} - \frac{3x^{\frac{11}{2}}}{\frac{11}{2}} + \frac{x^{-5}}{-5} - \frac{3x^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= \frac{6}{11}x^{\frac{11}{6}} - \frac{6}{11}x^{\frac{11}{2}} - \frac{1x^{-5}}{5} - 6x^{\frac{1}{2}} + C$$

Table of some of the Most General Antiderivatives

where a is a constant!

Function, f(x)	Most General Antiderivative, F(x)
a	$ax + C$
$ax^n \ (n \neq -1)$	$\frac{a}{n+1}x^{n+1} + C$
$\frac{a}{x} \ (x \neq 0)$	$a \ln\lvert x \rvert + C$
ae^{kx}	$\frac{a}{k}e^{kx} + C$
a^{kx}	$\frac{a^x}{k \ln a} + c$
$a \cos kx$	$\frac{a}{k} \sin kx + C$
$a \sin kx$	$-\frac{a}{k} \cos kx + C$
$a \sec^2 kx$	$\frac{a}{k} \tan kx + C$
$a \sec kx \tan kx$	$\frac{a}{k} \sec kx + C$
$a \csc kx \cot kx$	$-\frac{a}{k} \csc kx + C$
$a \csc^2 kx$	$-\frac{a}{k} \cot kx + C$
$\frac{a}{\sqrt{1-(kx)^2}}$	$\frac{a}{k} \sin^{-1} kx + C$
$\frac{a}{1+(kx)^2}$	$\frac{a}{k} \tan^{-1} kx + C$

Examples:

Determine the general antiderivative:

$\int 5e^x dx$

Note: Constants do not change these but powers do

$= 5e^x + C$

$f(x) = \frac{10}{x}$

$F(x) = 10\ln|x| + C$

All of these have a linear power of *x* (that is *x* is to the power of one).

Examples:

Determine the general antiderivative:

$$\int e^{10x}dx$$

If there is a constant in front of the linear *x* then divide by that constant (do not add one to the constant for these simple integrals).

$$= \frac{e^{10x}}{10} + C$$

$$= \frac{1}{10}e^{10x} + C$$

$$\int (e^{5x} - 4e^{6x} + \sin 12x - \sec^2 8x)dx$$

$$= \frac{e^{5x}}{5} - \frac{4e^{6x}}{6} - \frac{\cos 12x}{12} - \frac{\tan 8x}{8} + C$$

$$= \frac{1}{5}e^{5x} - \frac{2}{3}e^{6x} - \frac{1}{12}\cos 12x - \frac{1}{8}\tan 8x + C$$

$$\int x^3 + 9x^{-5} + \frac{2}{x} + 7e^{-2x}dx$$

$$= \frac{x^4}{4} + \frac{9x^{-4}}{-4} + 2\ln|x| + \frac{7e^{-2x}}{-2} + C$$

$$= \frac{1}{4}x^4 - \frac{9}{4x^4} + 2\ln|x| - \frac{7}{2}e^{-2x} + C$$

$$f(x) = \cos 5x - x^2\csc^2 x^3 + 5x\sin 2x^2$$

Practice Problems...

Page 408

#4, 5, 6, 7

Antiderivatives involving chain rule...

Remember how the Chain Rule works...

$$f(x) = [g(x)]^n$$

$$f'(x) = n[g(x)]^{n-1} g'(x)$$

Identifying a unique solution for an antiderivative

Examples:

Determine the function with the given derivative whose graph satisfies the initial condition provided.

1. $f'(x) = 2x - \cos x + 1, \quad f(0) = 3$

2. $f''(x) = 12x^2 + 6x - 4, \quad f(0) = 4 \text{ and } f(1) = 1$

$$f(x) = 2\sqrt[4]{x^5} - \frac{3}{x^2} + xe^{-8x^2} - \frac{2x}{1+x^4} + \frac{2}{5x} + 3x^3\cos 5x^2$$